C3: Parth ac Amrediad

Haf 2005

10 $f(x) = x^2 + 1$ Parth $(0, \infty)$

$g(x) = 2x - 3$ Parth $(5, \infty)$

(a) Amrediad $f(x) = (1, \infty)$.

Amrediad $g(x) = (7, \infty)$

(b) Mae $f(1) = 1^2 + 1$

$= 2$

Nid yw 2 ym mharth $g(x)$ felly ni ellir ffurfio $gf(1)$.

(c) $fg(x) = 3x^2 - 6x + 17$

$(g(x))^2 + 1 = 3x^2 - 6x + 17$

$(2x-3)^2 + 1 = 3x^2 - 6x + 17$

$(2x-3)(2x-3) + 1 = 3x^2 - 6x + 17$

$4x^2 - 6x - 6x + 9 + 1 = 3x^2 - 6x + 17$

$4x^2 - 12x + 10 = 3x^2 - 6x + 17$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

Unai $x - 7 = 0$ neu $x + 1 = 0$

$x = 7$ $x = -1$

Nid yw -1 ym mharth $f(x)$ felly'r unig ateb yw

$\underline{x = 7}$

Gaeaf 2006

(9) $f(x) = e^{x}$ Parth $(-\infty, \infty)$

$g(x) = \ln(x^2-4)$ Parth $(2, \infty)$

(a) Parth $fg = (2, \infty)$

(b) $fg(x) = 5$

$e^{g(x)} = 5$

$e^{\ln(x^2-4)} = 5$

$x^2 - 4 = 5$

$x^2 = 9$

$x = \pm\sqrt{9}$

$x = \pm 3$

Nid yw -3 yn mharth $g(x)$ felly'r unig ateb yw $\underline{x=3}$

Haf 2006

(8) $f(x) = x - \frac{1}{x}$ Parth $[1, \infty)$

(a) $f(x) = x - x^{-1}$

$f'(x) = 1 - (-1)x^{-2}$

$f'(x) = 1 + x^{-2}$

$f'(x) = 1 + \frac{1}{x^2}$

Mae x^2 o hyd yn bositif gyda'r parth yn $[1, \infty)$.
Felly mae $1 + \frac{1}{x^2}$ bob amser yn bositif.

Gan fod graddiant $f(x)$ bob amser yn bositif mae gwerth lleiaf $f(x)$ yn digwydd pan fydd $x=1$. Y gwerth lleiaf yw $f(1) = 1 - \frac{1}{1}$

$= 0$

(b) Amrediad $f(x) = [0, \infty)$

(c) $g(x) = 3x^2 + 2$ Parth $[0, \infty)$

$$gf(x) = \frac{3}{x^2} + 2$$

$$3(f(x))^2 + 2 = \frac{3}{x^2} + 8$$

$$3\left(x - \frac{1}{x}\right)^2 + 2 = \frac{3}{x^2} + 8$$

$$3\left(x - \frac{1}{x}\right)\left(x - \frac{1}{x}\right)^2 + 2 = \frac{3}{x^2} + 8$$

$$3\left(x^2 - 1 - 1 + \frac{1}{x^2}\right) + 2 = \frac{3}{x^2} + 8$$

$$3x^2 - 6 + \frac{3}{x^2} + 2 = \frac{3}{x^2} + 8$$

$$3x^2 - 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 4.$$

$$x = \pm\sqrt{4}$$

$$x = \pm 2.$$

Nid yw -2 ym mharth f na g felly'r unig ateb yw $\underline{x = 2}$.

