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Hyperbolic Functions

(Haf 2006)

1. *(a)* Using the exponential definitions of sinh x and cosh x, show that

$$\cosh 2x = 2\sinh^2 x + 1. \qquad [3]$$

(b) Solve the equation

$$\cosh 2x = 3\sinh x$$

giving your answers correct to three significant figures. [6]

(Haf 2007)

5. *(a)* Given that

$$a\cosh x + b\sinh x \equiv r\cosh(x + \alpha) \text{ where } a > b > 0, r > 0,$$

show that

$$\alpha = \frac{1}{2}\ln\left(\frac{a+b}{a-b}\right)$$

and find an expression for r in terms of a and b. [8]

(b) Hence, or otherwise, solve the equation

$$5\cosh x + 3\sinh x = 4,$$

giving your answer as a natural logarithm. [4]

(Haf 2008)

4. *(a)* Using appropriate definitions in terms of exponential functions, show that

$$\operatorname{sech}^2 x \equiv 1 - \tanh^2 x. \qquad [4]$$

(b) Solve the equation

$$5\operatorname{sech}^2 x = 11 - 13\tanh x$$

giving your answer as a natural logarithm. [8]

(Haf 2009)

1. Solve the equation

$$\cosh 2\theta = 6\sinh\theta - 3.$$

Give your answers in the form $\ln\left(p + \sqrt{q}\right)$, where p, q are positive integers. [7]

(Haf 2010)

1. The function f is defined for $x \geqslant 0$ by

$$f(x) = \sinh 2x - 14\sinh x + 8x.$$

(a) Show that

$$f'(x) = 2(2\cosh^2 x - 7\cosh x + 3). \quad [2]$$

(b) Show that there is one stationary point on the graph of f. Find its x-coordinate, giving your answer correct to two decimal places. [5]

(c) Obtain an expression for $f''(x)$ and **hence** classify the stationary point as either a maximum or a minimum. [3]

(Haf 2011)

1. Find the positive root of the equation

$$3\tanh^2\theta = 5\operatorname{sech}\theta + 1,$$

giving your answer in the form $\ln(a + \sqrt{b})$, where a, b are positive integers. [8]

(Haf 2012)

1. Show that

$$\int_0^1 x\sinh x \,\mathrm{d}x = \frac{1}{\mathrm{e}}. \quad [6]$$

2. Consider the equation

$$\cosh^2 x = \sinh x + k$$

where k is a constant.

(a) Find the range of values of k for which the equation has no real solution. [4]

(b) Find the positive root of the equation when $k = 3$, giving your answer in the form $\ln(a + \sqrt{b})$, where a, b are positive integers. [3]

(Haf 2013)

1. Determine the two positive roots of the equation

$$\cosh 2x - 7\cosh x + 7 = 0,$$

giving your answers correct to two decimal places. [6]

(Haf 2014)

1. *(a)* Starting with the exponential definition of $\sinh x$, show that

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right).$$ [4]

(b) Solve the equation

$$\cosh 2x = 2\sinh x + 5,$$

giving your answers in the form $\ln\left(a + \sqrt{b}\right)$ where a, b are integers. [5]

(Haf 2015)

1. *(a)* Express $5\cosh\theta + 3\sinh\theta$ in the form $r\cosh(\theta + \alpha)$, $r > 0$, where the values of r and α are to be found. [4]

(b) Hence solve the equation

$$5\cosh\theta + 3\sinh\theta = 10.$$ [4]

(Haf 2016)

4. The function f is defined on the domain [0, ∞) by

$$f(\theta) = \cosh 2\theta - 8\cosh\theta.$$

Consider the equation $f(\theta) = k$, where k is a constant.

(a) Show that the equation has no real roots if $k < -9$. [4]

(b) Solve the equation when $k = -8$, giving your answers correct to two decimal places. [3]

(c) Determine

(i) the value of k for which the equation has a repeated root,

(ii) the set of values of k for which the equation has exactly one real root. [5]

(Haf 2017)

1. Solve the equation

$$2\sinh\theta + \cosh\theta = 2.$$

Give your answer correct to three significant figures. [7]

(Haf 2018)

1. The function f is defined by

$$f(x) = \cosh^3 x - 2\sinh x.$$

(a) Obtain an expression for $f'(x)$. [2]

(b) Determine the x-coordinate of the stationary point on the graph of f. Give your answer correct to three significant figures. [5]