

## Hafaliadau Differol

Haf 2006 (2)  $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 5t - 14$

— (1)

Ceisio  $x = Ae^{mt}$  fel bod  $m$  yn bodloni  $am^2 + bm + c = 0$   
 $a=1$   $b=2$   $c=10$

Hafaliad ategol / Auxiliary equation

$$m^2 + 2m + 10 = 0$$

Hafaliad cwadrateg  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-2 \pm \sqrt{2^2 - 4(1)(10)}}{2 \times 1}$$

$$m = \frac{-2 \pm \sqrt{-36}}{2}$$

$$m = \frac{-2 \pm i\sqrt{36}}{2}$$

$$m = \frac{-2 \pm 6i}{2}$$

Unai  $m = -1 + 3i$  neu  $m = -1 - 3i$

Dau ddatrwydd cymhlyg felly'r ffwythiant cyflenwol  
(complementary function) yw  $x = e^{-t}(A \cos 3t + B \sin 3t)$

Ar gyfer yr integryn neilltuoel (particular integral)  
wrth gymharu efo  $5t - 14$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$

$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i (1):  $0 + 2a + 10(at + b) = 5t - 14$   
 $(2a + 10b) + 10at = 5t - 14$

Yn cymharu cyfernodau  $t$ :  $10a = 5$  felly  $a = \frac{1}{2}$

Yn cymharu'r cysonion  $2a + 10b = -14$   
 $2\left(\frac{1}{2}\right) + 10b = -14$   
 $10b = -15$   
 $b = -\frac{3}{2}$

Felly'r integryn neilltuoel yw  $x = \frac{1}{2}t - \frac{3}{2}$

Datrysiaid Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuoel  
 $x = e^{-t}(A \cos 3t + B \sin 3t) + \frac{1}{2}t - \frac{3}{2}$

Amnewid  $x = 4\frac{1}{2}$ ,  $t=0$  i mewn:

$$4\frac{1}{2} = e^{-0}(A \cos 0 + B \sin 0) + \frac{1}{2}(0) - \frac{3}{2}$$

$$4\frac{1}{2} = 1(A(1) + B(0)) - \frac{3}{2}$$

$$4\frac{1}{2} = A - \frac{3}{2}$$

$A = 6$

Differu  $\frac{dx}{dt} = (-1)e^{-t}(A \cos 3t + B \sin 3t)$   
 $+ e^{-t}(-3A \sin 3t + 3B \cos 3t) + \frac{1}{2}$

Amnewid  $\frac{dx}{dt} = 3\frac{1}{2}$ ,  $t=0$  i mewn:

$$3\frac{1}{2} = (-1)e^{-0}(A \cos 0 + B \sin 0)$$

$$+ e^{-0}(-3A \sin(0) + 3B \cos(0)) + \frac{1}{2}$$

$$3\frac{1}{2} = (-1)(A) + (1)(3B) + \frac{1}{2}$$

$$3\frac{1}{2} = -A + 3B + \frac{1}{2}$$

$$3 = -A + 3B$$

$$3 = -6 + 3B$$

$$9 = 3B$$

$B = 3$

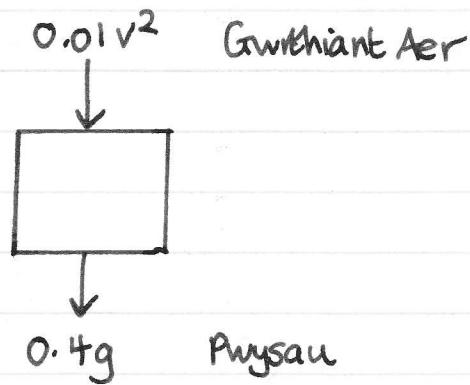
felly'r datrysiaid cyffredinol yw

$$x = e^{-t}(6 \cos 3t + 3 \sin 3t) + \frac{1}{2}t - \frac{3}{2}$$

## Hafaliadau Differol

Haf 2006

(3)



(a) Dewis i fyny yn bositif.

$$F = ma \text{ ar y bêl: } -0.01v^2 - 0.4g = 0.4a$$

$$\text{Ond } a = \frac{dv}{dt}$$

$$a = \frac{dx}{dt} \times \frac{dv}{dx} \quad \text{Rheol y gadwynn (chain rule)}$$

$$a = v \times \frac{dv}{dx}$$

$$\text{Felly } -0.01v^2 - 0.4g = 0.4v \left( \frac{dv}{dx} \right)$$

$$-v^2 - 40g = 40v \left( \frac{dv}{dx} \right)$$

$$40v \left( \frac{dv}{dx} \right) = -v^2 - 392$$

$$40v \frac{dv}{dx} = -(392 + v^2)$$

QED.

(b) Yn gwahanu newidynnau (separating variables)

$$40v dv = -(392 + v^2) dx$$

$$\frac{40v}{-(392 + v^2)} dv = dx$$

$$\int \frac{40v}{-(392 + v^2)} dv = \int dx$$

$$-40 \int \frac{v}{392 + v^2} dv = \int dx$$

$$-40 \left( \frac{1}{2} \right) \ln |392 + v^2| = x + K$$

$$-20 \ln |392 + v^2| = x + K$$

Os yw  $t=0s$ , yna  $v=17ms^{-1}$  ag  $x=0m$   
 Felly  $-20 \ln |392 + 17^2| = 0 + K$   
 $K = -20 \ln |392 + 289|$   
 $K = -20 \ln |681|.$

Felly  $-20 \ln |392 + v^2| = x - 20 \ln |681|$   
 $x = 20 \ln |681| - 20 \ln |392 + v^2|$   
 $x = 20 \ln \left| \frac{681}{392 + v^2} \right|$

Ar yr uchder mwyaf mae  $v=0ms^{-1}$

Felly  $x = 20 \ln \left| \frac{681}{392 + 0^2} \right|$

$x = 11.05m$  yn gywir i 2 le degol.

(c) Wrth ddychwelyd i 0 bydd buanedd y bêl yn llai na  $17ms^{-1}$  gan fod egni wedi cael ei ddefnyddio yn erbyn gwrthiant aer (bydd gan y bêl lai o egni cinetig wrth ddychwelyd i'r dddear).

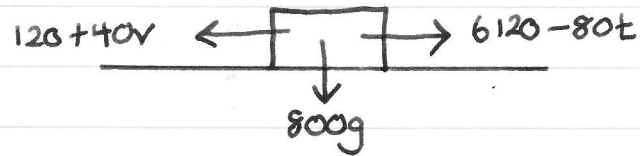


## Hafaliadau Differol

Cyfeiriad teithio  
→

Haf 2007

5



(a)

Ail Ddeddf Newton:  $F = ma$

$$6120 - 80t - (120 + 40v) = 800a$$

$$6000 - 40v - 80t = 800a$$

$$150 - v - 2t = 20a$$

Rhannu efo 40

$$150 - \frac{dx}{dt} - 2t = 20 \frac{d^2x}{dt^2}$$

$$20 \frac{d^2x}{dt^2} = 150 - \frac{dx}{dt} - 2t$$

Cyfnewid ochrau

$$20 \frac{d^2x}{dt^2} + \frac{dx}{dt} = 150 - 2t$$

QED

(b) Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$

$$a = 20 \quad b = 1 \quad c = 0$$

Hafaliad ategol / Auxiliary equation

$$20m^2 + m = 0$$

$$m(20m + 1) = 0$$

Unai  $m = 0$  neu  $20m + 1 = 0$

$$m = \underline{\underline{-\frac{1}{20}}}$$

Dau ddatrysiaid real felly'r ffwythiant cyflenwol  
(complementary function) yw  $x = Ae^{0t} + Be^{-\frac{1}{20}t}$   
 $x = A + Be^{-\frac{1}{20}t}$

Ar gyfer yr integryn neilltuol (particular integral)  
wrth gymharu efo  $150 - 2t$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$

$$\frac{d^2x}{dt^2} = 0$$

$$\text{Yn amnewid i } 20 \frac{d^2x}{dt^2} + \frac{dx}{dt} = 150 - 2t$$

$$20(0) + a = 150 - 2t$$

$$a = 150 - 2t$$

Dim gwybodaeth i ddarganfod b.