Gaeaf 2007

(10) $f(x) = x + 5$ Parth $(-\infty, \infty)$

$g(x) = |2x+1| + 2$ Parth $(-\infty, \infty)$

$fg(x) > 10$

$g(x) + 5 > 10$

$|2x+1| + 2 + 5 > 10$

$|2x+1| > 3$

Unai $2x + 1 > 3$ neu $2x + 1 < -3$

$2x > 2$ $2x < -4$

$x > 1$ neu $x < -2$

Haf 2007

(8) $f(x) = e^x$ Parth $[0, \infty)$

$g(x) = x^2 + 1$ Parth $(-\infty, \infty)$

(a) Amrediad $f(x) = [1, \infty)$

Amrediad $g(x) = [1, \infty)$

(b) $gf(x) = (f(x))^2 + 1$

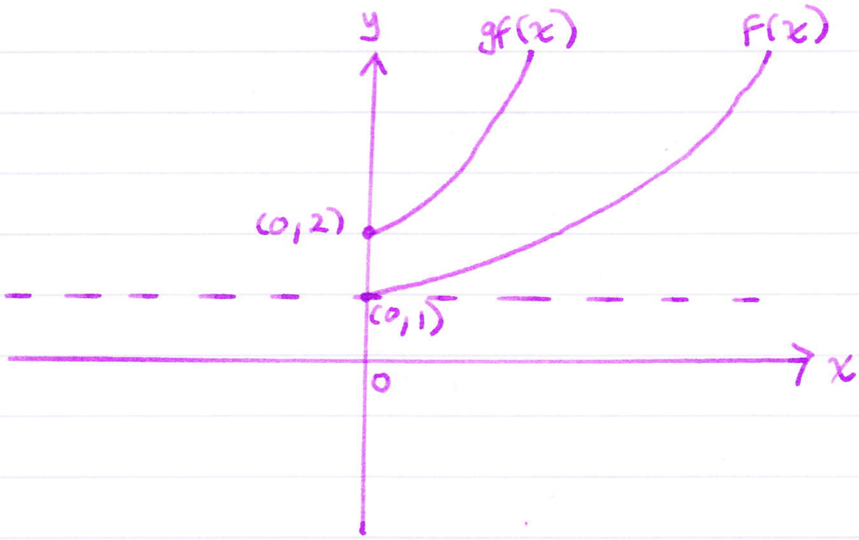
$= (e^x)^2 + 1$

$= e^{2x} + 1$

(c) Parth $gf(x) = [0, \infty)$

Amrediad $gf(x) = [2, \infty)$

(ch)



Gaeaf 2008

(9) $f(x) = \ln x$ Parth $(0, \infty)$
$g(x) = e^{4x}$ Parth $(-\infty, \infty)$

(a) $fg(x) = \ln(g(x))$
$= \ln(e^{4x})$
$= 4x$

(b) $gf(x) = e^{4f(x)}$.
$= e^{4\ln x}$
$= e^{\ln x^4}$
$= x^4$

Haf 2008

(10) $f(x) = 2e^{x}$ Parth $(-\infty, \infty)$
$g(x) = 3\ln x$ Parth $[1, \infty)$

(a) $f(-1) = 2e^{-1}$
$= 0.7357588823\ldots$

Nid yw $0.7357\cdots$ ym mharth $g(x)$ felly nid yw $gf(-1)$ yn bodoli.

(b) $fg(x) = 2e^{g(x)}$
$= 2e^{3\ln x}$
$= 2e^{\ln x^3}$
$= 2x^3$

Parth $fg(x) = [1, \infty)$
Amrediad $fg(x) = [2, \infty)$

<u>Gaeaf 2009</u>

10 $f(x) = 2x - K$ Parth $[1, \infty)$

(a) Amrediad $K = [2-K, \infty)$

$g(x) = 3x^2 + 4$ Parth $[0, \infty)$

(b) I ffurfio'r ffwythiant $gf(x)$ ar gyfer bob rhif ym mharth $f(x)$ rhaid bod $2-K \geq 0$
$2 \geq K$
$K \leq 2$

Felly'r gwerth mwyaf ar gyfer K yw 2.

(c) $gf(2) = 31$.
$g[f(2)] = 31$
$g[2 \times 2 - K] = 31$
$g[4-K] = 31$
$3(4-K)^2 + 4 = 31$
$3(4-K)(4-K) + 4 = 31$
$3(16-4K-4K+K^2) + 4 = 31$
$3(16-8K+K^2) + 4 = 31$
$48 - 24K + 3K^2 + 4 = 31$
$3K^2 - 24K + 48 + 4 - 31 = 0$
$3K^2 - 24K + 21 = 0$

$K^2 - 8K + 7 = 0$
$(K-1)(K-7) = 0$
Unai $K-1=0$ neu $K-7=0$
$K=1$ $K=7$
ond rhaid bod $K \leq 2$
felly'r ateb yw <u>$K=1$</u>

