WJEC CBAC

GCE AS/A level

976/01

MATHEMATICS C4
Pure Mathematics

P.M. FRIDAY, 18 June 2010

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{8 - x - x^2}{x(x-2)^2}.$$

(a) Express $f(x)$ in terms of partial fractions. [4]

(b) Use your result to part *(a)* to find the value of $f'(1)$. [3]

2. Find the equation of the normal to the curve

$$5x^2 + 4xy - y^3 = 5$$

at the point $(1, -2)$. [5]

3. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$2\cos 2\theta = 9\cos\theta + 7.$$ [5]

(b) (i) Express $5\sin x - 12\cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Use your results to part (i) to find the least value of

$$\frac{1}{5\sin x - 12\cos x + 20}.$$

Write down a value for x for which this least value occurs. [6]

4. The region R is bounded by the curve $y = \sin x$, the x-axis and the lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$. Find the volume generated when R is rotated through four right-angles about the x-axis. Give your answer correct to three decimal places. [5]

5. Expand $\left(1 - \frac{x}{4}\right)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2.
State the range of values of x for which your expansion is valid.
Hence, by writing $x = 1$ in your expansion, show that

$$\sqrt{3} \approx \frac{111}{64}.$$ [5]

6. The parametric equations of the curve C are

$$x = \frac{2}{t},\ y = 4t.$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$y = -2p^2x + 8p.$$ [4]

(b) The tangent to C at the point P passes through the point (2, 3). Show that P can be one of two points. Find the coordinates of each of these two points. [4]

7. *(a)* Find $\int x^3 \ln x \, dx$. [4]

(b) Use the substitution $u = 2x - 3$ to evaluate $\int_1^2 x(2x-3)^4 \, dx$. [5]

8. The value, £V, of a car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^2.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that $V = 12000$ when $t = 0$, show that

$$V = \frac{12000}{at+1},$$

where a is a constant. [4]

(c) The value of the car at the end of two years is £9000. Find the value of the car at the end of four years. [4]

9. The position vectors of the points A and B are given by

$$\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k},$$
$$\mathbf{b} = \mathbf{i} - 4\mathbf{j} + 8\mathbf{k},$$

respectively.

(a) Find the angle between the vectors **a** and **b**. [4]

(b) (i) Write down the vector **AB**.

(ii) Find the vector equation of the line AB. [3]

(c) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} - 4\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Show that the lines AB and L intersect and find the position vector of the point of intersection. [6]

10. Prove by contradiction the following proposition.

If a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$.

The first line of the proof is given below.

Assume that positive real numbers a, b exist such that $a + b < 2\sqrt{ab}$. [3]