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De Moivre's Theorem

(Haf 2006)

2. Find the three cube roots of the complex number i. Give your answers in the form $x + \mathrm{i}y$. [9]

6. *(a)* Use mathematical induction to prove that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

where n is a positive integer. [6]

(b) Use the result in *(a)* with $n = 5$ to show that

$$\sin 5\theta = a\sin^5\theta - b\sin^3\theta + c\sin\theta$$

where a, b, c are positive integers to be found. [7]

(Haf 2007)

2. Find the two square roots of the complex number $1 + \sqrt{3}\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$. [6]

7. *(a)* Given that

$$z = \cos\theta + \mathrm{i}\sin\theta,$$

use de Moivre's Theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

for all positive integers n. [3]

(b) Hence by expanding $\left(z + \frac{1}{z}\right)^5$, show that

$$\cos^5\theta = a\cos 5\theta + b\cos 3\theta + c\cos\theta$$

where a, b and c are constants to be determined. [5]

(Haf 2008)

6. *(a)* Given that

$$z = \cos\theta + i\sin\theta,$$

show that

$$z^n - z^{-n} = 2i\sin n\theta. \quad [3]$$

(b) Expand $(z - z^{-1})^3$ and hence show that

$$\sin^3\theta = a\sin 3\theta + b\sin\theta$$

where the values of the constants a and b are to be determined. [5]

8. *(a)* Find the modulus and argument of the complex number 8i. [2]

(b) Hence find the three cube roots of 8i, giving your answers in the form $x + iy$. [8]

(Haf 2009)

3. Giving your answers in the form $r(\cos\theta + i\sin\theta)$, find the fourth roots of the complex number $-8 + 8\sqrt{3}\,i$. [8]

7. *(a)* Given that

$$z = \cos\theta + i\sin\theta,$$

show that

$$z^n + z^{-n} = 2\cos n\theta. \quad [3]$$

(b) Hence solve the equation

$$z^2 - 2z + 3 - 2z^{-1} + z^{-2} = 0. \quad [7]$$

(Haf 2010)

2. *(a)* Given that $3 + 4i = r(\cos\theta + i\sin\theta)$, where $0 < \theta < \frac{\pi}{2}$, find the values of r and θ. [2]

(b) Hence find the three cube roots of $3 + 4i$ in the form $x + iy$. Give the values of x and y correct to three significant figures. [7]

5. Write down de Moivre's Theorem for $n = 5$. Hence show that, for $\sin\theta \neq 0$,

$$\frac{\sin 5\theta}{\sin\theta} = A\cos^4\theta + B\cos^2\theta + C,$$

where A, B, C are constants to be determined.

Deduce the limiting value of $\frac{\sin 5\theta}{\sin\theta}$ as θ tends to zero. [8]

(Haf 2011)

4. Given that $z = -1 + \mathrm{i}$,

(a) find the modulus and argument of z, [3]

(b) find the three cube roots of z in the form $x + \mathrm{i}y$, giving x and y correct to three decimal places, [7]

(c) find the smallest positive integer n for which z^n is a positive real number. [2]

5. *(a)* Given that $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$$

and find a similar expression for $z^n + \dfrac{1}{z^n}$. [5]

(b) Hence by expanding $\left(z - \dfrac{1}{z}\right)^4$, show that

$$\sin^4\theta = a\cos 4\theta + b\cos 2\theta + c,$$

where a, b, c are constants whose values should be determined. [5]

(Haf 2012)

8. *(a)* Using mathematical induction, prove that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

for positive integral values of n. [7]

(b) (i) The complex number w is a cube root of the complex number z. Show that $w\left(\cos\dfrac{2\pi}{3} + \mathrm{i}\sin\dfrac{2\pi}{3}\right)$ is another cube root of z.

(ii) Write down the real cube root of -8. Using the result in (i), or otherwise, find the two complex cube roots of -8, giving your answers in the form $x + \mathrm{i}y$. [7]

(Haf 2013)

3. *(a)* Find the four fourth roots of -1, giving your answers in the form $x + \mathrm{i}y$. [6]

(b) (i) Plot the points corresponding to these roots on an Argand diagram.

(ii) The points are joined up to form a square. Find the area of the square. [3]

8. Using de Moivre's Theorem, show that

$$\cos 5\theta = a\cos^5\theta + b\cos^3\theta + c\cos\theta,$$

where a, b, c are constants whose values are to be determined. [6]

(Haf 2014)

4. The complex number z is given by $1+\mathrm{i}\sqrt{3}$.

(a) Find the modulus and the argument of z. [2]

(b) Find the three cube roots of z, giving your answers in the form $x + \mathrm{i}y$ with x and y correct to three decimal places. [6]

6. Using de Moivre's Theorem, show that for $\sin\theta \neq 0$,

$$\frac{\sin 6\theta}{\sin\theta} = a\cos^5\theta + b\cos^3\theta + c\cos\theta,$$

where a, b, c are constants whose values are to be determined.

Hence determine the limiting value of $\dfrac{\sin 6\theta}{\sin\theta}$ as θ tends to π. [8]

(Haf 2015)

3. The complex number $z = 2\left(\cos\left(\frac{3\pi}{4}\right) + \mathrm{i}\sin\left(\frac{3\pi}{4}\right)\right)$.

(a) Find the three cube roots of z, giving your answers in the form $x + \mathrm{i}y$, with x and y correct to three decimal places. [6]

(b) Find the smallest positive integer n for which z^n is

(i) real,

(ii) imaginary. [3]

(Haf 2016)

2. *(a)* (i) Evaluate $(3 - \mathrm{i})^2$, giving your answer in the form $a + \mathrm{i}b$.

(ii) Using your result, show that

$$(3 - \mathrm{i})^4 = 28 - 96\mathrm{i}.$$ [3]

(b) Hence write down the four 4th roots of $28 - 96\mathrm{i}$. [3]

3. *(a)* Use de Moivre's Theorem to prove that, for $\sin\theta \neq 0$,

$$\frac{\sin 4\theta}{\sin\theta} = 4\cos\theta\left(1 - 2\sin^2\theta\right).$$ [4]

(b) Hence evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin 4\theta}{\sin\theta}\,\mathrm{d}\theta.$$

Give your answer correct to three significant figures. [4]

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(Haf 2017)

3. Find the three cube roots of the complex number $-8\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$ where x, y are either integers or surds. [8]

4. *(a)* Given that $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

and find a similar expression for $z^n - \frac{1}{z^n}$. [4]

(b) By expanding $\left(z + \frac{1}{z}\right)^5$, show that

$$\cos^5\theta = a\cos 5\theta + b\cos 3\theta + c\cos\theta,$$

where a, b, c are constants whose values should be determined. [5]

(c) Hence evaluate the integral

$$\int_0^{\frac{\pi}{2}} \cos^5\theta \, \mathrm{d}\theta \, .$$ [4]

(Haf 2018)

1. Find the three cube roots of the complex number $3 + 4\mathrm{i}$. Give your answers in Cartesian form with the real and imaginary parts correct to two decimal places. [8]

6. *(a)* Given that $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$$

and find a similar expression for $z^n + \frac{1}{z^n}$. [4]

(b) Hence show that

$$\sin^3\theta\cos\theta = a\sin 4\theta + b\sin 2\theta,$$

where a, b are constants whose values are to be determined. [5]