

C3: Differu: Rheolau

Haf 2005

(4)

(a) $x^2 + 2xy + 3y^2 = 12$

$$2x + (2x)\left(\frac{dy}{dx}\right) + (2)y + 6y\left(\frac{dy}{dx}\right) = 0$$

$$2x + 2y + \frac{dy}{dx}(2x + 6y) = 0$$

$$\frac{dy}{dx}(2x + 6y) = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 6y}$$

$$\frac{dy}{dx} = -\frac{x + y}{x + 3y}$$

(b) $x = 2t^4, y = 3t^2$

(i) $\frac{dx}{dt} = 8t^3 \quad \frac{dy}{dt} = 6t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 6t \times \frac{1}{8t^3}$$

$$\frac{dy}{dx} = \frac{3}{4t^2}$$

(ii) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$$= \frac{dt}{dx} \times \frac{d}{dt}\left(\frac{dy}{dx}\right)$$

$$= \frac{1}{8t^3} \times \frac{d}{dt}\left(\frac{3}{4}t^{-2}\right)$$

$$= \frac{1}{8t^3} \times (-2)\left(\frac{3}{4}\right)t^{-3}$$

$$= \frac{1}{8t^3} \times -\frac{3}{2t^3} \rightarrow = \frac{-3}{16t^6}$$

Graef 2006

③

$$(a) \quad y^4 + x^3y = x^2 + 4x - 3$$

$$4y^3 \frac{dy}{dx} + x^3 \left(\frac{dy}{dx} \right) + (3x^2)y = 2x + 4$$

$$\frac{dy}{dx} (4y^3 + x^3) = 2x + 4 - 3x^2y$$

$$\frac{dy}{dx} = \frac{2x + 4 - 3x^2y}{4y^3 + x^3}$$

$$\text{OS gw } x=2, y=1 \text{ gna } \frac{dy}{dx} = \frac{2 \times 2 + 4 - 3 \times 2^2 \times 1}{4 \times 1^3 + 2^3}$$

$$\frac{dy}{dx} = \frac{-4}{12}$$

$$\frac{dy}{dx} = -\frac{1}{3}$$

$$(b) \quad x = 2t^3 \quad y = 3t^4$$

$$(i) \quad \frac{dx}{dt} = 6t^2 \quad \frac{dy}{dt} = 12t^3$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 12t^3 \times \frac{1}{6t^2}$$

$$\frac{dy}{dx} = 2t$$

$$(ii) \quad \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$= \frac{1}{6t^2} \times \frac{d}{dt} (2t)$$

$$= \frac{2}{6t^2} = \frac{1}{3t^2}$$

Haf 2006

(3) (a) $x = \cos t \quad y = \sin 2t$

$$\frac{dx}{dt} = -\sin t \quad \frac{dy}{dt} = 2\cos 2t$$
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos 2t}{-\sin t}$$
$$\frac{dy}{dx} = -\frac{2\cos 2t}{\sin t}$$

(b) $x^4 + 2x^2y + y^2 = 21$

$$4x^3 + 2x^2(\frac{dy}{dx}) + (4x)y + 2y(\frac{dy}{dx}) = 0$$
$$\frac{dy}{dx}(2x^2 + 2y) + 4x^3 + 4xy = 0$$
$$\frac{dy}{dx}(2x^2 + 2y) = -4x^3 - 4xy$$
$$\frac{dy}{dx} = \frac{-4x^3 - 4xy}{2x^2 + 2y}$$
$$\frac{dy}{dx} = \frac{-2x^3 - 2xy}{x^2 + y}$$
$$\frac{dy}{dx} = \frac{-2x(x^2 + y)}{(x^2 + y)}$$
$$\frac{dy}{dx} = -2x$$

CYNLLUN MARCIO
YN ANGHYWIR

Graef 2007

⑥ (a) $x^3 + x^2y + y^4 = 17$

$$3x^2 + x^2\left(\frac{dy}{dx}\right) + (2x)y + 4y^3\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} \left(x^2 + 4y^3 \right) + 3x^2 + 2xy = 0$$

$$\frac{dy}{dx} \left(x^2 + 4y^3 \right) = -3x^2 - 2xy$$

$$\frac{dy}{dx} = \frac{-3x^2 - 2xy}{x^2 + 4y^3}$$

(b) $x = t^3$ $y = t^2 + 1$

$$\frac{dx}{dt} = 3t^2 \quad \frac{dy}{dt} = 2t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 2t \times \frac{1}{3t^2}$$

$$\frac{dy}{dx} = \frac{2}{3t}$$

(ii) $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$

$$= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$= \frac{1}{3t^2} \times \frac{d}{dt} \left(\frac{2}{3}t^{-1} \right)$$

$$= \frac{1}{3t^2} \times (-1) \left(\frac{2}{3} \right) t^{-2}$$

$$= \frac{1}{3t^2} \times -\frac{2}{3t^2}$$

$$= \frac{-2}{9t^4}$$

Haf 2007

(3) a) $x = 5t^2 \quad y = t^5 + \frac{20t^3}{3}$

(i) $\frac{dx}{dt} = 10t \quad \frac{dy}{dt} = 5t^4 + 3\left(\frac{20}{3}\right)t^2$
 $\frac{dy}{dt} = 5t^4 + 20t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (5t^4 + 20t^2) \times \frac{1}{10t}$$

$$\frac{dy}{dx} = \frac{1}{2}t^3 + 2t$$

(ii) $\frac{dy}{dx} = 1 \Rightarrow 1 = \frac{1}{2}t^3 + 2t$
 $2 = t^3 + 4t$
 $0 = t^3 + 4t - 2$
 $t^3 + 4t - 2 = 0 \quad \checkmark$