$$\text{Felly ceisiwn } x = at^2 + bt$$

$$\frac{dx}{dt} = 2at + b$$

$$\frac{d^2x}{dt^2} = 2a$$

$$\text{Yn amnewid i } 20 \frac{d^2x}{dt^2} + \frac{dx}{dt} = 150 - 2t$$

$$20(2a) + 2at + b = 150 - 2t$$

$$40a + b + (2a)t = 150 - 2t$$

$$\text{Yn cymharu cyfernodau } t: 2a = -2$$

$$a = -1$$

$$\text{Yn cymharu'r cysonion: } 40a + b = 150$$

$$40(-1) + b = 150$$

$$b = 190$$

$$\text{Felly'r integryn neilltuel yw } x = -t^2 + 190t$$

Datrysiaid Cyffredinol = Ffuythiant Cyflenwol + Integryn Neilltuel

$$x = A + Be^{-\frac{1}{20}t} - t^2 + 190t$$

Amnewid  $x=0$ ,  $t=0$  i mewn:

$$0 = A + Be^{-\frac{1}{20}(0)} - (0)^2 + 190(0)$$

$$0 = A + B \quad \text{--- (1)}$$

$$\text{Differu } \frac{dx}{dt} = \left(-\frac{1}{20}\right)Be^{-\frac{1}{20}t} - 2t + 190$$

Amnewid  $t=0$ ,  $v = \frac{dx}{dt} = 0$  i mewn:

$$0 = \left(-\frac{1}{20}\right)Be^{-\frac{1}{20}(0)} - 2(0) + 190$$

$$0 = -\frac{1}{20} B + 190$$

$$\frac{1}{20} B = 190$$

$$\underline{B = 3800}$$

Yn amnewid i mewn i ①:  $0 = A + 3800$

$$\underline{A = -3800}$$

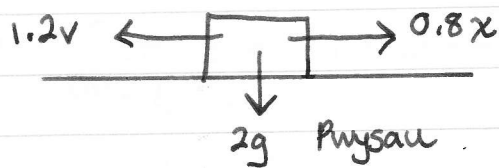
Felly'r datrysiad cyffredinol yw

$$x = -3800 + 3800e^{-\frac{1}{20}t} - t^2 + 190t$$

## Hafaliadau Differol

→ Cyfeiriad ber'chio

Haf 2008 (2)



(i) Ail Ddeddf Newton:  $F = ma$

$$0.8x - 1.2v = 2a$$

$$0.8x - 1.2 \frac{dx}{dt} = 2 \frac{d^2x}{dt^2}$$

$$0.4x - 0.6 \frac{dx}{dt} = \frac{d^2x}{dt^2}$$

Rhannu efo 2

$$2x - 3 \frac{dx}{dt} = \frac{d^2x}{dt^2}$$

Lluosi efo 5

$$5 \frac{d^2x}{dt^2} + 3 \frac{dx}{dt} - 2x = 0.$$

QED

(ii) Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$   
 $a = 5$   $b = 3$   $c = -2$

Hafaliad ategol / Auxiliary equation

$$5m^2 + 3m - 2 = 0$$

Hafaliad cwadratig  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-3 \pm \sqrt{3^2 - 4(5)(-2)}}{2(5)}$$

$$m = \frac{-3 \pm \sqrt{49}}{10}$$

Unai  $m = 0.4$  neu  $m = -1$

Dau ddatrysiaid real felly'r ffwythiant cyflenwol  
(complementary function) yw  $x = Ae^{0.4t} + Be^{-t}$

Does dim angen integryn neilltuoel yma gan fod  
ochr dde yr hafaliad gwreiddiol yn sero.

Os yw  $t=0$  yna mae  $x=0$ ,  $v=7$

Yn amnewid i  $x = Ae^{0.4t} + Be^{-t}$

$$0 = Ae^{(0)} + Be^{(0)}$$

$$0 = A + B \quad \text{--- (1)}$$

Differu  $\frac{dx}{dt} = 0.4Ae^{0.4t} + (-1)Be^{-t}$

Amnewid  $7 = 0.4Ae^{(0)} - 1Be^{(0)}$

$$7 = 0.4A - B$$

$$B = 0.4A - 7 \quad \text{--- (2)}$$

Yn amnewid o (2) i mewn i (1):

$$0 = A + 0.4A - 7$$

$$7 = 1.4A$$

$$\underline{A = 5}$$

Felly yn amnewid yn ôl i (1):  $0 = A + B$

$$0 = 5 + B$$

$$\underline{B = -5}$$

Felly'r datrysiad cyffredinol yw

$$x = 5e^{0.4t} - 5e^{-t}$$

(iii) Mae  $e^{0.4t}$  yn cynyddu wrth i  $t$  gynyddu

(ar brofi efo TABLE MODE ar gyfrifiannell)

Mae  $e^{-t}$  yn lleihau wrth i  $t$  gynyddu.

Felly mae  $5e^{0.4t} - 5e^{-t}$

$$= 5(e^{0.4t} - e^{-t}) \text{ yn cynyddu wrth i } t$$

gynyddu gan ei fod yn lusrif (5) o rif sydd

yn cynyddu fynnu rhif sydd yn lleihau.

Felly os cynyddir  $t$  cawn rif mwy fynnu rhif llai

nag o'r blaen.

$$(b) \quad 5 \frac{d^2x}{dt^2} + 3 \frac{dx}{dt} - 2x = 20t - 70 \quad \text{--- (1)}$$

Fel ôr blaen, cawn  $x = Ae^{0.4t} + Be^{-t}$   
ar gyfer yr hafaliad ategol (auxiliary equation).

Ar gyfer yr integryn neilltuol (particular integral)  
wrth gymharu efo  $20t - 70$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$
$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i mewn i (1):

$$5(0) + 3(a) - 2x = 20t - 70$$

$$3a - 2(at + b) = 20t - 70$$

$$3a - 2b \quad -2at = 20t - 70$$

Yn gymharu cyfernodau  $t$ :  $-2a = 20$

$$\underline{a = -10}$$

Yn gymharu cysonion:  $3a - 2b = -70$

$$-30 - 2b = -70$$

$$-2b = -40$$

$$\underline{b = 20}$$

Datrysiad Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuol

$$x = Ae^{0.4t} + Be^{-t} - 10t + 20$$

(Does dim gwybodaeth i ffeindio  $A$  a  $B$ ).

## Hafaliadau Differol

Haf 2008 (4)  $a + \frac{v^2}{90} + 10 = 0$

$$\text{Nawr } a = \frac{dv}{dt}$$

$$a = \frac{dx}{dt} \times \frac{dv}{dx} \quad \text{Rheol y gadwyn}$$

$$a = v \times \frac{dv}{dx}$$

Felly  $v \frac{dv}{dx} + \frac{v^2}{90} + 10 = 0$

$$v \frac{dv}{dx} = -\left(10 + \frac{v^2}{90}\right)$$

$$90v \frac{dv}{dx} = -(900 + v^2)$$

$$\frac{90v}{-(900 + v^2)} dv = dx$$

$$\int \frac{90v}{-(900 + v^2)} dv = \int dx$$

$$-90 \int \frac{v}{900 + v^2} dv = \int dx$$

$$-90 \left(\frac{1}{2}\right) \ln|900 + v^2| = x + K$$

$$-45 \ln|900 + v^2| = x + K$$

Os yw  $t=0$ , yna  $x=0$  a  $v=15$

Felly  $-45 \ln|900 + 15^2| = 0 + K$

$$-45 \ln|900 + 225| = K$$

$$K = -45 \ln|1125|$$

Felly  $-45 \ln|900 + v^2| = x - 45 \ln|1125|$

$$x = 45 \ln|1125| - 45 \ln|900 + v^2|$$

$$x = 45 \ln \left| \frac{1125}{900 + v^2} \right|$$

(b) Ceir yr uchder mwyaf pan fo  $v=0 \text{ ms}^{-1}$ .

Felly

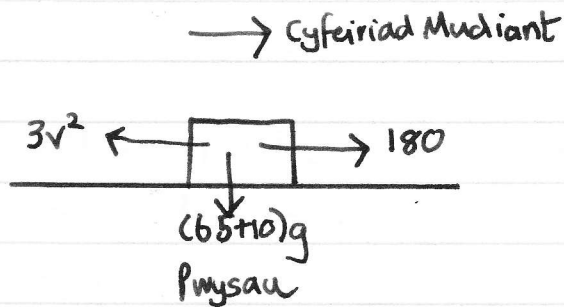
$$x = 45 \ln \left| \frac{1125}{900 + 0^2} \right|$$

$$x = 10.04 \text{ m}$$

yn gywir i ddau le degol.

