Hen Gwestiynau Arholiad

Differential Equations

(Haf 2006)

2. Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 5t - 14,$$

such that $x = 4\frac{1}{2}$ and $\frac{dx}{dt} = 3\frac{1}{2}$ when $t = 0$. [12]

3. A ball of mass 0·4 kg is thrown vertically upwards from a point O with initial speed 17 ms^{-1}. When the ball is at a height of x m above O and its speed is v ms^{-1}, the air resistance acting on the ball has magnitude 0·01 v^2 N.

(a) Show that, as the ball is ascending, v satisfies the differential equation

$$40v\frac{dv}{dx} = -(392 + v^2).$$ [3]

(b) Find, correct to two decimal places, the greatest height of the ball. [7]

(c) State, with a reason, whether the speed of the ball when it returns to O is

(i) greater than 17 ms^{-1},

(ii) less than 17 ms^{-1},

(iii) equal to 17 ms^{-1}. [2]

(Haf 2007)

5. An experimental vehicle, of mass 800 kg, is propelled from rest along a straight horizontal track by means of a horizontal force of variable magnitude $(6120 - 80t)$ N, where t s is the time after projection. The vehicle expriences a resistance of magnitude $(120 + 40v)$ N, where v ms^{-1} is the speed of the vehicle at time t s. The distance of the vehicle from its starting position at time t s is x m.

(a) Show that x satisfies the differential equation

$$20\frac{d^2x}{dt^2} + \frac{dx}{dt} = 150 - 2t.$$ [5]

(b) Find an expression for x in terms of t. [12]

(Haf 2008)

2. *(a)* An experimental vehicle, of mass 2 kg, is designed such that, after crossing the starting line at a speed of 7 ms^{-1}, it moves in a straight horizontal line under a propulsive force of magnitude $0{\cdot}8x$ N and a resistive force of magnitude $1{\cdot}2v$ N, where x m is the distance from the starting line and v ms^{-1} is the speed of the vehicle at time t s.

(i) Show that x satisfies the differential equation

$$5\frac{d^2x}{dt^2} + 3\frac{dx}{dt} - 2x = 0 \ . \qquad [3]$$

(ii) By solving the above equation, find an expression for x in terms of t. [6]

(iii) Show that x increases with t. [2]

(b) Find the general solution of the differential equation

$$5\frac{d^2x}{dt^2} + 3\frac{dx}{dt} - 2x = 20t - 70. \qquad [5]$$

4. A particle is projected vertically upwards with initial speed 15 ms^{-1} from a point O. In the subsequent motion,

$$a + \frac{v^2}{90} + 10 = 0 \ ,$$

where a ms^{-2} is the acceleration and v ms^{-1} is the speed when the height of the particle is x m above O.

(a) Find an expression for x in terms of v. [7]

(b) Find, correct to two decimal places, the greatest height of the particle above O. [2]

(Haf 2009)

3. A cyclist, of mass 65 kg, starts from rest and rides his bicycle, of mass 10 kg, along a straight horizontal road. The cyclist produces a constant forward force of 180 N and experiences a variable resistance to motion of magnitude $3v^2$ N, where v ms^{-1} is the speed of the bicycle. Show that v satisfies the differential equation

$$25v\frac{dv}{dx} = 60 - v^2,$$

where x is the distance from the start of motion.
Calculate the speed of the cyclist when he has cycled a distance of 20 m. Give your answer correct to two decimal places. [10]

5. A particle P, of mass 2 kg, moves along a horizontal x-axis so that at time t s, its speed is v ms^{-1}. The particle moves under the action of a force $(156 - 52x)$ N, where x is the x-co-ordinate of P, and a resistive force of magnitude $4v$ N. Initially, the particle P is at the origin O and its velocity is 3 ms^{-1}.

(a) Show that x satisfies the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 26x = 78. \qquad [2]$$

(b) Find an expression for x in terms of t and determine the value of x when $t = 0{\cdot}5$. [12]

(Haf 2010)

3. Find the solution of the differential equation

$$4\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 9x = 18t - 87,$$

such that $x = 5$ and $\frac{dx}{dt} = 10$ when $t = 0$. [12]

5. An object, of mass 150 kg, descends vertically and experiences a total resistance to motion of $10v^2$ N, where v is the speed of the object at time t seconds. At time $t = 0$ it passes point A with speed 30 ms^{-1}. The distance from point A of the object at time t seconds is s metres.

(a) Show that s satisfies the differential equation

$$15v\frac{dv}{ds} = 15g - v^2.$$ [3]

(b) Find an expression for s in terms of v. [6]

(c) Given that the object hits the ground with a speed of 14 ms^{-1}, calculate the height of the point A. [2]

(d) Find an expression for v^2 in terms of s. [3]

(Haf 2011)

2. A particle, of mass 8 kg, moves along the x-axis. At time $t = 0$, the particle is at O and its velocity is 3 ms^{-1}. At time t s, the velocity of the particle is v ms^{-1} and it moves under the action of a propulsive force of magnitude $4v$ N and a resistive force of magnitude $(4 - 16t)$ N.

