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Further Integration

(Haf 2007)

1. Use the substitution $x = y^2$ to evaluate the integral

$$\int_1^4 \frac{dx}{\sqrt{x(9-x)}},$$

giving your answer correct to two significant figures. [6]

(Haf 2008)

3. *(a)* Using the substitution $u = x^2$, evaluate the integral

$$\int_0^{\sqrt{3}} \frac{x\,dx}{(9+x^4)},$$

giving your answer in the form $\frac{\pi}{k}$, where k is an integer. [5]

(b) Evaluate the integral

$$\int_0^1 \frac{dx}{\sqrt{25-9x^2}}.$$ [4]

(Haf 2009)

2. Using the substitution $u = \tan x$, evaluate the integral

$$\int_0^{\frac{\pi}{6}} \frac{\sec^2 x}{\sqrt{3-\sec^2 x}}\,dx \quad .$$

Explain briefly why the integral could not be evaluated if the upper limit were changed to $\frac{\pi}{3}$. [7]

(Haf 2010)

1. Using the substitution $u = x\sqrt{x}$, evaluate the integral

$$\int_0^2 \frac{\sqrt{x}}{\sqrt{9-x^3}}\,dx.$$

Give your answer correct to three decimal places. [5]

(Haf 2011)

1. Using the substitution $u = \sqrt{x}$, evaluate the integral

$$\int_1^4 \frac{1}{(9+x)\sqrt{x}}\,\mathrm{d}x.$$

Give your answer correct to four decimal places. [5]

7. *(a)* Differentiate the following integral with respect to x.

$$\int_0^x \sin(\mathrm{e}^t)\,\mathrm{d}t \qquad [1]$$

(b) By putting $u = x^2$ and using the chain rule, differentiate the following integral with respect to x.

$$\int_0^{x^2} \sin(\mathrm{e}^t)\,\mathrm{d}t \qquad [2]$$

(Haf 2012)

2. Using the substitution $u = \mathrm{e}^x$, evaluate the integral

$$\int_0^1 \frac{1}{\left(\mathrm{e}^x + 4\mathrm{e}^{-x}\right)}\mathrm{d}x.$$

Give your answer correct to three decimal places. [6]

(Haf 2013)

1. Using the substitution $u = x^2$, evaluate the integral

$$\int_1^2 \frac{x}{\sqrt{25 - x^4}}\mathrm{d}x.$$

Give your answer correct to three significant figures. [5]

(Haf 2014)

2. Using the substitution $u = \sin^2 x$, evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{4 - \sin^4 x}}\,\mathrm{d}x.$$

Give your answer in the form $\frac{\pi}{k}$, where k is a positive integer. [5]

(Haf 2015)

5. Differentiate the following integrals with respect to x.

(a) $\displaystyle\int_0^x \mathrm{e}^{\sqrt{u}}\,\mathrm{d}u$ [1]

(b) $\displaystyle\int_0^{x^2} \mathrm{e}^{\sqrt{u}}\,\mathrm{d}u$ [3]

(c) $\displaystyle\int_x^{x^2} \mathrm{e}^{\sqrt{u}}\,\mathrm{d}u$ [2]

(Haf 2016)

1. Using the substitution $u = x^2$, evaluate the integral

$$\int_0^{\sqrt{2}} \frac{x}{\sqrt{16 - x^4}}\,\mathrm{d}x\,,$$

giving your answer in the form $\frac{\pi}{n}$, where n is a positive integer. [6]

(Haf 2017)

2. Evaluate the integral

$$\int_0^2 \frac{2x^2 + 5}{x^2 + 4}\,\mathrm{d}x\,,$$

giving your answer in the form $a + b\pi$, where a, b are constants to be determined. [5]

(Haf 2018)

3. *(a)* Express $3 + 2x - x^2$ in the form $a - (x - b)^2$, where a, b are positive integers. [2]

(b) Hence evaluate the integral

$$\int_0^2 \frac{1}{\sqrt{3 + 2x - x^2}}\,\mathrm{d}x\,,$$

giving your answer in the form $\frac{\pi}{n}$, where n is a positive integer to be determined. [3]