## Hafaliadau Differol

Haf 2009 (3)



Ail Ddeddf Newton:  $F = ma$

$$180 - 3v^2 = 75a$$

$$60 - v^2 = 25a$$

Rhannu efo 3

Ond  $a = \frac{dv}{dt}$

$$a = \frac{dx}{dt} \times \frac{dv}{dx} \quad \text{Rheol y gadwyn}$$

$$a = v \times \frac{dv}{dx}$$

Felly  $60 - v^2 = 25v \frac{dv}{dx}$

$$25v \frac{dv}{dx} = 60 - v^2$$

QED

Yn gwahanu newidynnau (separating variables)

$$25v \, dv = (60 - v^2) \, dx$$

$$\frac{25v}{60 - v^2} \, dv = dx$$

$$\int \frac{25v}{60 - v^2} \, dv = \int dx$$

$$25 \int \frac{v}{60 - v^2} \, dv = \int dx$$

$$25 \left(-\frac{1}{2}\right) \ln|60 - v^2| = x + K$$

$$-12\frac{1}{2} \ln|60 - v^2| = x + K$$

Os yw  $t = 0$  yna  $x = 0$ ,  $v = 0$



$$\text{Felly } -12\frac{1}{2} \ln|60-v^2| = 0 + K$$

$$K = -12\frac{1}{2} \ln|60|$$

$$\text{Ac yna } -12\frac{1}{2} \ln|60-v^2| = x - 12\frac{1}{2} \ln|60|$$

$$x = 12\frac{1}{2} \ln|60| - 12\frac{1}{2} \ln|60-v^2|$$

$$x = 12\frac{1}{2} \ln \left| \frac{60}{60-v^2} \right|$$

Beth yw'r buanedd pan fydd y beiciwr wedi teithio 20m?

$$20 = 12\frac{1}{2} \ln \left| \frac{60}{60-v^2} \right|$$

$$1.6 = \ln \left| \frac{60}{60-v^2} \right|$$

$$e^{1.6} = \frac{60}{60-v^2}$$

$$(60-v^2)e^{1.6} = 60$$

$$60-v^2 = \frac{60}{e^{1.6}}$$

$$60 - 60e^{-1.6} = v^2$$

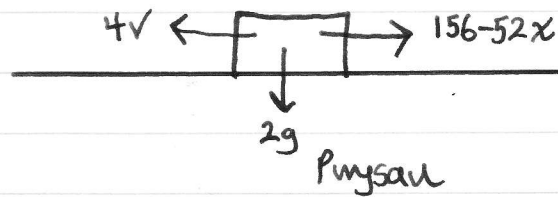
$$v = \sqrt{60 - 60e^{-1.6}}$$

$$\underline{v = 6.92 \text{ ms}^{-1}} \text{ yn gywir i ddau le degol.}$$

## Hafaliadau Differol

Haf 2009 (5)

→ Cyfeiriad Teitbio



(a) Ail Ddeddf Newton:  $F=ma$

$$156-52x-4v = 2a$$

$$156-52x-4\frac{dx}{dt} = 2\frac{d^2x}{dt^2}$$

$$78-26x-2\frac{dx}{dt} = \frac{d^2x}{dt^2}$$

Rhamnu o 2

$$0 = \frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 26x - 78$$

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 26x = 78.$$

QED

①

(b) Ceisio  $x = Ae^{mt}$  fel bod  $m$  yn bodloni  $am^2 + bm + c = 0$

$$a=1 \quad b=2 \quad c=26$$

Hafaliad ategol/auxiliary equation

$$m^2 + 2m + 26 = 0$$

Hafaliad cwadratig  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-2 \pm \sqrt{2^2 - 4(1)(26)}}{2 \times 1}$$

$$m = \frac{-2 \pm \sqrt{-100}}{2}$$

$$m = \frac{-2 \pm 10i}{2}$$

$$\text{Unai } \underline{m = -1 + 5i} \quad \text{neu } \underline{m = -1 - 5i}$$

Dau ddatrysiaid cymhlyg felly'r ffwythiant cyflenwol

(Complementary function) yw  $x = e^{-t}(A \cos 5t + B \sin 5t)$ .

Ar gyfer yr integryn neilltuoel (particular integral)  
wrth gymharu efo 78 ceisiwn  $x = a$

$$\frac{dx}{dt} = 0$$

$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i ①:  $(0) + 2(0) + 2b(a) = 78$

$$2ba = 78$$

$$\underline{a = 3}$$

Felly'r integryn neilltuoel yw  $x = 3$

Datrysiad Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuoel

$$x = e^{-t}(A \cos 5t + B \sin 5t) + 3 \quad \text{--- ②}$$

Os yw  $t = 0s$ , yna  $x = 0m$ ,  $v = 3ms^{-1}$ .

Amnewid i mewn i ②:  $0 = e^{-0}(A \cos(0) + B \sin(0)) + 3$

$$0 = 1(A(1) + B(0)) + 3$$

$$0 = A + 3$$

$$\underline{A = -3}$$

Differu  $\frac{dx}{dt} = (-1)e^{-t}(A \cos 5t + B \sin 5t) + e^{-t}(-5A \sin 5t + 5B \cos 5t)$

Amnewid  $3 = (-1)e^0(A \cos(0) + B \sin(0)) + e^0(-5A \sin(0) + 5B \cos(0))$

$$3 = (-1)(A + 0) + (1)(-5(0) + 5(B))$$

$$3 = -A + 5B$$

$$3 = 3 + 5B$$

$$0 = 5B$$

$$\underline{B = 0}$$

Felly'r datrysiad cyffredinol yw  $x = e^{-t}(-3 \cos 5t) + 3$

$$x = -3e^{-t}(\cos 5t) + 3$$

Os yw  $t = 0.5$  yna  $x = -3e^{-0.5}(\cos(2.5)) + 3$

$$\underline{x = 4.46m} \quad \text{yn gywir i 2le degol.}$$

## Hafaliadau Differol

Haf 2010 (3)  $4 \frac{d^2x}{dt^2} - 12 \frac{dx}{dt} + 9x = 18t - 87$  — (1)

Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$   
 $a = 4$   $b = -12$   $c = 9$

Hafaliad ategol / Auxiliary equation

$$4m^2 - 12m + 9 = 0$$

Hafaliad cwadratig  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{12 \pm \sqrt{(-12)^2 - 4(4)(9)}}{2 \times 4}$$

$$m = \frac{12 \pm \sqrt{0}}{8}$$

$$m = 1.5 \quad (\text{dwywaith}).$$

Un gwreiddyn wedi'i ailadrodd felly'r ffynthiant cyflenwol (complementary function) yw  $x = e^{1.5t}(A + Bt)$

Ar gyfer yr integryn neilltowl (particular integral) wrth gymharu efo  $18t - 87$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$

$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i (1):  $4(0) - 12(a) + 9(at + b) = 18t - 87$   
 $-12a + 9b + 9at = 18t - 87$

Yn cymharu cyfernodau t:  $9a = 18$

$$a = 2$$

Yn cymharu cysonion:  $-12a + 9b = -87$

$$-12(2) + 9b = -87$$

$$9b = -63$$

$$b = -7$$

Felly'r integryn neilltuoel yw  $x = 2t - 7$

Datrysiad Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuoel

$$x = e^{1.5t}(A + Bt) + 2t - 7 \quad \text{--- (2)}$$

Os yw  $t=0$ , yna  $x=5$  a  $\frac{dx}{dt} = 10$ .

Amnewid i (2):  $5 = e^0(A + B(0)) + 2(0) - 7$

$$5 = 1(A + 0) + 0 - 7$$

$$5 = A - 7$$

$$\underline{A = 12}$$

Differu  $\frac{dx}{dt} = (1.5)e^{1.5t}(A + Bt) + e^{1.5t}(B) + 2$

Amnewid  $10 = 1.5e^{(0)}(A + B(0)) + e^0(B) + 2$

$$10 = 1.5A + B + 2$$

$$10 = 1.5(12) + B + 2$$

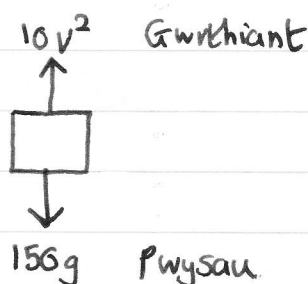
$$10 = 18 + B + 2$$

$$\underline{B = -10}$$

Felly'r datrysiad cyffredinol yw  $x = e^{1.5t}(12 - 10t) + 2t - 7$

## Hafaliadau Differol

Haf 2010 (5)