(5)

$$3y^2 + x^2y^3 + x^4 - x^2 - 11 = 0$$

$$6y\left(\frac{dy}{dx}\right) + x^2(3y^2 \times \frac{dy}{dx}) + (2x)y^3 + 4x^3 - 2x = 0$$

$$\frac{dy}{dx}(6y + 3x^2y^2) + 2xy^3 + 4x^3 - 2x = 0$$

$$\frac{dy}{dx}(6y + 3x^2y^2) = 2x - 2xy^3 - 4x^3$$

$$\frac{dy}{dx} = \frac{2x - 2xy^3 - 4x^3}{6y + 3x^2y^2}$$

os yw $x=2, y=-1$ yna $\frac{dy}{dx} = \frac{2 \times 2 - 2 \times 2 \times (-1)^3 - 4 \times 2^3}{6 \times -1 + 3 \times 2^2 \times (-1)^2}$

$$\frac{dy}{dx} = \frac{-24}{6}$$

$$\frac{dy}{dx} = -4$$

Gaeaf 2008

③ (a) $x = e^t \quad y = e^{2t} + 5$

$$\frac{dx}{dt} = 4e^3 \quad \frac{dy}{dt} = 2e^{2t}$$
$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
$$\frac{dy}{dx} = 2e^{2t} \times \frac{1}{4e^3}$$
$$\frac{dy}{dx} = \frac{e^{2t}}{2e^3}$$

(b) $x^4 + \sin y + x^2y^3 = 9$

$$4x^3 + \cos y \left(\frac{dy}{dx} \right) + x^2(3y^2) \frac{dy}{dx} + (2x)y^3 = 0$$
$$\frac{dy}{dx} (\cos y + 3x^2y^2) + 4x^3 + 2xy^3 = 0$$
$$\frac{dy}{dx} (\cos y + 3x^2y^2) = -4x^3 - 2xy^3$$
$$\frac{dy}{dx} = \frac{-4x^3 - 2xy^3}{\cos y + 3x^2y^2}$$
$$\frac{dy}{dx} = \frac{-2x(2x^2 + y^3)}{\cos y + 3x^2y^2}$$

Haf 2008

$$\textcircled{3} \quad x^2 + xsiny + y^3 = \pi^3 + 1$$

$$2x + x(\cos y) \frac{dy}{dx} + \sin y + (3y^2) \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x\cos y + 3y^2) + 2x + \sin y = 0$$

$$\frac{dy}{dx}(x\cos y + 3y^2) = -2x - \sin y$$

$$\frac{dy}{dx} = \frac{-2x - \sin y}{x\cos y + 3y^2}$$

$$\text{os yw } x=1, y=\pi \text{ yna } \frac{dy}{dx} = \frac{-2 \times 1 - \sin \pi}{1 \times \cos \pi + 3\pi^2}$$

RADIAN
MODE

$$\frac{dy}{dx} = \frac{-2}{-1 + 3\pi^2}$$

$$\frac{dy}{dx} \approx -0.0699 ; 4 \text{ lie deg of}$$

\textcircled{4}

$$x = \ln t \quad y = e^{2t}$$

$$\text{(a)} \quad \frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 2e^{2t}$$

$$\text{(b)} \quad \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$\frac{dy}{dx} = 2e^{2t} \times t$$

$$= t \times \frac{d}{dt} \left(2te^{2t} \right)$$

$$\frac{dy}{dx} = 2te^{2t} \quad \checkmark$$

$$= t \times (2t(2e^{2t}) + 2e^{2t})$$

$$= 2e^{2t}(2t^2 + t)$$

$$= 2te^{2t}(2t + 1)$$

Gaeaf 2009

③

$$(a) \quad x^2 + 3xy + 2y^2 - 2x = 13$$

$$2x + 3x\left(\frac{dy}{dx}\right) + (3)y + 4y\left(\frac{dy}{dx}\right) - 2 = 0$$

$$\frac{dy}{dx}(3x + 4y) + 2x + 3y - 2 = 0$$

$$\frac{dy}{dx}(3x + 4y) = 2 - 2x - 3y$$

$$\frac{dy}{dx} = \frac{2 - 2x - 3y}{3x + 4y}$$

$$\text{os yw } x=1, y=2 \quad \text{yna} \quad \frac{dy}{dx} = \frac{2 - 2 \times 1 - 3 \times 2}{3 \times 1 + 4 \times 2}$$

$$\frac{dy}{dx} = \frac{-6}{11}$$

$$(b) \quad x = 2e^t + 6 \quad y = 4e^{2t} + 3e^t + 1$$

$$\frac{dx}{dt} = 2e^t \quad \frac{dy}{dt} = 8e^{2t} + 3e^t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (8e^{2t} + 3e^t) \times \frac{1}{2e^t}$$

$$\frac{dy}{dx} = \frac{8e^{2t}}{2e^t} + \frac{3e^t}{2e^t}$$

$$\frac{dy}{dx} = 4e^t + \frac{3}{2}$$

$$\text{os yw } \frac{dy}{dx} = b \quad \text{yna} \quad b = 4e^t + \frac{3}{2}$$

