Hen Gwestiynau Arholiad

Real Functions

(Haf 2006)

1. The function f is defined as follows.

$$f(x) = x \qquad \text{for } x < 0,$$

$$f(x) = \sin x \qquad \text{for } x \geqslant 0.$$

Determine whether or not

(i) the function f,

(ii) its derivative f'

is continuous when $x = 0$. [5]

3. The function f is defined on the domain $(-\infty, 0) \cup (0, \infty)$ by

$$f(x) = \frac{1}{x(x^2 + 1)}.$$

(a) Show that f is strictly decreasing over the interval $(0, \infty)$. [3]

(b) State, giving a reason, whether f is even or odd or neither even nor odd. [2]

(c) State the equation of each of the asymptotes on the graph of f. [2]

(d) Sketch the graph of f. [2]

(Haf 2007)

6. The function f is defined by

$$f(x) = \frac{x^2 + 4}{x}.$$

(a) Find the coordinates of the stationary points on the graph of f. [4]

(b) Find the equation of each of the two asymptotes. [2]

(c) Sketch the graph of f. [2]

(d) Find $f(A)$ where A is the interval [1, 5]. [4]

8. The function f is defined on the domain $(0, 2)$ by

$$f(x) = 4x^2 \qquad \text{for } 0 < x < 1,$$

$$f(x) = (x+1)^2 \qquad \text{for } 1 \leqslant x < 2.$$

(a) Determine whether or not f is continuous when $x = 1$. [2]

(b) Show that f is a strictly increasing function. [2]

(c) Obtain an expression for $f^{-1}(x)$ on each part of its domain. [6]

(Haf 2008)

1. For each of the following functions state, with a reason, whether it is even, odd or neither even nor odd.

(a) $\dfrac{x}{x^2+1}$ [2]

(b) $e^x + 1$ [2]

2. The function f is defined by

$$f(x) = 1 + ax^3 \qquad \text{for } x < 2,$$

$$f(x) = bx^2 - 3 \qquad \text{for } x \geqslant 2.$$

Given that both f and its derivative f' are continuous at $x = 2$, find the values of the constants a and b. [6]

(Haf 2009)

1. The functions f, g and h are defined as follows:

$$f(x) = \sin x$$

$$g(x) = |x|$$

$$h(x) = \frac{1}{x}$$

(a) State, with a reason, which one of the above functions is not continuous. [2]

(b) State, with a reason, whether

(i) g is even or odd,

(ii) h is even or odd. [4]

8. The function f is defined by

$$f(x) = \frac{x(x+3)}{x-1} \ .$$

(a) Show that $f(x)$ can be written in the form

$$ax + b + \frac{c}{x-1}$$

where a, b, c are constants to be found. [3]

(b) Find the coordinates of the stationary points on the graph of f. [4]

(c) State the equation of each of the asymptotes on the graph of f and sketch the graph of f. [4]

(d) Find $f^{-1}(A)$, where A is the interval $[0, 10]$. [5]

(Haf 2010)

6. The function f is defined by

$$f(x) = \frac{x}{(x-1)^2}.$$

(a) Find the coordinates of the stationary point on the graph of f. [4]

(b) State the equation of each of the asymptotes of the graph of f. [2]

(c) Sketch the graph of f. [2]

(d) Find $f^{-1}(A)$, where A is the interval $[0, 2]$. [5]

7. Let f be a function with domain $(-a, a)$ and define functions g and h as follows.

$$g(x) = f(x) + f(-x)$$
$$h(x) = f(x) - f(-x)$$

(a) Show that g is an even function and h is an odd function. Hence show that f can be expressed as the sum of an even function and an odd function. [3]

(b) Given that, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$,

$$f(x) = \ln(1 + \sin x),$$

(i) find and simplify an expression for $g(x)$,

(ii) show that

$$h(x) = 2\ln(\sec x + \tan x).$$ [7]

(Haf 2011)

3. The piecewise function f is defined by

$$f(x) = -x^2 + 6x - 7 \qquad (x \leqslant 2),$$

$$f(x) = x^2 - 2x + 4 \qquad (x > 2).$$

(a) Determine whether or not f is continuous for all values of x. [2]

(b) Determine whether or not f is a strictly increasing function. [4]

(c) The interval [1, 3] is denoted by A. Determine $f(A)$. [3]

(Haf 2012)

1. The piecewise function f is defined by

$$f(x) = ax^2 - 8 \quad (x \leqslant 2),$$
$$f(x) = x^3 - bx \quad (x > 2),$$

where a and b are constants.