↓ = positif

(a) Ail Ddeddf Newton:  $F = ma$

$$150g - 10v^2 = 150a$$

$$15g - v^2 = 15a$$

Rhannu efo 10

$$\text{Ond } a = \frac{dv}{dt}$$

$$a = \frac{ds}{dt} \times \frac{dv}{ds}$$

Rheol y  
gadwyn

$$a = v \times \frac{dv}{ds}$$

Felly

$$15g - v^2 = 15v \frac{dv}{ds}$$

$$15v \frac{dv}{ds} = 15g - v^2 \quad \text{QED.}$$

(b) Yn gwahanu newidynnau (separating variables)

$$15v \, dv = (15g - v^2) \, ds$$

$$\frac{15v}{15g - v^2} \, dv = ds$$

$$\int \frac{15v}{15g - v^2} \, dv = \int ds$$

$$15 \int \frac{v}{15g - v^2} \, dv = \int ds$$

$$15 \left( \frac{1}{2} \right) \ln |15g - v^2| = s + K$$

Os yw  $t=0$ , yna  $s=0$ ,  $v=30$ .

$$\text{Felly } 15 \left( \frac{1}{2} \right) \ln |15g - 30^2| = 0 + K$$

$$K = -7.5 \ln |753|$$

$$\text{Felly } -7.5 \ln |15g - v^2| = s - 7.5 \ln |-753|$$

$$7.5 \ln |-753| - 7.5 \ln |15g - v^2| = s$$

$$s = 7.5 \ln \left| \frac{-753}{15g - v^2} \right|$$

(c) Os yw  $v = 14 \text{ ms}^{-1}$  yna

$$s = 7.5 \ln \left| \frac{-753}{15g - 14^2} \right|$$

$s = 20.49 \text{ m}$  yw uchder A, yn gywir i 2 le degol

$$\text{(ch)} \quad s = 7.5 \ln \left| \frac{-753}{15g - v^2} \right|$$

$$\frac{s}{7.5} = \ln \left| \frac{-753}{15g - v^2} \right|$$

$$e^{\frac{s}{7.5}} = \frac{-753}{15g - v^2}$$

$$e^{\frac{2}{15}s} (15g - v^2) = -753$$

$$15g e^{\frac{2}{15}s} - v^2 e^{\frac{2}{15}s} = -753$$

$$v^2 e^{\frac{2}{15}s} = 753 + 15g e^{\frac{2}{15}s}$$

$$v^2 = e^{-\frac{2}{15}s} (753) + 15g$$

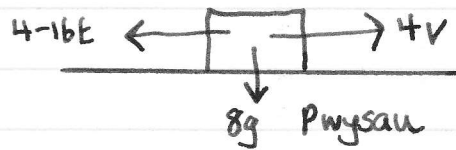
$$v^2 = 753 e^{-\frac{2}{15}s} + 147$$

## Hafaliadau Differol

→ Cyfeiriad berthio

Haf 2011

(2)



(a) Ail Ddeddf Newton  $F=ma$

$$4v - (4 - 16t) = 8a$$

$$4v - 4 + 16t = 8a$$

$$4 \frac{dx}{dt} - 4 + 16t = 8 \frac{d^2x}{dt^2}$$

$$\frac{dx}{dt} - 1 + 4t = 2 \frac{d^2x}{dt^2}$$

$$4t - 1 = 2 \frac{d^2x}{dt^2} - \frac{dx}{dt}$$

$$2 \frac{d^2x}{dt^2} - \frac{dx}{dt} = 4t - 1.$$

Rhannu efo 4

QED

①

(b) Ceisio  $x = Ae^{mt}$  fel bod m yn bodlonir hafaliad  $am^2 + bm + c = 0$

$$a=2 \quad b=-1 \quad c=0$$

Hafaliad ategol / Auxiliary equation

$$2m^2 - m = 0$$

$$m(2m - 1) = 0$$

Unai  $m=0$  neu  $2m-1=0$

$$\underline{m = \frac{1}{2}}$$

Dau ddatrysiaid real felly'r ffwythiant cyflennol

(complementary function) yw  $x = Ae^{0t} + Be^{\frac{1}{2}t}$

$$x = A + Be^{\frac{1}{2}t}$$

Ar gyfer yr integryn neilltuol (particular integral)

with gymharu efo  $4t - 1$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$

$$\frac{d^2x}{dt^2} = 0$$

Amnewid i mewn i ①:  $2(0) - a = 4t - 1$

$$-a = 4t - 1$$



Dim gwybodaeth i ddarganfod b.

Felly ceisiwn  $x = at^2 + bt$   
 $\frac{dx}{dt} = 2at + b$   
 $\frac{d^2x}{dt^2} = 2a$

Yn amnewid:  $2(2a) - (2at + b) = 4t - 1$   
 $4a - 2at - b = 4t - 1$

Yn cymharu cyfernodau t:  $-2a = 4$   
 $a = -2$

Yn cymharu cysonion:  $4a - b = -1$   
 $4(-2) - b = -1$   
 $-b = -1 + 8$   
 $b = -7$

Felly'r integryn neilltuol yw  $x = -2t^2 - 7t$

Dabrysiad cyffredinol = Ffynthiant Cyflennol + Integryn Neilltuol  
 $x = A + Be^{\frac{1}{2}t} - 2t^2 - 7t$

Os yw  $t=0$ , yna  $x=0$  a  $v=3$ .

Amnewid:  $0 = A + Be^0 - 2(0) - 7(0)$   
 $0 = A + B$

Differu:  $\frac{dx}{dt} = \frac{1}{2}Be^{\frac{1}{2}t} - 4t - 7$

Amnewid:  $3 = \frac{1}{2}Be^0 - 4(0) - 7$

$3 = \frac{1}{2}B - 7$

$B = 20$   $\longrightarrow$  Felly  $0 = A + B$

$0 = A + 20$

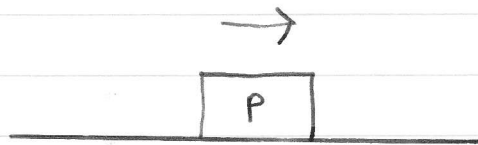
$A = -20$

Felly'r dabrysiad cyffredinol yw  $x = -20 + 20e^{\frac{1}{2}t} - 2t^2 - 7t$

## Hafaliadau Differol

Haf 2011

(4)



$$a = -\frac{9}{2x^2}$$

(tuag at 0 felly cyflymiad negatif)

Ond  $a = \frac{dv}{dt}$

$$a = \frac{dx}{dt} \times \frac{dv}{dx}$$

Rhed y gadwyn

$$a = v \times \frac{dv}{dx}$$

Felly  $v \frac{dv}{dx} = -\frac{9}{2x^2}$

$$v dv = -\frac{9}{2x^2} dx$$

$$\int v dv = \int -\frac{9}{2x^2} dx$$

$$\int v dv = -\frac{9}{2} \int \frac{1}{x^2} dx$$

$$\frac{v^2}{2} = -\frac{9}{2} (-x^{-1}) + K$$

Os yw  $x = \frac{3}{4}$ , yna  $v = 3$

Felly  $\frac{3^2}{2} = -\frac{9}{2} \left( -\left(\frac{3}{4}\right)^{-1} \right) + K$

$$\frac{9}{2} = -\frac{9}{2} \left( -\frac{4}{3} \right) + K$$

$$\frac{9}{2} = 6 + K$$

$$K = -\frac{3}{2}$$

Felly  $\frac{v^2}{2} = -\frac{9}{2} (-x^{-1}) - \frac{3}{2}$

$$v^2 = 9x^{-1} - 3$$

Os yw P yn llonydd yna  $v = 0$ .