Haf 2009

(9) $f(x) = 3e^{2x}$ Parth $(-\infty, \infty)$

$g(x) = \ln 4x$ Parth $(0, \infty)$

(a) Parth $fg(x) = (0, \infty)$

Amrediad $g(x) = (-\infty, \infty)$

Amrediad $fg(x) = (0, \infty)$

(b)
$$fg(x) = 12$$
$$3e^{2g(x)} = 12$$
$$3e^{2\ln 4x} = 12$$
$$e^{2\ln 4x} = 4$$
$$e^{\ln(4x)^2} = 4$$
$$e^{\ln 16x^2} = 4$$
$$16x^2 = 4$$
$$x^2 = \frac{1}{4}$$
$$x = \pm\sqrt{\frac{1}{4}}$$
$$x = \pm\frac{1}{2}$$

Ond parth $fg(x) = (0, \infty)$ felly $\underline{x = \frac{1}{2}}$ yn unig.

Gaeaf 2010

(10) $f(x) = x^2 - 1$ Parth $(0, \infty)$

$g(x) = 2x - 1$ Parth $(2, \infty)$

(a) Amrediad $f(x) = (-1, \infty)$

Amrediad $g(x) = (3, \infty)$

(b) $f(1) = 1^2 - 1$

$= 0$

Nid yw 0 ym mharth $g(x)$ felly nid yw'n bosibl ffurfio $gf(1)$.

(c) (i) $fg(x) = (g(x))^2 - 1$

$= (2x-1)^2 - 1$

$= (2x-1)(2x-1) - 1$

$= 4x^2 - 2x - 2x + 1 - 1$

$= 4x^2 - 4x$

$= 4x(x-1)$

(ii) Parth $fg(x) = (2, \infty)$

Amrediad $fg(x) = (8, \infty)$

<u>Haf 2010</u>

10 $f(x) = \sqrt{x+4}$ Parth $[-3, \infty)$

$g(x) = 2x^2 - 3$ Parth $(-\infty, \infty)$

(a) Amrediad $f(x) = [1, \infty)$

Amrediad $g(x) = [-3, \infty)$

(b) $gf(x) = 2(f(x))^2 - 3$

$= 2(\sqrt{x+4})^2 - 3$

$= 2(x+4) - 3$

$= 2x + 8 - 3$

$= 2x + 5$

(c) $fg(x) = 17$

$\sqrt{g(x)+4} = 17$

$\sqrt{2x^2 - 3 + 4} = 17$

$\sqrt{2x^2 + 1} = 17$

$2x^2 + 1 = 289$

$2x^2 = 288$

$x^2 = 144$

$x = \pm\sqrt{144}$

$x = \pm 12$

Gaeaf 2011

(10) $f(x) = e^x$ Parth $[0, \infty)$
$g(x) = 4x^3 + 7$ Parth $(-\infty, \infty)$

(a) $gf(x) = 4(f(x))^3 + 7$
$= 4(e^x)^3 + 7$
$= 4e^{3x} + 7$

$gf(0) = 4e^{3\times 0} + 7$
$= 4\times 1 + 7$
$= 11$

(b) Parth $gf(x) = [0, \infty)$
Amrediad $gf(x) = [11, \infty)$

(c) (i) $gf(x) = 18$
$4e^{3x} + 7 = 18$
$4e^{3x} = 11$
$e^{3x} = 2.75$
$3x = \ln(2.75)$
$x = \frac{\ln(2.75)}{3}$
$x = 0.337$ i 3 lle degol

(ii) $gf(x) = K$
$4e^{3x} + 7 = K$
Nid oes datrysiad os yw $K < 11$
gan mai amrediad $gf(x)$ yw $[11, \infty)$.
Felly nid oes datrysiad os yw ee. $K = 10$.