$$4.5 = 4e^t$$

$$1.125 = e^t$$

$$\ln(1.125) = \ln(e^t)$$

$$t = \ln(1.125)$$

$$t = 0.118 \text{ i } 3.11^\circ \text{ deg}$$

Haf 2009

③

(a) $x^3 + y^2 + x \tan 2y = 8$

$$3x^2 + 2y\left(\frac{dy}{dx}\right) + x(2\sec^2(2y))\left(\frac{dy}{dx}\right) + \tan 2y = 0$$

$$\frac{dy}{dx}\left(2y + 2x\sec^2(2y)\right) + 3x^2 + \tan 2y = 0$$

$$\frac{dy}{dx}\left(2y + 2x\sec^2(2y)\right) = -3x^2 - \tan 2y$$

$$\frac{dy}{dx} = \frac{-3x^2 - \tan 2y}{2y + 2x\sec^2(2y)}$$

(b) $x = 3t + t^2$ $y = \frac{1+4t}{3+2t}$

$$\frac{dx}{dt} = 3+2t \quad \frac{dy}{dt} = \frac{(3+2t)(4) - (1+4t)(2)}{(3+2t)^2}$$

$$\frac{dy}{dt} = \frac{12+8t - 2-8t}{(3+2t)^2}$$

$$\frac{dy}{dt} = \frac{10}{(3+2t)^2}$$

(ii) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{dy}{dx} = \frac{10}{(3+2t)^2} \times \frac{1}{(3+2t)}$$

$$\frac{dy}{dx} = \frac{10}{(3+2t)^3}$$

Gceaf 2010

(3)

$$(a) \quad y^3 + 2x^3y = 3x^2 + 4x - 3$$

$$3y^2\left(\frac{dy}{dx}\right) + 2x^3\left(\frac{dy}{dx}\right) + 6x^2(y) = 6x + 4$$

$$\frac{dy}{dx} (3y^2 + 2x^3) + 6x^2y = 6x + 4$$

$$\frac{dy}{dx} (3y^2 + 2x^3) = 6x + 4 - 6x^2y$$

$$\frac{dy}{dx} = \frac{6x + 4 - 6x^2y}{3y^2 + 2x^3}$$

$$\text{OS yw } x=2, y=1 \text{ yma } \frac{dy}{dx} = \frac{6 \times 2 + 4 - 6 \times 2^2 \times 1}{3 \times 1^2 + 2 \times 2^3}$$

$$\frac{dy}{dx} = -\frac{8}{19}$$

$$(b) \quad x = 3t^2 \quad y = 4t^3 + t^6$$

$$\frac{dx}{dt} = 6t \quad \frac{dy}{dt} = 12t^2 + 6t^5$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (12t^2 + 6t^5) \times \frac{1}{6t}$$

$$\frac{dy}{dx} = 2t + t^4$$

$$(ii) \quad \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$= \frac{1}{6t} \times \frac{d}{dt} (2t + t^4)$$

$$= \frac{1}{6t} \times (2 + 4t^3)$$

$$\rightarrow \frac{d^2y}{dx^2} = \frac{1}{3t} + \frac{2t^2}{3}$$

Haf 2010

(3)

(a) $y^4 + 4x^2y = 3x^3 - 5x$

$$4y^3\left(\frac{dy}{dx}\right) + 4x^2\left(\frac{dy}{dx}\right) + (8x)y = 9x^2 - 5$$

$$\frac{dy}{dx}(4y^3 + 4x^2) + 8xy = 9x^2 - 5$$

$$\frac{dy}{dx}(4y^3 + 4x^2) = 9x^2 - 8xy - 5$$

$$\frac{dy}{dx} = \frac{9x^2 - 8xy - 5}{4y^3 + 4x^2}$$

(b) $x = 4t + \cos 2t$ $y = \sin 3t$

$$\frac{dx}{dt} = 4 + 2\sin 2t \times (-1) \quad \frac{dy}{dt} = 3\cos 3t$$

$$\frac{dx}{dt} = 4 - 2\sin 2t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 3\cos 3t \times \frac{1}{4 - 2\sin 2t}$$

$$\frac{dy}{dx} = \frac{3\cos 3t}{4 - 2\sin 2t}$$

OS yw $t = \frac{\pi}{12}$ yna $\frac{dy}{dx} = \frac{3\cos(3 \times \frac{\pi}{12})}{4 - 2\sin(2 \times \frac{\pi}{12})}$

$$\frac{dy}{dx} = \frac{\frac{3\sqrt{2}}{2}}{3}$$

$$\frac{dy}{dx} = \frac{\sqrt{2}}{2}$$

$$\frac{dy}{dx} = \frac{\sqrt{2} \times \sqrt{2}}{2 \times \sqrt{2}}$$

$$\frac{dy}{dx} = \frac{2}{2\sqrt{2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{2}} \quad \checkmark$$

Graaf 2011

③

$$(a) \quad x^4 + 3x^2y - 2y^2 = 15$$

$$4x^3 + 3x^2(\frac{dy}{dx}) + (6x)y - 4y(\frac{dy}{dx}) = 0$$

$$\frac{dy}{dx}(3x^2 - 4y) + 4x^3 + 6xy = 0$$

$$\frac{dy}{dx}(3x^2 - 4y) = -4x^3 - 6xy$$

$$\frac{dy}{dx} = \frac{-4x^3 - 6xy}{3x^2 - 4y}$$

$$(b) \quad x = \ln t \quad y = t^3 - 7t$$

$$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 3t^2 - 7$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (3t^2 - 7) \times \frac{1}{t}$$

$$\frac{dy}{dx} = 3t^3 - 7t$$

$$(ii) \quad \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{dt}{dx} \times \frac{d}{dt}\left(\frac{dy}{dx}\right)$$

$$= t \times \frac{d}{dt}(3t^3 - 7t)$$

$$= t \times (9t^2 - 7)$$

$$= 9t^3 - 7t.$$

$$\text{Os yw } t = \frac{1}{3} \text{ gna } \frac{d^2y}{dx^2} = 9\left(\frac{1}{3}\right)^3 - 7\left(\frac{1}{3}\right)$$

$$\frac{d^2y}{dx^2} = -2$$

Haf 201)