(a) Show that x satisfies the differential equation

$$2\frac{d^2x}{dt^2} - \frac{dx}{dt} = 4t - 1.$$ [3]

(b) Find an expression for x in terms of t. [12]

4. A particle P moves along the x-axis. When the displacement of P from the origin O is x m, its acceleration is of magnitude $\left(\frac{9}{2x^2}\right)$ ms^{-2} and is directed towards O.
When $x = \frac{3}{4}$, the velocity of P is 3 ms^{-1}. Find the speed of P when $x = 2$ and the value of x when P comes to rest. [10]

(Haf 2012)

3. Find the solution of the second order differential equation

$$2\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 2x = 6t + 5$$

such that $x = 3$ and $\frac{dx}{dt} = 2$ when $t = 0$. [12]

4. A particle P, of mass 0·5 kg, moves along the positive x-axis away from the origin O. At time t s, the displacement of P from O is x m and its speed is v ms^{-1}. The particle is moving under the action of a force of magnitude $\frac{4}{2x+1}$ N acting in the direction of motion. As P passes point A, where $OA = 3$ m, its speed is 4 ms^{-1}.

(a) Find an expression for v^2 in terms of x, and hence calculate the speed of P when it is 10 m from O. [8]

(b) Find the distance of P from O when its speed is 6 ms^{-1}. [3]

(Haf 2013)

3. *(a)* A particle P, of mass 2 kg, moves along the horizontal x-axis under the action of a force directed towards the origin O. The magnitude of the force is equal to $8x$ N, where x m is the displacement of P from O. The particle is also subjected to a resistive force which is equal to $10v$ N, where v ms^{-1} is the speed of P at time t s. When $t = 0$ s, the particle P is at $x = 2$ m and it is moving away from O with speed 3 ms^{-1}.

(i) Show that the equation of motion of the particle is

$$\frac{d^2x}{dt^2} = -4x - 5\frac{dx}{dt}.$$

(ii) Find an expression for x in terms of t. [10]

(b) Find the general solution of the second order differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 12t - 3.$$ [4]

(Haf 2014)

4. The reading x of the pointer on a set of kitchen scales at time t is modelled by the differential equation

$$2\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 1.$$

(a) Find the general solution of the equation for x. [5]

(b) Determine the limiting value of x. [2]

(c) Given that $x = 0·5$ and $\frac{dx}{dt} = 0$ when $t = 0$,

(i) find an expression for x in terms of t,

(ii) calculate the instantaneous reading of the scale when $t = \frac{\pi}{3}$.
Give your answer correct to three significant figures. [5]

(Haf 2015)

2. *(a)* An object of mass 0·5 kg is initially moving along the positive x-axis away from the origin O. The object moves under the action of a force of magnitude $6{\cdot}5x$ N which is directed towards O. The resistance to motion of the object is $2v$ N, where $v\,\mathrm{ms}^{-1}$ is the velocity of the object at time t seconds.

(i) Show that the equation of motion of the object is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 13x = 0.$$

(ii) Find an expression for x in terms of t given that $x = 6$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 3$ when $t = 0$.

Determine the approximate value of x when t is large. [9]

(b) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 13x = 91t + 15.$$ [4]

(Haf 2016)

3. Solve the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 6\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 27t,$$

where $x = \frac{\mathrm{d}x}{\mathrm{d}t} = 0$ when $t = 0$. Hence find the value of x when $t = 2$. [12]

(Haf 2017)

3. The function x satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 6\frac{\mathrm{d}x}{\mathrm{d}t} + (10-k)x = \frac{1}{50}k(k-5)(12t-26),$$

where k is a constant. When $t = 0$, $x = 8$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 16$. Find x in each of the following cases.

(a) $k = 5.$ [5]

(b) $k = 0.$ [5]

(c) $k = 10.$ [8]

(Haf 2018)

4. Solve the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} - 15x = 30t - 19,$$

where $x = 10$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = -31$ when $t = 0$. Hence find the value of x when $t = 1$. [13]

(Haf 2019)

1. A particle *P*, of mass 2 kg, moves along a horizontal x-axis. At time t seconds, its velocity is $v\,\text{ms}^{-1}$ and its displacement from the origin *O* is x metres. The particle moves under the action of a tractive force $(148 - 26x)$ N and a resistive force $(8v + 26t)$ N.

(a) Show that x satisfies the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 74 - 13t.$$ [2]

(b) Given that the particle *P* is at the origin *O* when $t = 0$ and is moving with velocity $5\,\text{ms}^{-1}$, find an expression for x in terms of t and determine the value of x when $t = 0{\cdot}5$. [13]