Haf 2011

(10) $f(x) = x^2 - 19$ Parth $(-\infty, 0)$

$g(x) = 1 - \frac{1}{2}x$ Parth $(6, \infty)$

(a) Amrediad $f(x) = (-19, \infty)$

Amrediad $g(x) = (-\infty, -2)$

$f(-2) = (-2)^2 - 19$

$= 4 - 19$

$= -15$

(b) Parth $fg(x) = (6, \infty)$

Amrediad $fg(x) = (-15, \infty)$

(c) (i) $fg(x) = (g(x))^2 - 19$

$= (1 - \frac{1}{2}x)^2 - 19$

$= (1 - \frac{1}{2}x)(1 - \frac{1}{2}x) - 19$

$= 1 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{4}x^2 - 19$

$= \frac{1}{4}x^2 - x - 18$

(ii) $fg(x) = 2x - 26$

$\frac{1}{4}x^2 - x - 18 = 2x - 26$

$\frac{1}{4}x^2 - 3x + 8 = 0$

$x^2 - 12x + 32 = 0$

$(x - 4)(x - 8) = 0$

Unai $x - 4 = 0$ neu $x - 8 = 0$

$x = 4$ $x = 8$

ond nid yw 4 ym mharth $fg(x)$

felly'r unig ateb yw $\underline{x = 8}$

Gaeaf 2012

(10) $f(x) = 3x + K$ Parth $[1, \infty)$

(a) Amrediad $f(x) = [3+K, \infty)$

$g(x) = x^2 - 6$ Parth $[-2, \infty)$

(b) I ffurfio $gf(x)$ rhaid bod $3+K \geqslant -2$
$K \geqslant -5$

Felly gwerth lleiaf K yw -5.

(c) (i) $gf(x) = (f(x))^2 - 6$
$= (3x+K)^2 - 6$
$= (3x+K)(3x+K) - 6$
$= 9x^2 + 3xK + 3xK + K^2 - 6$
$= 9x^2 + 6xK + K^2 - 6$

(ii) $gf(2) = 3$
$9(2)^2 + 6(2)K + K^2 - 6 = 3$
$36 + 12K + K^2 - 6 = 3$
$K^2 + 12K + 27 = 0$
$(K+3)(K+9) = 0$
unai $K+3=0$ neu $K+9=0$
$K=-3$ $K=-9$
ond rhaid bod $K \geqslant -5$ felly'r unig ateb yw $\underline{K=-3}$

Haf 2012

(10) $g(x) = \sqrt{3x^2+7}$ Parth $(-\infty, \infty)$

$\underline{gg(x) = 8}$

$\sqrt{3(g(x))^2+7} = 8$

$3(g(x))^2+7 = 64$

$3(g(x))^2 = 57$

$(g(x))^2 = 19$

$(\sqrt{3x^2+7})^2 = 19$

$3x^2+7 = 19$

$3x^2 = 12$

$x^2 = 4$

$x = \pm\sqrt{4}$

$\underline{x = \pm 2}$

Gaeaf 2013

(9) (a) $f(x) = x^2 - 25$ Parth $(-\infty, \infty)$

$g(x) = 2x - 3$ Parth $(0, \infty)$

(i) Parth $fg(x) = (0, \infty)$

(ii) Amrediad $fg(x) = [-25, \infty)$

Amrediad $g(x) = (-3, \infty)$. $f(-3) = -16$ and $f(0) = -25$

(iii) $fg(x) = (g(x))^2 - 25$

$= (2x-3)^2 - 25$

$= (2x-3)(2x-3) - 25$

$= 4x^2 - 6x - 6x + 9 - 25$

$= 4x^2 - 12x - 16$

(iv) $fg(x)=0$

$$4x^2-12x-16=0$$

$$x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

Unai $x-4=0$ neu $x+1=0$

$x=4$ $\qquad$ $x=-1$

Ond nid yw -1 ym mharth $fg(x)$ felly'r unig ateb yw $x=4$.

(b) $h(x)=\dfrac{2x+7}{5x-2}$

(i) $$hh(x)=\frac{2h(x)+7}{5h(x)-2}$$

$$=\frac{2\left(\frac{2x+7}{5x-2}\right)+7}{5\left(\frac{2x+7}{5x-2}\right)-2}$$

$$=\frac{\frac{4x+14}{5x-2}+7\left(\frac{5x-2}{5x-2}\right)}{\frac{10x+35}{5x-2}-2\left(\frac{5x-2}{5x-2}\right)}$$

$$=\frac{\frac{4x+14+35x-14}{5x-2}}{\frac{10x+35-10x+4}{5x-2}}$$

$$=\left(\frac{4x+14+35x-14}{\cancel{5x-2}}\right)\times\left(\frac{\cancel{5x-2}}{10x+35-10x+4}\right)$$