(3)

$$(a) 2x^3 + x^2 \cos y + y^4 + 2x - 25 = 0$$

$$6x^2 + x^2(-\sin(y)) \frac{dy}{dx} + (2x)\cos y + 4y^3 \left(\frac{dy}{dx} \right) + 2 = 0$$

$$\frac{dy}{dx} \left(-x^2 \sin(y) + 4y^3 \right) + 6x^2 + 2x \cos y + 2 = 0$$

$$\frac{dy}{dx} \left(-x^2 \sin(y) + 4y^3 \right) = -6x^2 - 2x \cos(y) - 2$$

$$\frac{dy}{dx} = \frac{-6x^2 - 2x \cos(y) - 2}{-x^2 \sin(y) + 4y^3}$$

$$\frac{dy}{dx} = \frac{6x^2 + 2x \cos(y) + 2}{x^2 \sin(y) - 4y^3}$$

$$(b) x = t^3$$

$$y = 2t^2 + 5t^4$$

$$\frac{dx}{dt} = 3t^2$$

$$\frac{dy}{dt} = 4t + 20t^3$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (4t + 20t^3) \times \frac{1}{3t^2}$$

$$\frac{dy}{dx} = \frac{4}{3t} + \frac{20t}{3}$$

$$(ii) \frac{dy}{dx} = 5 \Rightarrow 5 = \frac{4}{3t} + \frac{20t}{3}$$

$$15t = 4 + 20t^2 \quad (\text{muosi efo } 3t)$$

$$0 = 20t^2 - 15t + 4.$$

$$\begin{aligned} \text{Gwahanolyn: } b^2 - 4ac &= (-15)^2 - 4 \times 20 \times 4 \\ &= 225 - 320 \\ &= -95 \end{aligned}$$

Gan fod $b^2 - 4ac < 0$ nid oes unrhyw werth
real o t fel bod $\frac{dy}{dx} = 5$.

Graef 2012

③ (a) $x = 3t^2 \quad y = t^6 - 4t^3$

$$\frac{dx}{dt} = 6t \quad \frac{dy}{dt} = 6t^5 - 12t^2$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (6t^5 - 12t^2) \times \frac{1}{6t}$$

$$\frac{dy}{dx} = t^4 - 2t$$

(ii) $\frac{dy}{dx} = \frac{I}{2} \Rightarrow \frac{I}{2} = t^4 - 2t$

$$I = 2t^4 - 4t$$

$$0 = 2t^4 - 4t - 7$$

$$2t^4 - 4t - 7 = 0 \quad \checkmark$$

④ $x^2y^2 + x^4 + 6 = 2y^3 + 2x$

$$x^2(2y\frac{dy}{dx}) + (2x)y^2 + 4x^3 = 6y^2(\frac{dy}{dx}) + 2$$

$$\frac{dy}{dx}(2x^2y - 6y^2) = 2 - 4x^3 - 2xy^2$$

$$\frac{dy}{dx} = \frac{2 - 4x^3 - 2xy^2}{2x^2y - 6y^2}$$

os yw $x=2, y=3$ yna $\frac{dy}{dx} = \frac{2 - 4 \times 2^3 - 2 \times 2 \times 3^2}{2 \times 2^2 \times 3 - 6 \times 3^2}$

$$\frac{dy}{dx} = \frac{-66}{-30}$$

$$\frac{dy}{dx} = 2.2$$

Haf 2012

③

$$(a) x^3 - 4x^2y = 2y^3 - 3x - 2$$

$$3x^2 - 4x^2\left(\frac{dy}{dx}\right) - (8x)y = 6y^2\left(\frac{dy}{dx}\right) - 3$$

$$\frac{dy}{dx} \left(-4x^2 - 6y^2 \right) = -3 - 3x^2 + 8xy$$

$$\frac{dy}{dx} = \frac{-3 - 3x^2 + 8xy}{-4x^2 - 6y^2}$$

$$\frac{dy}{dx} = \frac{3 + 3x^2 - 8xy}{4x^2 + 6y^2}$$

$$\text{OS gw } x=3, y=1 \text{ gna } \frac{dy}{dx} = \frac{3 + 3 \times 3^2 - 8 \times 3 \times 1}{4 \times 3^2 + 6 \times 1^2}$$

$$\frac{dy}{dx} = \frac{6}{42}$$

$$\frac{dy}{dx} = \frac{1}{7}$$

$$(b) x = \sin at \quad y = \cos at$$

$$\frac{dx}{dt} = a \cos at \quad \frac{dy}{dt} = -a \sin at$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = -a \sin at \times \frac{1}{a \cos at}$$

$$\frac{dy}{dx} = -\tan at$$

$$(ii) \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$\frac{d^2y}{dx^2} = \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{a \cos at} \times \frac{d}{dt} (-\tan at)$$

$$\frac{d^2y}{dx^2} = \frac{1}{a \cos at} \times -a^2 \sec^2 at$$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos at} \times \frac{-1}{\cos^2 at}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{\cos^3 at}$$

