GCE AS/A level

0982/01

MATHEMATICS M3
Mechanics 3

P.M. THURSDAY, 21 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A car of mass 600 kg starts from rest and moves along a straight horizontal road. At time t s, the force acting on the car has magnitude $\frac{27000}{(t+3)^2}$ N acting in the direction of motion. Resistance to motion may be ignored.

(a) Find an expression for v ms^{-1}, the velocity of the car at time t s. Hence show that the speed of the car has a limiting value as t increases and find this limiting value. [7]

(b) Calculate the distance travelled by the car in the first 6 s of motion. Give your answer correct to two decimal places. [5]

2. The points O, A and B lie, in that order, on a straight line with $OA = 0{\cdot}6$ m and $OB = 0{\cdot}8$ m. A particle P performs Simple Harmonic Motion along the line with centre O. The speed of P at A is $0{\cdot}3\sqrt{3}$ ms^{-1} and its speed at B is $0{\cdot}2\sqrt{5}$ ms^{-1}.

(a) Show that the amplitude of the motion is 1·2 m and that the period is 4π s. [7]

(b) Determine the magnitude of the acceleration of P at A. [2]

(c) Calculate the time taken for P to move directly from A to B. Give your answer correct to 3 significant figures. [4]

(d) Given that P is at O at time $t = 0$, find the distance of P from O when $t = \frac{2\pi}{3}$. [2]

(e) Given that P is at O when $t = 0$, find the speed of P when $t = \frac{2\pi}{3}$. [3]

3. Find the solution of the second order differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 6t + 5$$

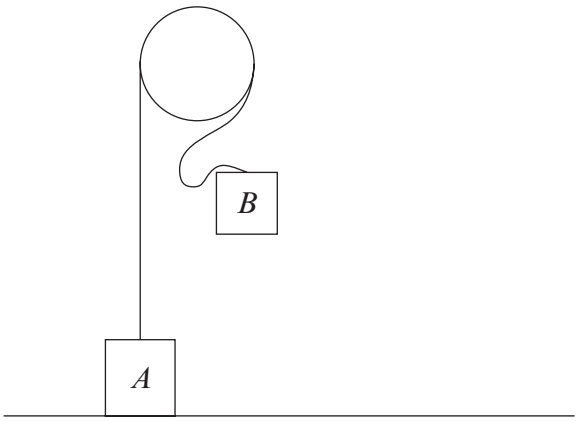
such that $x = 3$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$ when $t = 0$. [12]

4. A particle P, of mass 0·5 kg, moves along the positive x-axis away from the origin O. At time t s, the displacement of P from O is x m and its speed is v ms^{-1}. The particle is moving under the action of a force of magnitude $\frac{4}{2x+1}$ N acting in the direction of motion. As P passes point A, where $OA = 3$ m, its speed is 4 ms^{-1}.

(a) Find an expression for v^2 in terms of x, and hence calculate the speed of P when it is 10 m from O. [8]

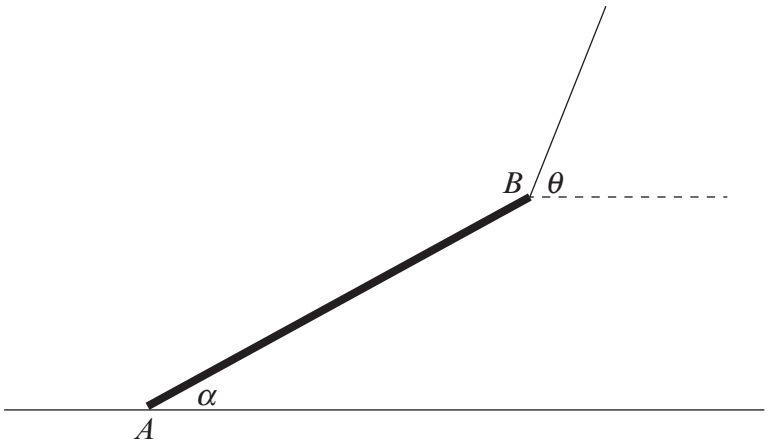
(b) Find the distance of P from O when its speed is 6 ms^{-1}. [3]

5. A particle A, of mass 5 kg, rests on a horizontal surface. It is attached to one end of a light inextensible string which passes over a smooth light fixed pulley. The other end of the string is attached to another particle B of mass 2 kg. Initially, the particles are held at rest with the string just taut. Particle B is then raised vertically and released from rest. After dropping for 0·5 s, the string becomes taut.



Find the speed with which particle A begins to rise and the impulsive tension in the string. [8]

6. The diagram shows a straight uniform beam AB of weight 2100 N and length 2 m resting in equilibrium with its end A on rough horizontal ground. A light cable, which is attached to the other end B, is holding the beam with the end B off the ground so that the beam makes an angle α with the ground, where $\tan\alpha = \frac{5}{12}$. The cable makes an angle θ with the horizontal.



The coefficient of friction between the end A and the ground is $\frac{3}{4}$. Given that the end A of the beam is about to slip,

(a) find the normal reaction of the ground on the beam at A, [6]

(b) calculate the tension in the cable and the value of the angle θ. [8]