$$=\frac{39x}{39}$$

$$=x. \quad ✓$$

(ii) Gan fod $hh(x)=x$

mae $h^{-1}(x)=h(x)$

$$h^{-1}(x)=\frac{2x+7}{5x-2}$$

Haf 2013

(11) $f(x) = \ln x$ Parth $(0, \infty)$

$g(x) = \tan x$ Parth $(0, \frac{\pi}{4}]$

(a) (i) Parth $fg(x) = (0, \frac{\pi}{4}]$

(ii) Amrediad $g(x) = (0, 1]$

Amrediad $fg(x) = (-\infty, 0]$

$g(\frac{\pi}{4}) = \tan(\frac{\pi}{4})$
$= 1$

$f(1) = \ln(1)$
$= 0$

(b) (i) $fg(x) = \ln(g(x))$
$= \ln(\tan x)$.

$fg(x) = -0.4$

$\ln(\tan x) = -0.4$

$\tan x = e^{-0.4}$

$(e^{-0.4} \approx 0.67)$

$x = \tan^{-1}(e^{-0.4})$

$\underline{x = 0.59}$ i 2 le degol

(ii) $fg(x) = K$

Nid oes datrysiad os nad yw K yn amrediad $fg(x)$. Gan fod amrediad $fg(x) = (-\infty, 0]$ nid oes datrysiad os yw $K > 0$, e.e. $K = 2$.

Gaeaf 2014

(10) $f(x) = \sqrt{x^2+5}$ parth $(0, \infty)$

$g(x) = \dfrac{-4}{x+1}$ parth $(-\infty, -2)$

(a) $g(x) = -4(x+1)^{-1}$

$g'(x) = -4(-1)(x+1)^{-2}(1)$

$= \dfrac{4}{(x+1)^2}$

Dim ots beth yw gwerth x, mae $(x+1)^2$ yn bositif, felly mae $g'(x)$ o hyd yn bositif. Gan fod graddiant $g(x)$ felly o hyd yn bositif, rhaid bod g yn ffwythiant cynyddol.

(b) $g(-2) = \dfrac{-4}{-2+1} = \dfrac{-4}{-1} = 4$

$g(-11) = \dfrac{-4}{-11+1} = \dfrac{-4}{-10} = \dfrac{4}{10}$

$g(-101) = \dfrac{-4}{-101+1} = \dfrac{-4}{-100} = \dfrac{4}{100}$.

Amrediad $g(x)$ yw $(0, 4)$.

(c) Parth $fg(x)$ yw parth $g(x)$, sef $(-\infty, -2)$

Amrediad $fg(x)$ yw $(\sqrt{5}, \sqrt{21})$

(gan fod $f(0) = \sqrt{0^2+5} = \sqrt{5}$ a $f(4) = \sqrt{4^2+5} = \sqrt{21}$.)

(ch) (i) $fg(x) = \sqrt{g(x)^2+5}$

$= \sqrt{\left(\dfrac{-4}{x+1}\right)^2+5}$

$= \sqrt{\dfrac{16}{(x+1)^2}+5}$

(ii) $\sqrt{fg(x)} = 3$

$$\sqrt{\frac{16}{(x+1)^2} + 5} = 3$$

$$\frac{16}{(x+1)^2} + 5 = 9$$

$$\frac{16}{(x+1)^2} = 4$$

$$16 = 4(x+1)^2$$

$$4 = (x+1)^2$$

$$(x+1)^2 = 4$$

$$(x+1)(x+1) = 4$$

$$x^2 + x + x + 1 = 4$$

$$x^2 + 2x + 1 = 4$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

Unai $x+3=0$ neu $x-1=0$

$\underline{\underline{x=-3}}$ $\qquad$ $x=1$

$\downarrow$

Ddim ym mharth $fg(x)$

NEU $x+1 = \pm\sqrt{4}$

$x+1 = \pm 2$

Unai $x+1=2$ neu $x+1=-2$

$x=1$ $\qquad$ $\underline{\underline{x=-3}}$

$\downarrow$

Ddim ym mharth $fg(x)$

C3 Haf 2014

(10) $f(x) = x^2 + Kx - 8$ Parth $[-2, \infty)$

$g(x) = Kx - 4$ Parth $[2, \infty)$

Mae $K > 0$.