$$\frac{d^2y}{dx^2} = -\sec^3 at$$

Graef 2013

(3)

$$(a) \quad x^3 + 5x^4y - 2y^3 + 7 = 0$$

$$3x^2 + 5x^4\left(\frac{dy}{dx}\right) + (20x^3)y - 6y^2\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} \left(5x^4 - 6y^2 \right) + 3x^2 + 20x^3y = 0$$

$$\frac{dy}{dx} \left(5x^4 - 6y^2 \right) = -3x^2 - 20x^3y$$

$$\frac{dy}{dx} = \frac{-3x^2 - 20x^3y}{5x^4 - 6y^2}$$

$$(b) \quad x = t^3 - 5 \quad y = t^4 + 7t^5$$

$$\frac{dx}{dt} = 3t^2 \quad \frac{dy}{dt} = 4t^3 + 35t^4$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (4t^3 + 35t^4) \times \frac{1}{3t^2}$$

$$\frac{dy}{dx} = \frac{4t}{3} + \frac{35t^2}{3}$$

$$(ii) \quad \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$= \frac{1}{3t^2} \times \frac{d}{dt} \left(\frac{4t}{3} + \frac{35t^2}{3} \right)$$

$$= \frac{1}{3t^2} \left(\frac{4}{3} + \frac{70t}{3} \right)$$

$$= \frac{4}{9t^2} + \frac{70}{9t}$$

$$(iii) \text{ osyw } x=3$$

$$y_n, y_n \quad x = t^3 - 5$$

$$3 = t^3 - 5$$

$$8 = t^3$$

$$t^3 = 8$$

$$t = \sqrt[3]{8}$$

$$t = 2$$

$$\text{Finally } \frac{d^2y}{dx^2} = \frac{4}{9x^2} + \frac{70}{9x^2}$$

$$= \frac{1}{9} + \frac{35}{9}$$

$$= 4$$

Haf 2013

$$\textcircled{3} \quad x^3 y^2 = 128$$

$$\text{a) } x^3 \left(2y \frac{dy}{dx} \right) + (3x^2)y^2 = 0$$

$$\frac{dy}{dx} (2x^3 y) = -3x^2 y^2$$

$$\frac{dy}{dx} = \frac{-3x^2 y^2}{2x^3 y}$$

$$\frac{dy}{dx} = -\frac{3y}{2x}$$

$$\text{b) } \frac{dy}{dx} = 3 \Rightarrow 3 = \frac{-3y}{2x}$$

$$6x = -3y$$

$$2x = -y$$

$$-y = 2x$$

$$y = -2x$$

Felly, ar gyfer $P = (a, b)$, mae $b = -2a$. \checkmark

$$\text{cii) } b = -2a \text{ --- (1)}$$

Yn amnewid $x=a, y=b$ i mewn i $x^3 y^2 = 128$:

$$a^3 b^2 = 128 \text{ --- (2)}$$

Yn amnewid am b o (1) a (2): $a^3 (-2a)^2 = 128$

$$a^3 (4a^2) = 128$$

$$a^5 = 32$$

$$a = \sqrt[5]{32}$$

$$\underline{a = 2}$$

Felly $b = -2a$

$$\underline{b = -4}$$

$$\textcircled{4} \quad x = \ln t \quad y = 5t^4$$

$$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 20t^3$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 20t^3 \times \frac{1}{t}$$

$$\frac{dy}{dx} = 20t^4.$$

dx

$$(b) \quad \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$= t \times \frac{d}{dt} (20t^4)$$

$$= t \times (80t^3)$$

$$= 80t^4.$$

$$\frac{d^2y}{dx^2} = 0.648 \Rightarrow$$

$$0.648 = 80t^4$$

$$0.0081 = t^4$$

$$t^4 = 0.0081$$

$$t = \pm \sqrt[4]{0.0081}$$

$$t = \pm 0.3$$

and $t > 0$ felly $t = 0.3$

Gaeaf 2014

$$③ \quad x^3 - 2x^2y + 3y^2 = 3$$

$$3x^2 - (2x^2)\left(\frac{dy}{dx}\right) - 4xy + 6y\left(\frac{dy}{dx}\right) = 0$$

$$3x^2 - 4xy + \frac{dy}{dx}(6y - 2x^2) = 0$$

$$\frac{dy}{dx}(6y - 2x^2) = 4xy - 3x^2$$

$$\frac{dy}{dx} = \frac{4xy - 3x^2}{6y - 2x^2}$$

$$\text{Os yw } x = -2, y = -1 \quad \text{yna} \quad \frac{dy}{dx} = \frac{4(-2)(-1) - 3(-2)^2}{6(-1) - 2(-2)^2}$$

$$= \frac{8 - 12}{-6 - 8}$$

$$= \frac{-4}{-14}$$

$$= \frac{2}{7}$$

Gaeaf 2014

④ $x = 2t^3, \frac{dy}{dx} = 2t + 4t^3$

a) $\frac{dx}{dt} = 6t^2$

b) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$$= \frac{dt}{dx} \times \frac{d}{dt}\left(\frac{dy}{dx}\right)$$

$$= \frac{1}{6t^2} \times \frac{d}{dt}(2t + 4t^3)$$

$$= \frac{1}{6t^2} \times (2 + 12t^2)$$

$$= \frac{2}{6t^2} + \frac{12t^2}{6t^2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{3t^2} + 2$$

A yw'n bosib cael $\frac{d^2y}{dx^2} = 2$?

$$\frac{1}{3t^2} + 2 = 2$$

$$\frac{1}{3t^2} = 0 \quad [\text{Lloynu 2}]$$

$$\frac{1}{t^2} = 0 \quad [\text{Lloosi efo 3}]$$

Dim oes pa mor fawr fydd t, ni fydd $\frac{1}{t^2}$ yn sera.
Felly nid oes gwerth ar gyfer t fel bod $\frac{d^2y}{dx^2} = 2$.

