GCE AS/A level

978/01

MATHEMATICS FP2

Further Pure Mathematics

A.M. THURSDAY, 24 June 2010

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Using the substitution $u = x\sqrt{x}$, evaluate the integral

$$\int_0^2 \frac{\sqrt{x}}{\sqrt{9 - x^3}}\,\mathrm{d}x.$$

Give your answer correct to three decimal places. [5]

2. *(a)* Given that $3 + 4\mathrm{i} = r(\cos\theta + \mathrm{i}\sin\theta)$, where $0 < \theta < \frac{\pi}{2}$, find the values of r and θ. [2]

(b) Hence find the three cube roots of $3 + 4\mathrm{i}$ in the form $x + \mathrm{i}y$. Give the values of x and y correct to three significant figures. [7]

3. Consider the equation

$$5\sin x - 5\cos x = 1.$$

Putting $t = \tan\left(\frac{x}{2}\right)$, show that

$$2t^2 + 5t - 3 = 0.$$

Hence find the general solution to the above trigonometric equation. [10]

4. The function f is defined by

$$f(x) = \frac{3x^2}{(x+2)(x^2+2)}.$$

(a) Express $f(x)$ in partial fractions. [4]

(b) Evaluate the integral

$$\int_1^2 f(x)\,\mathrm{d}x.$$

[6]

5. Write down de Moivre's Theorem for $n = 5$. Hence show that, for $\sin\theta \neq 0$,

$$\frac{\sin 5\theta}{\sin\theta} = A\cos^4\theta + B\cos^2\theta + C,$$

where A, B, C are constants to be determined.

Deduce the limiting value of $\frac{\sin 5\theta}{\sin\theta}$ as θ tends to zero. [8]

6. The function f is defined by

$$f(x) = \frac{x}{(x-1)^2}.$$

(a) Find the coordinates of the stationary point on the graph of f. [4]

(b) State the equation of each of the asymptotes of the graph of f. [2]

(c) Sketch the graph of f. [2]

(d) Find $f^{-1}(A)$, where A is the interval $[0, 2]$. [5]

7. Let f be a function with domain $(-a, a)$ and define functions g and h as follows.

$$g(x) = f(x) + f(-x)$$
$$h(x) = f(x) - f(-x)$$

(a) Show that g is an even function and h is an odd function. Hence show that f can be expressed as the sum of an even function and an odd function. [3]

(b) Given that, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$,

$$f(x) = \ln(1 + \sin x),$$

(i) find and simplify an expression for $g(x)$,

(ii) show that

$$h(x) = 2\ln(\sec x + \tan x).$$ [7]

8. A parabola has equation

$$x^2 + 8y = 0.$$

(a) Find the coordinates of the focus and the equation of the directrix. [3]

(b) (i) Show that the point $P\,(4p, -2p^2)$ lies on the parabola for all values of p.

(ii) Find the equation of the tangent to the parabola at the point P.

(iii) Given that this tangent passes through the point $(\lambda, 2)$, show that

$$2p^2 - \lambda p - 2 = 0\ .$$

Hence show that the two tangents to the parabola from any point on the line $y = 2$ are perpendicular. [7]