(a) $g(2) = 2K - 4$

Felly amrediad g yw $[2K-4, \infty)$

(b) (i) I ffurfio'r ffwythiant fg rhaid bod

$$2K - 4 \geqslant -2$$
$$2K \geqslant -2 + 4$$
$$2K \geqslant 2$$
$$K \geqslant 1$$

Felly gwerth lleiaf K yw 1.

(ii)
$$\begin{aligned} fg(x) &= g(x)^2 + Kg(x) - 8 \\ &= (Kx-4)^2 + K(Kx-4) - 8 \\ &= K^2x^2 - 8Kx + 16 + K^2x - 4K - 8 \\ &= K^2x^2 + (K^2 - 8K)x + 8 - 4K \\ &= K^2x^2 + K(K-8)x + 8 - 4K \end{aligned}$$

(iii) $fg(3) = 0$.

$$K^2(3^2) + K(K-8)(3) + 8 - 4K = 0$$
$$9K^2 + 3K^2 - 24K + 8 - 4K = 0$$
$$12K^2 - 28K + 8 = 0$$
$$3K^2 - 7K + 2 = 0$$
$$(3K - 1)(K - 2) = 0$$

Unai $3K - 1 = 0$ neu $K - 2 = 0$

$3K = 1$ $\quad$ $K = 2$

$K = \frac{1}{3}$

Ond gwerth lleiaf K yw 1 felly rhaid bod $\underline{\underline{K = 2}}$

C3 Haf 2015

10) a) Gadewch i $h = x^2 + 5$, $K = x^2 + 4$.
Y deilliadau yw $\frac{dh}{dx} = 2x$ a $\frac{dK}{dx} = 2x$.
Mae'r deilliadau'n hafal ond nid yw'r ffwythiannau h a K eu hunain yn hafal, felly mae'r gosodiad yn anghywir.

b) $f(x) = 2\ln(4x+5) + 3$ parth $[7, 60]$
$g(x) = e^x$ parth $[9, \infty)$

(i) $f(x) = 2\ln(4x+5)+3$
$$y = 2\ln(4x+5)+3$$
$$y-3 = 2\ln(4x+5)$$
$$\frac{y-3}{2} = \ln(4x+5)$$
$$e^{\frac{y-3}{2}} = 4x+5$$
$$\frac{e^{\frac{y-3}{2}} - 5}{4} = x$$

Felly $f^{-1}(x) = \dfrac{e^{\frac{x-3}{2}} - 5}{4}$

(ii) Parth $f^{-1}(x)$ yw amrediad $f(x)$.
$f(7) = 2\ln(4\times 7+5)+3$
$f(7) = 2\ln(33)+3$
$f(7) = 9.99301 5123$

$f(60) = 2\ln(4\times 60+5)+3$
$f(60) = 2\ln(245)+3$
$f(60) = 14.00251642$

I'r cyfanrif agosaf, y parth ar gyfer $f^{-1}(x)$ yw $[10, 14]$

(iii) $gf(x) = e^{f(x)}$
$$= e^{2\ln(4x+5)+3}$$
$$= e^{\ln(4x+5)^2+3}$$
$$= e^{\ln(4x+5)^2} \times e^3$$
$$= (4x+5)^2 \times e^3$$
$$= e^3(4x+5)^2$$

C3 Haf 2016

(9) $f(x) = e^{4-\frac{x}{3}} + 8$ Parth $(-\infty, 12]$

a) $y = e^{4-\frac{x}{3}} + 8$

$y - 8 = e^{4-\frac{x}{3}}$

$\ln(y-8) = 4 - \frac{x}{3}$

$\frac{x}{3} = 4 - \ln(y-8)$

$x = 12 - 3\ln(y-8)$

Felly $f^{-1}(x) = 12 - 3\ln(x-8)$

b) $f(12) = e^{4-\frac{12}{3}} + 8$

$= e^{4-4} + 8$

$= e^{0} + 8$

$= 1 + 8$

$= 9$

$f(-\infty) = e^{4-\frac{-\infty}{3}} + 8$

$= e^{4+\frac{\infty}{3}} + 8$

$\to e^{\infty} + 8$

$\to \infty$

Felly amrediad $f(x) = [9, \infty)$

parth $f^{-1}(x) = [9, \infty)$

C3 Haf 2017

(9) $f(x) = 4x + k$ Parth $[2, \infty)$

(a) $f(2) = 4(2) + k$
$= 8 + k$

$f(\infty) = 4(\infty) + k$
$= \infty + k$
$= \infty$

Amrediad f yw $[8+k, \infty)$

$g(x) = x^2 - 9$ Parth $[-3, \infty)$

(b) Parth $gf(x)$ yw amrediad $f(x)$.
I gael ei ffurfio, rhaid bod $8 + k \geq -3$
$k \geq -11$