(Rhaid cael $t = \infty$ er mwyn cael $\frac{1}{t^2} = 0$;
nid yw hun yn rif penodol.)

$$c) \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$2t + 4t^3 = \frac{dy}{dt} \times \frac{1}{6t^2}$$

$$6t^2(2t + 4t^3) = \frac{dy}{dt}$$

$$\frac{dy}{dt} = 12t^3 + 24t^5$$

$$y = \int \frac{dy}{dt} dt$$

$$y = \int 12t^3 + 24t^5 dt$$

$$y = 12 \int t^3 + 2t^5 dt$$

$$y = 12 \left[\frac{t^4}{4} + \frac{2t^6}{6} \right] + K$$

$$y = \frac{12t^4}{4} + \frac{24t^6}{6} + K$$

$$y = 3t^4 + 4t^6 + K$$

OS yw $t=1$, mae $y=10$, Felly

$$10 = 3(1)^4 + 4(1)^6 + K$$

$$10 = 3 + 4 + K$$

$$3 = K$$

Felly $y = 3t^4 + 4t^6 + 3$

C3 1taf 2014

③

$$(a) \quad y^4 - 2x^2 + 8xy^2 + 9 = 0$$

$$4y^3 \left(\frac{dy}{dx} \right) - 4x + (8x) 2y \frac{dy}{dx} + 8(y^2) = 0$$

$$\frac{dy}{dx} (4y^3 + 16xy) - 4x + 8y^2 = 0$$

$$\frac{dy}{dx} (y^3 + 4xy) - x + 2y^2 = 0$$

$$\frac{dy}{dx} (y^3 + 4xy) = x - 2y^2$$

$$\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy} \quad \checkmark$$

$$(b) \text{ Os } yw \quad \frac{dy}{dx} = 0 \text{ yna} \quad \frac{x - 2y^2}{y^3 + 4xy} = 0$$

$$x - 2y^2 = 0$$

[Lluosi efa $y^3 + 4xy$]

$$x = 2y^2$$

$$2y^2 = x$$

$$y^2 = \frac{x}{2}$$

In amnewid am y^2 i mewn i $y^4 - 2x^2 + 8xy^2 + 9 = 0$

$$\left(\frac{x}{2}\right)^2 - 2x^2 + 8x\left(\frac{x}{2}\right) + 9 = 0$$

$$\frac{x^2}{4} - 2x^2 + 4x^2 + 9 = 0$$

$$x^2 - 8x^2 + 16x^2 + 36 = 0$$

[Lluosi efa 4]

$$9x^2 + 36 = 0$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm \sqrt{-4}$$

Nid oes datrysiau real ar gyfer x felly nid oes pwynt ar C fel bod $\frac{dy}{dx} = 0$.

C3 Haf 2014

(4)

$$x = 2e^t - 5$$

$$\frac{dx}{dt} = 2e^t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = (-8e^{-t} + 3e^t) \times \frac{1}{2e^t}$$

$$\frac{dy}{dx} = \frac{-8e^{-t}}{2e^t} + \frac{3e^t}{2e^t}$$

$$\frac{dy}{dx} = -4e^{-2t} + \frac{3}{2}$$

os yw $\frac{dy}{dx} = -1$ yna $-1 = -4e^{-2t} + \frac{3}{2}$

$$-2.5 = -4e^{-2t}$$

$$\frac{-2.5}{-4} = e^{-2t}$$

$$e^{-2t} = 0.625$$

$$-2t = \ln(0.625)$$

$$t = \frac{\ln(0.625)}{-2}$$

$$\underline{\underline{t = 0.235}} \text{ ; 3 llc de gal}$$

C3 Haf 2015

3) C₁: $x^3 + 2x \cos y + y^2 = 1 + \frac{\pi^2}{4}$

$$3x^2 + 2x(-\sin y \cdot \frac{dy}{dx}) + 2\cos y + 2y(\frac{dy}{dx}) = 0$$

$$3x^2 - 2x \sin y (\frac{dy}{dx}) + 2\cos y + 2y(\frac{dy}{dx}) = 0$$

$$\frac{dy}{dx} (2y - 2x \sin y) = -3x^2 - 2\cos y$$

$$\frac{dy}{dx} = \frac{-3x^2 - 2\cos y}{2y - 2x \sin y}$$

os yw $x=1, y=\frac{\pi}{2}$ yna $\frac{dy}{dx} = \frac{-3(1^2) - 2\cos(\frac{\pi}{2})}{2(\frac{\pi}{2}) - 2(1)\sin(\frac{\pi}{2})}$
 $\frac{dy}{dx} = \frac{-3}{\pi - 2}$
 $\frac{dy}{dx} = -2.63$ i 2 le degol.