Amnewid:

$$0^2 = 9x^{-1} - 3$$

$$3 = 9x^{-1}$$

$$1 = \frac{3}{x}$$

$$\underline{x = 3\text{m}}$$

## Hafaliadau Differol

Haf 2012 (3)  $2 \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 2x = 6t + 5$  — (1)

Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$   
 $a = 2$   $b = 5$   $c = 2$

Hafaliad ategol / Auxiliary equation

$$2m^2 + 5m + 2 = 0$$

Hafaliad Cwadratig  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-5 \pm \sqrt{5^2 - 4(2)(2)}}{2 \times 2}$$

$$m = \frac{-5 \pm \sqrt{9}}{4}$$

Unai  $m = \underline{-\frac{1}{2}}$  neu  $m = \underline{-2}$

Dau ddatbrysiad real felly'r ffwythiant cyflenwol  
(complementary function) yw  $x = Ae^{-\frac{1}{2}t} + Be^{-2t}$

Ar gyfer yr integryn neilltuoel (particular integral)

wrth gymharu efo  $6t + 5$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$

$$\frac{d^2x}{dt^2} = 0.$$

Amnewid i meun i (1):  $2(0) + 5(a) + 2(at + b) = 6t + 5$

$$5a + 2at + 2b = 6t + 5$$

Yn cymharu cyfernodau  $t$ :  $2a = 6$

$$\underline{a = 3}$$

Yn cymharu cysonion:  $5a + 2b = 5$

$$5(3) + 2b = 5$$

$$\underline{b = -5}$$

Felly'r integryn neilltuoel yw  $x = 3t - 5$

Datrysiaid cyffredinol = Ffwythiant cyffredinol + Integryn Neilltuoel  
 $x = Ae^{-\frac{1}{2}t} + Be^{-2t} + 3t - 5$

Os yw  $x = 3$ , yna  $\frac{dx}{dt} = 2$  a  $t = 0$

Amnewid:  $3 = Ae^{(0)} + Be^{(0)} + 3(0) - 5$   
 $3 = A + B - 5$   
 $8 = A + B \quad \text{--- (2)}$

Diffennu:  $\frac{dx}{dt} = (-\frac{1}{2})Ae^{-\frac{1}{2}t} - 2Be^{-2t} + 3$

Amnewid:  $2 = (-\frac{1}{2})Ae^{(0)} - 2Be^{(0)} + 3$

$$2 = -\frac{1}{2}A - 2B + 3$$

$$-1 = -\frac{1}{2}A - 2B$$

$$-1 + \frac{1}{2}A = -2B$$

$$-2 + A = -4B$$

$$A = 2 - 4B \quad \text{--- (3)}$$

Yn amnewid am  $A$  o (3) i (2):  $8 = 2 - 4B + B$

$$8 = 2 - 3B$$

$$6 = -3B$$

$$\underline{B = -2} \rightarrow \text{Felly } 8 = A + B$$

$$8 = A - 2$$

$$\underline{A = 10}$$

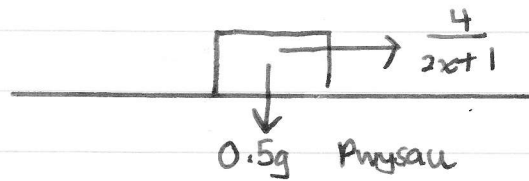
Felly'r datrysiaid cyffredinol yw

$$x = 10e^{-\frac{1}{2}t} - 2e^{-2t} + 3t - 5$$

## Hafaliadau Differol

→ Cyfeiriad Teithio

Haf 2012 (4)



(a) Ail Ddeddf Newton  $F = ma$   
 $\frac{4}{2x+1} = 0.5a$

Ond  $a = \frac{dv}{dt}$

$$a = \frac{dx}{dt} \times \frac{dv}{dx}$$

$$a = v \times \frac{dv}{dx}$$

Felly  $\frac{4}{2x+1} = 0.5 v \frac{dv}{dx}$

$$\frac{4 dx}{2x+1} = \frac{v}{2} dv$$

$$\frac{8}{2x+1} dx = v dv$$

$$\int \frac{8}{2x+1} dx = \int v dv$$

$$8 \int \frac{1}{2x+1} dx = \int v dv$$

$$8 \left(\frac{1}{2}\right) \ln|2x+1| = \frac{v^2}{2} + K$$

$$4 \ln|2x+1| = \frac{1}{2} v^2 + K$$

Os yw  $x = 3$ , yna  $v = 4$ .

Felly  $4 \ln|2(3)+1| = \frac{1}{2}(4^2) + K$

$$4 \ln|7| = 8 + K$$

$$K = 4 \ln|7| - 8$$

Felly  $4 \ln|2x+1| = \frac{1}{2}v^2 + 4 \ln|7| - 8$

$$8 \ln|2x+1| = v^2 + 8 \ln|7| - 16$$

$$v^2 = 8 \ln|2x+1| - 8 \ln|7| + 16$$

$$v^2 = 8 \ln \left| \frac{2x+1}{7} \right| + 16$$

Os yw  $x=10$  yna  $v^2 = 8 \ln \left| \frac{2(10)+1}{7} \right| + 16$

$$v^2 = 24.78889831 \dots$$

$$v = \underline{4.98 \text{ ms}^{-1}} \text{ yn gywir i 2 le degol}$$

(b) Os yw  $v=6$  yna  $6^2 = 8 \ln \left| \frac{2x+1}{7} \right| + 16$

$$36 - 16 = 8 \ln \left| \frac{2x+1}{7} \right|$$

$$\frac{20}{8} = \ln \left| \frac{2x+1}{7} \right|$$

$$e^{2.5} = \frac{2x+1}{7}$$

$$7e^{2.5} = 2x+1$$

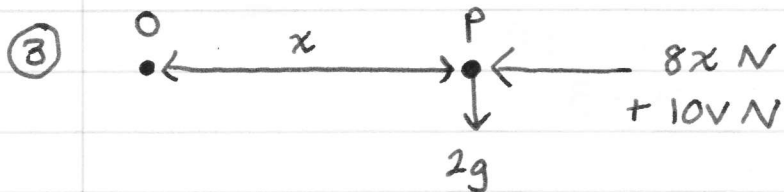
$$2x+1 = 7e^{2.5}$$

$$2x = 7e^{2.5} - 1$$

$$x = \frac{1}{2}(7e^{2.5} - 1)$$

$$x = \underline{42.14 \text{ m}} \text{ yn gywir i 2 le degol}$$

M3 Haf 2013



(a) (i)  $F = ma$  ar P, yn llorweddol,  $\rightarrow = +$  if :

$$-8x - 10v = 2a$$

$$-4x - 5v = a$$

$$a = -4x - 5v$$

$$\frac{d^2x}{dt^2} = -4x - 5 \frac{dx}{dt} \quad \checkmark$$

(ii)  $\frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 4x = 0$

ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$   
 $a = 1, b = 5, c = 4$

Hafalioda tegol / Auxiliary equation

$$m^2 + 5m + 4 = 0$$

$$(m+1)(m+4) = 0$$

Unai  $m = -1$  neu  $m = -4$ .

Dau ddatrysiaid real felly'r ffwythiant cyflennol  
(complementary function) yw  $x = Ae^{-t} + Be^{-4t}$

Does dim angen integryn neilltuoel yma gan fod ochr dde yr hafaliad gwreiddid yn sero.

Os yw  $t=0$  yna mae  $x = 2, v = 3$

Yn amnewid:  $x = Ae^{-t} + Be^{-4t}$

$$2 = Ae^{(0)} + Be^{(0)}$$

$$2 = A + B$$

①

Differu  $\frac{dx}{dt} = -Ae^{-t} - 4Be^{-4t}$   
Amnewid  $3 = -Ae^{(0)} - 4Be^{(0)}$

$$3 = -A - 4B$$

$$A = -4B - 3$$

②

Yn amnewid o ② i mewn i ①:

$$2 = -4B - 3 + B$$

$$5 = -3B$$

$$B = \underline{-\frac{5}{3}}$$

Felly yn amnewid yn ôl i ①:  $2 = A - \frac{5}{3}$

$$A = \underline{\frac{11}{3}}$$

Felly'r datrysiad cyffredinol yw  $x = \underline{\frac{11}{3}e^{-t} - \frac{5}{3}e^{-4t}}$

b)  $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 12t - 3$  — ①

fel ôr blaen, cawn  $x = Ae^{-t} + Be^{-4t}$

ar gyfer yr hafaliad ategol (auxiliary equation).

Ar gyfer yr integryn neilltuoel (particular integral)  
wrth gymharu ef  $12t - 3$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$

$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i mewn i ①:

$$(0) + 5(a) + 4x = 12t - 3$$

$$5a + 4(at + b) = 12t - 3$$

$$5a + 4b + 4at = 12t - 3$$

Yn cymharu cyfernodau  $t$ :  $4a = 12$

$$a = \underline{3}$$



Yn cymharu cysonion:  $5a + 4b = -3$

$$15 + 4b = -3$$

$$4b = -18$$

$$\underline{b = -4.5}$$

Datrysiad Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuo!

$$x = Ae^{-t} + Be^{-4t} + 3t - 4.5$$

(Does dim gwybodaeth i ffeindio A a B.)