Felly gwerth lleiaf k yw $\underline{\underline{-11}}$

(c) (i) $gf(x) = f(x)^2 - 9$
$= (4x+k)^2 - 9$
$= (4x+k)(4x+k) - 9$
$= 16x^2 + 4xk + 4xk + k^2 - 9$
$gf(x) = 16x^2 + 8xk + k^2 - 9$

(ii) $gf(2) = 7$
$16(2^2) + 8(2)k + k^2 - 9 = 7$
$16(4) + 16k + k^2 - 9 - 7 = 0$
$k^2 + 16k + 48 = 0$
$(k+12)(k+4) = 0$
Naill ai $k + 12 = 0$ neu $k + 4 = 0$
$k = -12$ $k = -4$

Ond $k \geq -11$ felly rhaid bod $\underline{\underline{k = -4}}$

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C3 Haf 2018

10) $f(x) = x^2 + 2x - 24$ Parth $(-\infty, \infty)$
$g(x) = 5 - 3x$ Parth $(0, \infty)$

a) Parth $fg = (0, \infty)$

b) $fg(x) = 200$
$f[5-3x] = 200$
$(5-3x)^2 + 2(5-3x) - 24 = 200$
$(5-3x)(5-3x) + 2(5-3x) - 24 = 200$
$25 - 15x - 15x + 9x^2 + 10 - 6x - 24 = 200$
$9x^2 - 36x + 11 = 200$
$9x^2 - 36x - 189 = 0$
$x^2 - 4x - 21 = 0$
$(x+3)(x-7) = 0$
Naill ai $x+3=0$ neu $x-7=0$
$x=-3$ $\underline{x=7}$
(Ddim yn y parth)

C3 Mai 2019

8) $f(x) = \dfrac{4x+3}{7-5x}$

a) Gadewch i $y = \dfrac{4x+3}{7-5x}$

$$7y - 5xy = 4x + 3$$
$$7y - 3 = 4x + 5xy$$
$$x(4+5y) = 7y - 3$$
$$x = \frac{7y-3}{4+5y}$$

Felly $f^{-1}(x) = \dfrac{7x-3}{4+5x}$

b) Parth $f^{-1}(x)$ yw amrediad $f(x)$

$$f(1) = \frac{4(1)+3}{7-5(1)}$$
$$f(1) = \frac{7}{2}$$
$$f(1) = 3.5$$

$$f(-\infty) = \frac{4(-\infty)+3}{7-5(\infty)} \qquad \left(\times \frac{\infty}{\infty}\right)$$
$$= \frac{-4 + \frac{3}{\infty}}{\frac{7}{\infty} + 5}$$
$$\rightarrow \frac{-4}{5}$$

Felly parth $f^{-1}(x)$ yw $(-0.8, 3.5]$

c) i) $f^{-1}(0.5) = \dfrac{7(0.5) - 3}{4 + 5(0.5)}$

$= \dfrac{3.5 - 3}{4 + 2.5}$

$= \dfrac{0.5}{6.5} = \dfrac{1}{13}$

ii) Os yw $f^{-1}(0.5) = \frac{1}{13}$

$\Rightarrow f\left(\frac{1}{13}\right) = 0.5$

Ar amnewid $1/13$ i mewn i $f(x)$ cawn:

$f\left(\frac{1}{13}\right) = \dfrac{4\left(\frac{1}{13}\right) + 3}{7 - 5\left(\frac{1}{13}\right)}$

$= \dfrac{\frac{4}{13} + 3}{7 - \frac{5}{13}}$

$= \dfrac{43/13}{86/13}$

$= \dfrac{43}{86} = 0.5$