b) C₂: $\frac{dy}{dx} = x^2 y$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$\frac{d^2y}{dx^2} = x^2 \left(\frac{dy}{dx} \right) + 2xy$$

$$\frac{d^2y}{dx^2} = x^2(x^2y) + 2xy$$

$$\frac{d^2y}{dx^2} = x^4y + 2xy$$

$$\frac{d^2y}{dx^2} = xy(x^3 + 2)$$

C3 Haf 2015

4) $x = \tan^{-1}(t)$, $y = \ln(t)$, $t > 0$

$$(a) \frac{dx}{dt} = \frac{1}{1+t^2} \quad \frac{dy}{dt} = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{1}{t} \times \frac{t^2 + 1}{1}$$

$$\frac{dy}{dx} = \frac{t^2 + 1}{t}$$

$$\frac{dy}{dx} = t + \frac{1}{t}$$

$$(b) \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{dt}{dx} \times \frac{d}{dt}\left(\frac{dy}{dx}\right)$$

$$= \frac{t^2 + 1}{1} \times \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$= (t^2 + 1) \times \frac{d}{dt}\left(t + t^{-1}\right)$$

$$= (t^2 + 1) \times (1 - t^{-2})$$

$$= (t^2 + 1)\left(1 - \frac{1}{t^2}\right)$$

$$= t^2 - \frac{t^2}{t^2} + 1 - \frac{1}{t^2}$$

$$= t^2 - t^{-2}$$

$$\text{Os yw } \frac{d^2y}{dx^2} = 0 \quad \text{yna} \quad t^2 - t^{-2} = 0$$

$$t^2 = t^{-2}$$

$$t^2 = \frac{1}{t^2}$$

$$t^4 = 1$$

$$t = \pm \sqrt[4]{1}$$

$$t = \pm 1$$

ond mae $t > 0$ felly $t = 1$

Beth yw gwerth x ? $x = \tan^{-1}(t)$

$$x = \tan^{-1}(1)$$

$$\therefore x = \underline{\underline{\frac{\pi}{4}}}$$

C3 Haf 2016

③ C: $x^2 + 3xy + 2y^3 - 2x = 21$.

Mae'r pwynt $P = (-5, 2)$ ar C.

Differn meun perthynas ag x :

$$2x + 3y + 3x\left(\frac{dy}{dx}\right) + 6y^2\left(\frac{dy}{dx}\right) - 2 = 0$$

$$2x + 3y - 2 + \frac{dy}{dx}(3x + 6y^2) = 0$$

$$\frac{dy}{dx}(3x + 6y^2) = 2 - 2x - 3y$$

$$\frac{dy}{dx} = \frac{2 - 2x - 3y}{3x + 6y^2}$$

Ar gyfer P, mae $x = -5$, $y = 2$

Felly mae $\frac{dy}{dx} = \frac{2 - 2(-5) - 3(2)}{3(-5) + 6(2^2)}$

$$\frac{dy}{dx} = \frac{2 + 10 - 6}{-15 + 24}$$

$$\frac{dy}{dx} = \frac{6}{9}$$

$$\frac{dy}{dx} = \frac{2}{3}$$

C3 Haf 2016

④ $x = 4 \sin 3t, \quad y = 2 \cos 3t$

a) $\frac{dx}{dt} = 12 \cos 3t \quad \frac{dy}{dt} = -6 \sin 3t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = -6 \sin 3t \times \frac{1}{12 \cos 3t}$$

$$\frac{dy}{dx} = \frac{-6 \sin 3t}{12 \cos 3t}$$

$$\frac{dy}{dx} = -\frac{1}{2} \tan 3t$$

b) (i) $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$

$$= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$= \frac{1}{12 \cos 3t} \times \frac{d}{dt} \left(-\frac{1}{2} \tan 3t \right)$$

$$= \frac{1}{12 \cos 3t} \times -\frac{1}{2} \left(3 \sec^2(3t) \right)$$

$$= \frac{-\sec^2(3t)}{8 \cos 3t}$$

$$= -\frac{1}{8} \sec^3(3t)$$

(ii) Yn defnyddio $y = 2 \cos 3t$

$$\frac{y}{2} = \cos 3t$$

$$\frac{y}{2} = \sec 3t$$

$$\left(\frac{y}{2}\right)^3 = \sec^3 3t$$

$$\frac{8}{y^3} = \sec^3 3t$$

Felly

$$\frac{d^2y}{dx^2} = -\frac{1}{8} \left(\frac{8}{y^3} \right)$$

$$\frac{d^2y}{dx^2} = -\frac{1}{y^3}$$

C3 Haf 2017

3) a) $x^4 - 3x^2y + 2y^3 - 4x = 7$

$$4x^3 - 3x^2 \left(\frac{dy}{dx} \right) - 6xy + 6y^2 \left(\frac{dy}{dx} \right) - 4 = 0$$

$$4x^3 - 6xy - 4 + \left(\frac{dy}{dx} \right) (-3x^2 + 6y^2) = 0$$

$$\frac{dy}{dx} (6y^2 - 3x^2) = 4 + 6xy - 4x^3$$

$$\frac{dy}{dx} = \frac{4 + 6xy - 4x^3}{6y^2 - 3x^2}$$

b) $x = 7t + 2t^2, \quad y = \frac{4+3t}{7+4t}$

$$\frac{dx}{dt} = 7 + 4t \quad \frac{dy}{dt} = \frac{(7+4t)(3) - (4+3t)(4)}{(7+4t)^2}$$

$$\frac{dy}{dt} = \frac{21 + 12t - 16 - 12t}{(7+4t)^2}$$

$$\frac{dy}{dt} = \frac{5}{(7+4t)^2}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{dt} \times \frac{dt}{dx} \\ &= \frac{5}{(7+4t)^2} \times \frac{1}{7+4t} \\ &= \frac{5}{(7+4t)^3}\end{aligned}$$