M3 Haf 2014

$$\textcircled{4} \quad 2 \frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 5x = 1 \quad \text{---} \textcircled{1}$$

(a) Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$   
 $a = 2 \quad b = 6 \quad c = 5$

Hafaliad ategol/ Auxiliary equation

$$2m^2 + 6m + 5 = 0$$

Hafaliad cwadratig  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-6 \pm \sqrt{6^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$m = \frac{-6 \pm \sqrt{-4}}{4}$$

$$m = \frac{-6 \pm \sqrt{(-1)(4)}}{4}$$

$$m = \frac{-6 \pm 2i}{4}$$

Unai  $m = -1.5 + 0.5i$  neu  $m = -1.5 - 0.5i$

Dau ddatrysiaid cymhlyg felly'r ffwythiant cyflenwol  
(complementary function) yw  $x = e^{-1.5t}(A \cos 0.5t + B \sin 0.5t)$

Ar gyfer yr integryn neilltuol (particular integral) wrth  
gymharu efo 1 ceisiwn  $x = a$

$$\frac{dx}{dt} = 0$$

$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i  $\textcircled{1}$ :  $2(0) + 6(0) + 5a = 1$

$$5a = 1$$

$$a = \frac{1}{5}$$

Felly'r integryn neilltuol yw  $x = \frac{1}{5}$

Datrysiaid Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuo  
 $x = e^{-1.5t} (A \cos 0.5t + B \sin 0.5t) + \frac{1}{5}$  (2)

(b) Fel mae  $t$  yn cynyddu mae  $e^{-1.5t}$  yn lleihau.  
Fel mae  $t \rightarrow \infty$  mae  $e^{-1.5t} \rightarrow 0$   
Felly gwerth terfannol  $x$  yw  $\frac{1}{5}$ .

(c) Os yw  $t=0s$ , yna  $x=0.5m$ ,  $\frac{dx}{dt} = 0$ .

Amnewid i mewn i (2):

$$0.5 = e^{-0} (A \cos(0) + B \sin(0)) + \frac{1}{5}$$

$$0.5 = 1 (A(1) + B(0)) + \frac{1}{5}$$

$$0.5 = A + \frac{1}{5}$$

$$A = 0.5 - 0.2$$

$$\underline{A = 0.3}$$

Differu  $\frac{dx}{dt} = (-1.5)e^{-1.5t} (A \cos 0.5t + B \sin 0.5t)$   
 $+ e^{-1.5t} (-0.5A \sin 0.5t + 0.5B \cos 0.5t)$

Amnewid  $0 = (-1.5)e^0 (A \cos(0) + B \sin(0))$

$$+ e^0 (-0.5A \sin(0) + 0.5B \cos(0))$$

$$0 = -1.5(A(1) + B(0)) + 1(-0.5A(0) + 0.5B(1))$$

$$0 = -1.5(A) + 1(0.5B)$$

$$0 = -1.5A + 0.5B$$

$$0.5B = 1.5A$$

$$0.5B = 1.5 \times 0.3$$

$$B = \frac{1.5 \times 0.3}{0.5}$$

$$\underline{B = 0.9}$$

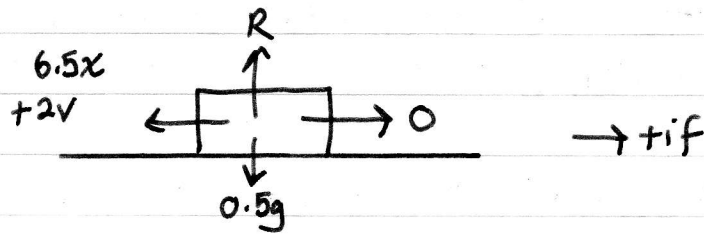
Felly'r datrysiaid cyffredinol yw

$$x = e^{-1.5t} (0.3 \cos 0.5t + 0.9 \sin 0.5t) + 0.2$$

(ii) Os yw  $t = \frac{\pi}{3}$  yna  $x = e^{-1.5(\frac{\pi}{3})} (0.3 \cos(\frac{\pi}{6}) + 0.9 \sin(\frac{\pi}{6})) + 0.2$   
 $x = 0.348$  i 3 ffigur ystyrlon

M2 Haf 2015

(2)



a)  $F=ma$  ar y gwrthrych, yn llorweddol.

$$-6.5x - 2v = 0.5a$$

$$-6.5x - 2 \frac{dx}{dt} = 0.5 \frac{d^2x}{dt^2}$$

$$-13x - 4 \frac{dx}{dt} = \frac{d^2x}{dt^2}$$

$$0 = \frac{d^2x}{dt^2} + 4 \frac{dx}{dt} + 13x$$

$$\frac{d^2x}{dt^2} + 4 \frac{dx}{dt} + 13x = 0 \quad \checkmark$$

ii) Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$

$$a=1 \quad b=4 \quad c=13$$

Hafaliad ategol/ Auxiliary equation

$$m^2 + 4m + 13 = 0$$

Hafaliad cwadratig  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(13)}}{2(1)}$$

$$m = \frac{-4 \pm \sqrt{-36}}{2}$$

$$m = \frac{-4 \pm \sqrt{36} \sqrt{-1}}{2}$$

$$m = \frac{-4 \pm 6i}{2}$$

$$m = -2 \pm 3i$$

Unai  $m = -2 + 3i$  neu  $m = -2 - 3i$

Dau ddatrysiad cymhlyg fellyr ffwythiant cyflenwol  
(complementary function) yw  $x = e^{-2t}(A \cos 3t + B \sin 3t)$

Does dim angen integryn neilltuol yma gan fod ochr  
dde yr hafaliad gwreiddiol yn sero.

Os yw  $t=0$  yna mae  $x=6$ ,  $\frac{dx}{dt} = 3$  ( $v=3$ )

Yn amnewid i  $x = e^{-2t}(A \cos 3t + B \sin 3t)$   
 $6 = e^{-2(0)}(A \cos(3 \times 0) + B \sin(3 \times 0))$   
 $6 = 1(A(1) + B(0))$   
 $A = 6$

Differu  $\frac{dx}{dt} = -2e^{-2t}(A \cos 3t + B \sin 3t) + e^{-2t}(-3A \sin 3t + 3B \cos 3t)$   
Amnewid  $3 = -2e^{-2(0)}(A \cos(0) + B \sin(0)) + e^{-2(0)}(-3A \sin(0) + 3B \cos(0))$   
 $3 = -2(1)(A(1) + B(0)) + 1(-3A(0) + 3B(1))$   
 $3 = -2A + 3B$   
 $3 = -2(6) + 3B$   
 $3 + 12 = 3B$   
 $3B = 15$   
 $B = 5$

Felly'r datrysiad cyffredinol yw  $x = e^{-2t}(6 \cos 3t + 5 \sin 3t)$

Fel mae  $t$  yn cynyddu, mae  $e^{-2t}$  yn lleihau  
(bueddu at sero) felly os yw  $t$  yn fawr bras  
werth  $x$  yw  $0$ .

$$(ii) \frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 91t + 15 \quad \text{--- (1)}$$

Fel yn rhan (a), ceisio  $x = Ae^{mt}$  i gael hafaliad ategol  $m^2 + 4m + 13 = 0$  efo datrysiadau  $m = -2 + 3i$  neu  $m = -2 - 3i$ .

Ar gyfer yr integryn neilltuoel (particular integral) wrth gymharu efo  $91t + 15$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$
$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i mewn i (1):

$$0 + 4a + 13x = 91t + 15$$

$$4a + 13(at + b) = 91t + 15$$

$$4a + 13b + 13at = 91t + 15$$

Yn cymharu cyfernodau  $t$ :

$$13a = 91$$

$$\underline{a = 7}$$

Yn cymharu cysonion:

$$4a + 13b = 15$$

$$4(7) + 13b = 15$$

$$13b = 15 - 28$$

$$13b = -13$$

$$\underline{b = -1}$$

Datrysiad Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuoel

$$\underline{x = e^{-2t}(A \cos 3t + B \sin 3t) + 7t - 1}$$

(Does dim gwybodaeth i ffeindio  $A$  a  $B$ )



M3 Haf 2016

$$\textcircled{3} \quad \frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 9x = 27t \quad \text{---} \textcircled{1}$$

Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$   
 $a=1 \quad b=6 \quad c=9$

Hafaliad ategol / Auxiliary equation

$$m^2 + 6m + 9 = 0$$

$$(m+3)(m+3) = 0$$

$$\underline{m = -3} \quad (\text{dwywaith})$$

Un gwreiddyn wedi'i ailadrodd fellyr ffwythiant cyflenwol  
(complementary function) yw  $x = e^{-3t}(A + Bt)$

Ar gyfer yr integryn neilltuol (particular integral) wrth  
gymharu efo  $27t$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$

$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i  $\textcircled{1}$ :  $0 + 6(a) + 9(at+b) = 27t$

$$6a + 9at + 9b = 27t$$

Yn gymharu cyfernodau  $t$ :  $9a = 27$

$$\underline{a = 3}$$

Yn gymharu cysonion:  $6a + 9b = 0$

$$6(3) + 9b = 0$$

$$9b = -18$$

$$\underline{b = -2}$$

Fellyr integryn neilltuol yw  $x = 3t - 2$

Datrysiaid Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuoel

$$x = e^{-3t}(A+Bt) + 3t - 2 \quad \text{--- (2)}$$

Os yw  $t = 0$ s, yna  $x = 0$  a  $\frac{dx}{dt} = 0$ .

