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Conic Sections

(Haf 2006)

4. A hyperbola has equation

$$2x^2 - 4x - y^2 - 4y = 4.$$

(a) Find the coordinates of the centre of the hyperbola. [4]

(b) Find the coordinates of the foci and the equations of the directrices. [5]

8. The line $y = m(x - 2)$ intersects the circle $x^2 + y^2 = 1$ at the points A and B.

(a) Show that the coordinates of M, the mid-point of AB, are

$$\left(\frac{2m^2}{1+m^2},\ -\frac{2m}{1+m^2}\right).$$

[5]

(b) Find the Cartesian equation of the locus of M as m varies. [6]

(Haf 2007)

5. The ellipse E has equation

$$16x^2 + 25y^2 = 400.$$

(a) Find the coordinates of the foci of E. [4]

(b) Show that the point P with coordinates $(5\cos\theta, 4\sin\theta)$ lies on E. [1]

(c) (i) Show that the equation of the normal to E at P is

$$4y\cos\theta - 5x\sin\theta + 9\sin\theta\cos\theta = 0.$$

(ii) This normal intersects the x-axis at Q and the y-axis at R. Show that the locus of M, the mid-point of QR, is an ellipse. [10]

(Haf 2008)

5. *(a)* Show that the equation of the normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ is

$$y + px = ap(2 + p^2).$$

[4]

(b) This normal meets the x-axis at Q and the mid-point of PQ is R.

(i) Find the coordinates of R.

(ii) The locus of R as p varies is a parabola. Find the equation of this parabola and the coordinates of its focus. [8]

(Haf 2009)

6. The ellipse E has equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad .$$

(a) Show that the equation of the tangent to E at the point $(a\cos\theta, b\sin\theta)$ is

$$bx\cos\theta + ay\sin\theta = ab.$$ [5]

(b) This tangent meets the coordinate axes at P and Q, and the mid-point of PQ is R. Find the Cartesian equation of the locus of R as θ varies. [7]

(Haf 2010)

8. A parabola has equation

$$x^2 + 8y = 0.$$

(a) Find the coordinates of the focus and the equation of the directrix. [3]

(b) (i) Show that the point $P\ (4p, -2p^2)$ lies on the parabola for all values of p.

(ii) Find the equation of the tangent to the parabola at the point P.

(iii) Given that this tangent passes through the point $(\lambda, 2)$, show that

$$2p^2 - \lambda p - 2 = 0 \ .$$

Hence show that the two tangents to the parabola from any point on the line $y = 2$ are perpendicular. [7]

(Haf 2011)

6. The ellipse E has equation

$$2x^2 + 3y^2 - 4x + 12y + 8 = 0.$$

Find

(a) the coordinates of the centre of E, [3]

(b) the eccentricity of E, [4]

(c) the coordinates of the foci of E, [2]

(d) the equations of the directrices of E. [2]

(Haf 2012)

7. A parabola has equation

$$y^2 - 2y - 8x + 25 = 0.$$

(a) Find

(i) the coordinates of the vertex,
(ii) the coordinates of the focus,
(iii) the equation of the directrix. [6]

(b) The line $y = mx$ cuts the parabola at the points P_1 and P_2.

(i) Obtain a quadratic equation whose roots are the x-coordinates of P_1 and P_2.
(ii) Hence find the gradients of the two tangents from the origin to the parabola. [7]

(Haf 2013)

5. The ellipse E has equation

$$x^2 + 2y^2 - 4x + 4y + 2 = 0.$$

(a) Find

(i) the coordinates of the centre,

(ii) the eccentricity,

(iii) the coordinates of the foci,

(iv) the equations of the directrices. [9]

(b) (i) Show that the y-axis is a tangent to E.

(ii) Find the gradient of the tangent, other than the y-axis, from the origin to E. [7]

(Haf 2014)

7. The ellipse E has equation

$$4x^2 + 9y^2 = 36.$$

(a) Find

(i) the eccentricity,

(ii) the coordinates of the foci. [4]

(b) (i) Show that the point $P(3\cos\theta, 2\sin\theta)$ lies on E for all values of θ.

(ii) Show that the equation of the tangent to E at P is

$$3y\sin\theta + 2x\cos\theta = 6.$$

(iii) This tangent meets the x-axis at R and the y-axis at S. The midpoint of RS is denoted by M. Determine the equation of the locus of M as θ varies. [11]

(Haf 2015)

6. The point $P(x, y)$ moves in such a way that its distance from the point $(0, 3)$ is equal to its distance from the line $y + 3 = 0$.

(a) Show that the locus of P is the curve C with equation $x^2 = 12y$. [2]

(b) (i) Show that the point $(6t, 3t^2)$ lies on C for all values of t.

(ii) Show that the equation of the tangent to C at the point $(6t, 3t^2)$ is

$$y = tx - 3t^2.$$

(iii) Find the values of t for which the tangent passes through the point $(0, -12)$.

(iv) Hence find the angle between the two tangents to C from the point $(0, -12)$. [9]

(Haf 2016)

6. *(a)* Show that the general hyperbola with equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

can be represented parametrically by $x = a\sec\theta$, $y = b\tan\theta$. [2]

(b) The equation of the hyperbola *H* is

$$x^2 - y^2 = 1.$$

(i) Show that the equation of the normal to *H* at the point *P*$(\sec\theta, \tan\theta)$ is

$$x\sin\theta + y = 2\tan\theta.$$

(ii) This normal meets the x-axis at the point Q. Show that the locus of the midpoint of *PQ* as θ varies is a hyperbola. Determine its eccentricity and the coordinates of its foci. [12]

(Haf 2017)

7. *(a)* The point *P*(x, y) moves in such a way that its distance from the point $(a, 0)$ is equal to its distance from the line $x = -a$. Show that the locus of *P* is the parabola with equation $y^2 = 4ax$. [3]

(b) Determine the equation of the normal at the point $(at^2, 2at)$ on the parabola. [4]

(c) This normal intersects the parabola again at the point $(as^2, 2as)$. Obtain an expression for s in terms of t. [5]

(Haf 2018)

7. The equation of the ellipse *E* is

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

(a) Determine

(i) the eccentricity of *E*,

(ii) the coordinates of the foci of *E*,

(iii) the equations of the directrices of *E*. [4]

(b) Determine the equation of the normal to *E* at the point $(3\cos\theta, 2\sin\theta)$, simplifying your answer. [5]

(c) This normal meets the x and y axes at the points *A* and *B* respectively. Show that the locus of the midpoint of *AB* as θ varies is an ellipse. [5]