(Felly $\kappa = 5, n = 3$)

$$\begin{aligned}
 \text{(ii)} \quad \frac{d^2y}{dx^2} &= \frac{d}{dx} \left(\frac{dy}{dx} \right) \\
 &= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right) \\
 &= \frac{1}{7+4t} \times \frac{d}{dt} \left(\frac{5}{(7+4t)^3} \right) \\
 &= \frac{1}{7+4t} \times \frac{d}{dt} \left(5(7+4t)^{-3} \right) \\
 &= \frac{1}{7+4t} \left(-15(7+4t)^{-4} (4) \right) \\
 &= \frac{-60}{(7+4t)^5}
 \end{aligned}$$

C3 Itaf 2018

$$3) \quad x^5 + 4xy^2 - 2y^3 - 17 = 0$$

$$5x^4 + 4x\left(2y \times \frac{dy}{dx}\right) + 4y^2 - 6y^2 \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(8xy - 6y^2) + 5x^4 + 4y^2 = 0$$

$$\frac{dy}{dx}(8xy - 6y^2) = -4y^2 - 5x^4$$

$$\frac{dy}{dx} = \frac{-4y^2 - 5x^4}{8xy - 6y^2}$$

$$\text{ncu } \frac{dy}{dx} = \frac{5x^4 + 4y^2}{6y^2 - 8xy}$$

C3 Haf 2018

4) $x = \ln t, y = 4t^4 - 3t^2$ ar gyfer $t > 0$.

a) $\frac{dx}{dt} = \frac{1}{t}$ $\frac{dy}{dt} = 16t^3 - 6t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = 16t^3 - 6t \times \left(\frac{t}{1}\right)$$

$$\frac{dy}{dx} = 16t^4 - 6t^2$$

b) i) $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$

$$= \frac{dt}{dx} \times \frac{d}{dt} \left(\frac{dy}{dx} \right)$$

$$= t \times \frac{d}{dt} (16t^4 - 6t^2)$$

$$= t \times (64t^3 - 12t)$$

$$= 64t^4 - 12t^2$$

ii) $\frac{d^2y}{dx^2} = 1$

$$64t^4 - 12t^2 = 1$$

$$64t^4 - 12t^2 - 1 = 0$$

Gadewch i $u = t^2$.

Felly $64u^2 - 12u - 1 = 0$

$$64x - 1 = -64 \quad \begin{array}{c|cc} A & L \\ \hline -12 & -64 \end{array}$$

$$64u^2 - 16u + 4u - 1 = 0$$

$$16u(4u - 1) + 1(4u - 1) = 0$$

$$(16u + 1)(4u - 1) = 0 \quad \downarrow$$

$$(16u+1)(4u-1)=0$$

Naill ai $16u+1=0$ neu $4u-1=0$

$$16u = -1 \quad 4u = 1$$

$$u = -\frac{1}{16} \quad u = \frac{1}{4}$$

$$E^2 = -\frac{1}{16} \quad E^2 = \frac{1}{4}$$

$$E = \pm \sqrt{-\frac{1}{16}} \quad E = \pm \sqrt{\frac{1}{4}}$$

Dim gwerthoedd
real $E = \pm \frac{1}{2}$

ond mae $E > 0$

Felly rhaid bod $E = \frac{1}{2}$

Hwn yw'r unig werth fel bod $\frac{d^2y}{dx^2} = 1$.

(3 May 2019)

$$3). \text{ a) } 2x^4 - x^2 \sin y + y^5 - 4x + 17 = 0$$

$$8x^3 - x^2 \cos y \left(\frac{dy}{dx} \right) - 2x(\sin y) + \frac{dy}{dx}(5y^4) - 4 = 0$$

$$\frac{dy}{dx}(5y^4 - x^2 \cos y) + 8x^3 - 2x \sin y - 4 = 0$$

$$\frac{dy}{dx}(5y^4 - x^2 \cos y) = 2x \sin y + 4 - 8x^3$$

$$\frac{dy}{dx} = \frac{2x \sin y + 4 - 8x^3}{5y^4 - x^2 \cos y}$$

$$\text{b) } x = 4e^{3t} - 3e^{-2t}, y = 2e^t - 7e^{3t}$$

$$\text{~n) } \frac{dx}{dt} = 12e^{3t} + 6e^{-2t} \quad \frac{dy}{dt} = 2e^t - 21e^{3t}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2e^t - 21e^{3t}}{12e^{3t} + 6e^{-2t}} \\ &= 2e^t - 21e^{3t} \times \frac{1}{12e^{3t} + 6e^{-2t}} \end{aligned}$$

$$\frac{dy}{dx} = \frac{2e^t - 21e^{3t}}{12e^{3t} + 6e^{-2t}}$$

$$\text{nn) } 1) \frac{dy}{dx} = -2 \Rightarrow \frac{2e^t - 21e^{3t}}{12e^{3t} + 6e^{-2t}} = -2$$

$$\therefore 2e^t - 21e^{3t} = -24e^{7t} - 12e^{-2t}$$

$$2e^t + 3e^{3t} + 12e^{-2t} = 0$$

Ond, mae'r rhaid i'r odr chwch yd yn > 0
gan yd $e^x > 0$ ar gyffwrth yd gwerth x.

Felly nid oes gwerth ar gyffwrth t i'r beth

$$\frac{dy}{dx} = -2$$