Amnewid i (2):  $0 = e^{-3(0)}(A+B(0)) + 3(0) - 2$

$$0 = 1(A+0) - 2$$

$$0 = A - 2$$

$$\underline{A = 2}$$

Differu  $\frac{dx}{dt} = -3e^{-3t}(A+Bt) + e^{-3t}(B) + 3$

Amnewid  $0 = -3e^{-3(0)}(A+B(0)) + e^{-3(0)}(B) + 3$

$$0 = -3(A) + 1(B) + 3$$

$$0 = -3(2) + B + 3$$

$$6 - 3 = B$$

$$\underline{B = 3}$$

Felly'r datrysiaid cyffredinol yw  $x = e^{-3t}(2+3t) + 3t - 2$

Os yw  $t = 2$  mae  $x = e^{-3(2)}(2+3(2)) + 3(2) - 2$

$$x = e^{-6}(8) + 6 - 2$$

$$x = 8e^{-6} + 4$$

$$\underline{x = 4.0198} \text{ i 4 lle degol}$$



### M3 Haf 2017

$$3) \frac{d^2x}{dt^2} - 6 \frac{dx}{dt} + (10-K)x = \frac{1}{50} K(K-5)(12t-26)$$

$$a) K=5 \Rightarrow \frac{d^2x}{dt^2} - 6 \frac{dx}{dt} + (10-5)x = \left(\frac{1}{50}\right)^5 (5-5)(12t-26)$$

$$\frac{d^2x}{dt^2} - 6 \frac{dx}{dt} + 5x = 0$$

Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$   
 $a=1$   $b=-6$   $c=5$

Hafaliad ategol/ Auxiliary equation

$$m^2 - 6m + 5 = 0$$

$$(m-1)(m-5) = 0$$

Naill ai  $m-1=0$  neu  $m-5=0$

$$\underline{m=1}$$

$$\underline{m=5}$$

Dau ddatbysiad real felly'r ffwythiant cyflenwol  
(complementary function) yw  $x = Ae^t + Be^{5t}$

Does dim angen integryn neilltuol yma gan fod ochr dde yr hafaliad gwreiddiol yn sero.

O s yw  $t=0$  yna mae  $x=8$ ,  $\frac{dx}{dt} = 16$ .

Yn amnewid i  $x = Ae^t + Be^{5t}$

$$8 = Ae^0 + Be^{5 \times 0}$$

$$8 = A + B$$

$$8 - B = A \quad \text{--- (1)}$$

Differu  $\frac{dx}{dt} = Ae^t + 5Be^{5t}$   
 Amnewid  $16 = Ae^0 + 5Be^{5 \times 0}$   
 $16 = A + 5B$   
 $16 - 5B = A$  — (2)

(1), (2)  $\Rightarrow$   $8 - B = 16 - 5B$   
 $5B - B = 16 - 8$   
 $4B = 8$   
 $B = 2$  Felly  $A = 8 - 2$   
 $A = 6$

Felly'r datrysiad cyffredinol yw  $x = 6e^t + 2e^{5t}$

b)  $k=0 \Rightarrow \frac{d^2x}{dt^2} - 6\frac{dx}{dt} + (10-0)x = \begin{pmatrix} 1 \\ 50 \end{pmatrix} \begin{pmatrix} 0 \\ 0-5 \end{pmatrix} (12t-26)$   
 $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 10x = 0$

Ceisio  $x = Ae^{mt}$  fel bod  $m$  yn bodloni  $am^2 + bm + c = 0$   
 $a=1$   $b=-6$   $c=10$

Hafaliad ategol / Auxiliary equation

$$m^2 - 6m + 10 = 0$$

Hafaliad cwadrateg:  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2 \times 1}$$

$$m = \frac{6 \pm \sqrt{-4}}{2}$$

$$m = \frac{6 \pm \sqrt{4 \times -1}}{2}$$

$$m = \frac{6 \pm 2\sqrt{-1}}{2}$$



$$m = 3 \pm \sqrt{-1}$$

$$m = 3 \pm i$$

Naill ai  $m = 3 + i$  neu  $m = 3 - i$

Dau ddabrysiad cymhlyg felly'r ffwythiant cyflenwol  
(Complementary function) yw  $x = e^{3t}(A \cos t + B \sin t)$

Os yw  $t=0$  yna mae  $x = 8$ ,  $\frac{dx}{dt} = 16$ .

Yn amnewid i  $x = e^{3t}(A \cos t + B \sin t)$

$$8 = e^{3 \times 0}(A \cos 0 + B \sin 0)$$

$$8 = 1(A + 0)$$

$$\underline{8 = A}$$

Differu  $\frac{dx}{dt} = e^{3t}(-A \sin t + B \cos t) + 3e^{3t}(A \cos t + B \sin t)$

Amnewid  $16 = e^{3 \times 0}(-A \sin 0 + B \cos 0) + 3e^{3 \times 0}(A \cos 0 + B \sin 0)$

$$16 = 1(-8(0) + B(1)) + 3 \times 1(8(1) + B(0))$$

$$16 = B + 3(8)$$

$$16 - 24 = B$$

$$\underline{B = -8}$$

Felly'r ffwythiant cyflenwol yw  $x = e^{3t}(8 \cos t - 8 \sin t)$

$$x = 8e^{3t}(\cos t - \sin t)$$

Does dim angen integryn neilltuol yma gan fod ochr dde yr hafaliad gwreiddiol yn sero.

Felly'r datrysiad cyffredinol yw  $x = 8e^{3t}(\cos t - \sin t)$

$$c) \quad k=10 \Rightarrow \frac{d^2x}{dt^2} - 6 \frac{dx}{dt} + (10-10)x = \left(\frac{1}{50}\right)^{10} (10-5) (12t-26)$$

$$\frac{d^2x}{dt^2} - 6 \frac{dx}{dt} = 12t - 26 \quad \text{--- (1)}$$

Ceisio  $x = Ae^{mt}$  fel bod m yn bodloni  $am^2 + bm + c = 0$   
 $a=1 \quad b=-6 \quad c=0$

Itafaliad ategol / Auxiliary equation

$$m^2 - 6m = 0$$

$$m(m-6) = 0$$

Naill ai  $m=0$  neu  $m-6=0$

$$\underline{m=6}$$

Dau ddatbrysiad real fellyr ffwythiant cyflenwol  
 (complementary function) yw  $x = Ae^{0t} + Be^{6t}$   
 $x = A + Be^{6t}$

Ar gyfer yr integryn neilltuol (particular integral)  
 wrth gymharu efo  $12t-26$  ceisiwn  $x = at + b$

$$\frac{dx}{dt} = a$$

$$\frac{d^2x}{dt^2} = 0$$

Yn amnewid i mewn i (1):

$$0 - 6a = 12t - 26$$

$$a = -2t + \frac{13}{3}$$

ond mae  $a$  yn gysonyn, felly ni ellir ei ddiffinio yn nhermau  $t$ .