Given that f and its derivative f' are continuous when $x = 2$, find the values of a and b. [5]

5. *(a)* The function f is defined by

$$f(x) = x^2 \sin x.$$

Determine whether f is an even function or an odd function. [3]

(b) The function g is defined by

$$g(x) = x^n \sin x,$$

where n is a positive integer. Determine the set of values of n for which g is

(i) an even function,
(ii) an odd function. [3]

6. The function f is defined by

$$f(x) = \frac{2}{x-3} + x - 6.$$

(a) Determine the coordinates of the points where the graph of f intersects the coordinate axes. [5]

(b) Find the coordinates of the stationary points on the graph of f. [5]

(c) State the equation of each of the asymptotes on the graph of f. [2]

(d) Sketch the graph of f. [2]

(Haf 2013)

4. The function f is defined on the domain $x > 1$ by

$$f(x) = \frac{2x+3}{x-1}.$$

(a) Show that f is a strictly decreasing function. [3]

(b) Given that $S = [4, 5]$, determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [6]

7. The function f is defined by

$$f(x) = \frac{(2x^2+1)^2}{x^3}.$$

(a) Determine whether f is even, odd or neither even nor odd. [3]

(b) Find the x-coordinates of the stationary points on the graph of f. [4]

(c) State the equation of each of the asymptotes on the graph of f. [2]

(d) Sketch the graph of f and its asymptotes. [2]

(Haf 2014)

3. The function f is defined by

$$f(x) = e^{2x} \qquad \text{for } x < 0,$$

$$f(x) = (x+1)^2 \qquad \text{for } x \geqslant 0.$$

Determine whether or not

(a) f is continuous when $x = 0$, [3]

(b) the derivative f' is continuous when $x = 0$. [3]

8. The function f is defined by

$$f(x) = \frac{(x+4)(x-2)}{(x-4)}.$$

(a) Write down the coordinates of the points of intersection of the graph of f and the coordinate axes. [1]

(b) Determine the equation of

(i) the vertical asymptote on the graph of f,

(ii) the asymptote that is not parallel to a coordinate axis. [4]

(c) Find the coordinates of the stationary points on the graph of f. [4]

(d) Sketch the graph of f and its asymptotes. [3]

(e) The set $S = [-7, 3]$. Determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [6]

(Haf 2015)

2. The function f is defined by

$$f(x) = ax^3 + bx \qquad \text{for } x \leqslant -1,$$

$$f(x) = x^2 - x + 2 \qquad \text{for } x > -1.$$

(a) Given that f and its derivative are both continuous at $x = -1$, determine the values of the constants a and b. [6]

(b) The equation $f(x) = 0$ has exactly one root. Determine its value. [2]

7. The function f is defined by

$$f(x) = \frac{1}{x-1} - \frac{4}{x-2}.$$

(a) Write down the equations of the vertical asymptotes on the graph of f. [1]

(b) Find the points of intersection of the graph of f with the coordinate axes. [3]

(c) Find the coordinates of the stationary points on the graph of f and classify each point as a maximum or a minimum. [8]

(d) Sketch the graph of f. [2]

(e) The set $S = [-1, 0]$. Determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [6]

(Haf 2016)

7. The function f is defined by

$$f(x) = \frac{x^3 - 8}{x^3 - 1}.$$

(a) Write down the equations of the asymptotes on the graph of f. [2]

(b) Find the points of intersection of the graph of f with the coordinate axes. [2]

(c) Find the coordinates of the stationary point on the graph of f and identify it as a maximum, a minimum or a point of inflection. [5]

(d) Sketch the graph of f, including the asymptotes. [3]

(e) The set $S = [-2, 2]$. Determine

(i) $f(S)$.

(ii) $f^{-1}(S)$. [6]

(Haf 2017)

1. The function f is defined on the domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ by

$$f(x) = \sec x + x\tan x.$$

Determine whether f is even, odd or neither even nor odd. [3]

8. The function f is defined by

$$f(x) = x + 3 + \frac{1}{x+1}.$$

(a) Find the equation of

(i) the vertical asymptote on the graph of f,

(ii) the asymptote that is not parallel to a coordinate axis. [2]

(b) Find the coordinates of the stationary points on the graph of f. [5]

(c) (i) Obtain an expression for $f''(x)$.

(ii) Hence classify each of the stationary points as a maximum or a minimum. [3]

(d) Sketch the graph of f, including the asymptotes. [3]

(e) The set S is given by $S = [4, 5]$. Determine $f^{-1}(S)$. [4]

(Haf 2018)

2. The function f is defined by

$$f(x) = \sqrt{-x} \qquad \text{for } x < 0,$$

$$f(x) = -\sqrt{x} \qquad \text{for } x \geqslant 0.$$

Determine whether f is an even function, an odd function or neither even nor odd. [3]

8. The function f is defined by

$$f(x) = \frac{1 + x + x^2}{1 - x + x^2}\ .$$

(a) Find the equation of the asymptote on the graph of f. [1]

(b) (i) Find the coordinates of the two stationary points on the graph of f.

(ii) By considering the signs of $f'(x)$ in the vicinity of these stationary points, classify each as a maximum or a minimum. [8]

(c) Sketch the graph of f. [2]

(d) The set $S = (2, 3)$. Determine $f^{-1}(S)$. [5]