Felly, ar gyfer yr integryn neilltuol ceisiwn  $x = at^2 + bt$

$$\frac{dx}{dt} = 2at + b$$

$$\frac{d^2x}{dt^2} = 2a$$



Yn amnewid i mewn i ①:

$$2a - 6(2at + b) = 12t - 26$$

$$2a - 12at - 6b = 12t - 26$$

$$-12at + (2a - 6b) = 12t - 26$$

Yn cymharu cyfernodau t:  $-12a = 12$

$$a = -1$$

Yn cymharu cysonion:  $2a - 6b = -26$

$$-2 - 6b = -26$$

$$-6b = -24$$

$$\underline{b = 4}$$

Felly'r integryn neilltuoel (particular integral)

$$\text{yw } x = -t^2 + 4t.$$

Datbrysiad Cyffredinol = Ffwrthiant Cyflenwol + Integryn Neilltuoel

$$x = A + Be^{6t} - t^2 + 4t \quad \text{--- ②}$$

Os yw  $t=0$ , mae  $x=8$  a  $\frac{dx}{dt} = 16$ .

$$\text{Amnewid i ②: } 8 = A + Be^{0t} - 0^2 + 4 \times 0$$

$$8 = A + B$$

$$8 - B = A \quad \text{--- ③}$$

Differu ②:  $\frac{dx}{dt} = 6Be^{6t} - 2t + 4$

$$\text{Amnewid } 16 = 6Be^{6 \times 0} - 2 \times 0 + 4$$

$$16 = 6B + 4$$

$$12 = 6B$$

$$\underline{B = 2}$$

$$\text{Felly } A = 8 - 2$$

$$\underline{A = 6}$$

Felly'r datbrysiad cyffredinol yw  $x = 6 + 2e^{6t} - t^2 + 4t$

M3 Itaf 2018

$$4) \quad \frac{d^2x}{dt^2} + 2 \frac{dx}{dt} - 15x = 30t - 19 \quad \text{--- (1)}$$

Ceisio  $x = Ae^{mt}$  fel bod  $m$  yn bodloni  $am^2 + bm + c = 0$   
 $a = 1 \quad b = 2 \quad c = -15$

Hafaliad ategol / Auxiliary equation  
 $m^2 + 2m - 15 = 0$

Hafaliad cwadratig  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-2 \pm \sqrt{2^2 - 4(1)(-15)}}{2 \times 1}$$

$$m = \frac{-2 \pm \sqrt{64}}{2}$$

$$m = \frac{-2 \pm 8}{2}$$

$$m = -1 \pm 4$$

$$\text{Naill ai } m = -1 + 4 \quad \text{neu } m = -1 - 4$$

$$\underline{m = 3}$$

$$\underline{m = -5}$$

Dan ddatbrysiad real fellyr ffwythiant cyflenwol  
(complementary function) yw  $x = Ae^{3t} + Be^{-5t}$

Ar gyfer yr integryn neilltuel (particular integral)  
wrth gymharu efo  $30t - 19$  ceisiwn  $x = at + b$   
 $\frac{dx}{dt} = a$   
 $\frac{d^2x}{dt^2} = 0$

$$\text{Amnewid i mewn i (1): } 0 + 2(a) - 15(at + b) = 30t - 19$$
$$2a - 15at - 15b = 30t - 19$$

$$\text{Yn cymharu cyfernodau } t: -15a = 30$$

$$a = -2$$

$$\text{Yn cymharu cysonion: } 2a - 15b = -19$$

$$2(-2) - 15b = -19$$

$$-4 - 15b = -19$$

$$-15b = -15$$

$$b = 1$$

Felly'r integryn neilltuoel yw  $x = -2t + 1$

Datbysiad Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuoel

$$x = Ae^{3t} + Be^{-5t} - 2t + 1$$

Os yw  $t=0$ , mae  $x=10$  a  $\frac{dx}{dt} = -31$ .

$$\text{Amnewid: } 10 = Ae^{3(0)} + Be^{-5(0)} - 2(0) + 1$$

$$10 = A + B + 1$$

$$9 = A + B \quad \text{--- (2)}$$

$$\text{Differu: } \frac{dx}{dt} = 3Ae^{3t} - 5Be^{-5t} - 2$$

$$\text{Amnewid: } -31 = 3Ae^{3(0)} - 5Be^{-5(0)} - 2$$

$$-31 = 3A - 5B - 2$$

$$-29 = 3A - 5B \quad \text{--- (3)}$$

(2)  $\Rightarrow 9 - B = A$ . Yn amnewid i mewn i (3):

$$-29 = 3(9 - B) - 5B$$

$$-29 = 27 - 3B - 5B$$

$$-56 = -8B$$

$$B = 7$$

$$\text{Felly } A = 9 - B$$

$$A = 9 - 7$$

$$\underline{A = 2}$$

Felly'r datrysiad cyffredinol yw

$$\underline{x = 2e^{3t} + 7e^{-5t} - 2t + 1}$$

Os yw  $t=1$ , mae  $x = 2e^{3(1)} + 7e^{-5(1)} - 2(1) + 1$

$$x = 2e^3 + 7e^{-5} - 2 + 1$$

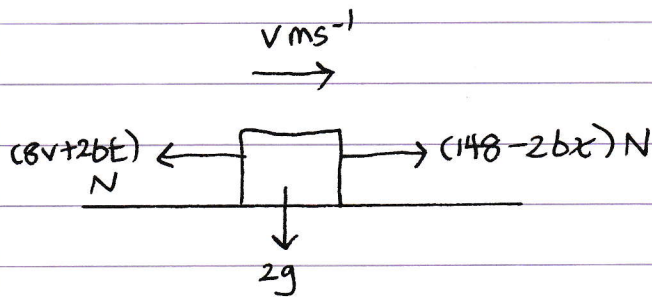
$$\underline{x = 2e^3 + 7e^{-5} - 1}$$

$$\left( \begin{array}{l} x = 39.21823948 \dots \\ x = 39.22 \text{ i 2 le degol} \end{array} \right)$$



### M3 Haf 2019

1)



a)  $F = ma$  ar y gronyn, yn llorweddol,  $\rightarrow = +$  if:

$$(148 - 26x) - (8v + 26t) = 2a$$

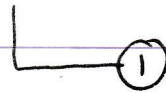
$$148 - 26x - 8v - 26t = 2a$$

$$148 - 26t = 2a + 8v + 26x$$

$$2a + 8v + 26x = 148 - 26t$$

$$a + 4v + 13x = 74 - 13t$$

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 74 - 13t \quad \checkmark$$



b) Ceisio  $x = Ae^{mt}$  fel bod  $m$  yn bodloni  $am^2 + bm + c = 0$

$$a = 1 \quad b = 4 \quad c = 13$$

Hafaliad ategol / Auxiliary equation

$$m^2 + 4m + 13 = 0$$

Hafaliad cwadratig  $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(13)}}{2 \times 1}$$

$$m = \frac{-4 \pm \sqrt{-36}}{2}$$

$$m = \frac{-4 \pm 6i}{2}$$

$$m = -2 \pm 3i$$

Naill ai  $m = -2 + 3i$  neu  $m = -2 - 3i$

Dau ddatrysiad cymhlyg felly'r ffwythiant cyflenwol  
(complementary function) yw  $x = e^{-2t}(A \cos 3t + B \sin 3t)$

Argyfer yr integryn neilltuol (particular integral)  
wrth gymharu efo  $74 - 13t$  ceisiwn  $x = at + b$   
 $\frac{dx}{dt} = a$   
 $\frac{d^2x}{dt^2} = 0$

Yn amnewid i ①:  $(0) + 4(a) + 13(at + b) = 74 - 13t$   
 $4a + 13at + 13b = 74 - 13t$

Yn gymharu cyfernodau t:  $13a = -13$   
 $a = -1$

Yn gymharu cysonion:  $4a + 13b = 74$   
 $4(-1) + 13b = 74$   
 $13b = 74 + 4$   
 $b = 6$

Felly'r integryn neilltuol yw  $x = -t + 6$

Datrysiad Cyffredinol = Ffwythiant Cyflenwol + Integryn Neilltuol  
 $x = e^{-2t}(A \cos 3t + B \sin 3t) - t + 6$  ②

Os yw  $t = 0$ , mae  $x = 0$  m,  $v = 5 \text{ ms}^{-1} = \frac{dx}{dt}$ .

Amnewid i ②:  $0 = e^{-2(0)}(A \cos(0) + B \sin(0)) - 0 + 6$   
 $0 = 1(A + 0) + 6$   
 $-6 = A$

Diffenu  $\frac{dx}{dt} = (-2)e^{-2t}(A \cos 3t + B \sin 3t)$   
 $+ e^{-2t}(-3A \sin 3t + 3B \cos 3t) - 1$

Amnewid  $5 = (-2)e^{-2(0)}(A \cos(0) + B \sin(0))$   
 $+ e^{-2(0)}(-3A \sin(0) + 3B \cos(0)) - 1$

$$5 = (-2)(A+0) + (1)(0+3B) - 1$$

$$5 = -2A + 3B - 1$$

$$6 = -2A + 3B$$

$$6 = -2(-6) + 3B$$

$$-6 = 3B$$

$$\underline{B = -2}$$

Felly'r datbysiad cyffredinol yw

$$x = e^{-2t}(-6\cos 3t - 2\sin 3t) - t + 6$$

$$x = 6 - t - 2e^{-2t}(3\cos 3t + \sin 3t)$$

OS yw  $t = 0.5$  mae

$$x = 6 - 0.5 - 2e^{-2(0.5)}(3\cos(1.5) + \sin(1.5))$$

$$x = 4.60994763$$

$$\underline{x = 4.61 \text{ m}} \quad ; \quad 2 \text{ leddegol}$$