C4: Integru Pellach

Haf 2005

7 (a) $\int_0^1 x(2x-1)^9\,dx$

$u = 2x - 1$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{du}{2} = dx$

$u = 2x - 1$

$u + 1 = 2x$

$\frac{u+1}{2} = x$

Terfannau:

[0] $u = 2 \times 0 - 1$

$u = -1$

[1] $u = 2 \times 1 - 1$

$u = 1$

$$\int_{-1}^{1} \left(\frac{u+1}{2}\right)\left(u\right)^9 \frac{du}{2}$$

$$= \frac{1}{4}\int_{-1}^{1} (u+1)(u)^9\,du$$

$$= \frac{1}{4}\int_{-1}^{1} u^{10} + u^9\,du$$

$$= \frac{1}{4}\left[\frac{u^{11}}{11} + \frac{u^{10}}{10}\right]_{-1}^{1}$$

$$= \frac{1}{4}\left[\left(\frac{1^{11}}{11} + \frac{1^{10}}{10}\right) - \left(\frac{(-1)^{11}}{11} + \frac{(-1)^{10}}{10}\right)\right]$$

$$= \frac{1}{4}\left[\frac{1}{11} + \frac{1}{10} - \left(\frac{-1}{11} + \frac{1}{10}\right)\right]$$

$$= \frac{1}{4}\left[\frac{1}{11} + \frac{1}{10} + \frac{1}{11} - \frac{1}{10}\right]$$

$$= \frac{1}{4} \times \frac{2}{11}$$

$$= \frac{1}{22}$$

(b) (i) $\int x \cos 2x \, dx$

Gadewch i $u = x$ $\quad \frac{dv}{dx} = \cos 2x$

$\frac{du}{dx} = 1$ $\quad v = \frac{1}{2}\sin 2x$

$$\int x\cos 2x\,dx = x\left(\tfrac{1}{2}\sin 2x\right) - \int 1\left(\tfrac{1}{2}\sin 2x\right)dx$$

$$= \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + K$$

(ii) $\int x\cos^2 x\,dx.$

Nawr $\cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

$\frac{\cos 2x + 1}{2} = \cos^2 x$

Felly $\int x\cos^2 x\,dx = \int x\left(\frac{\cos 2x + 1}{2}\right)dx$

$$= \int \frac{x\cos 2x}{2} + \frac{x}{2}\,dx$$

$$= \frac{1}{2}\int x\cos 2x\,dx + \frac{1}{2}\int x\,dx$$

$$= \frac{1}{2}\left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right] + \frac{1}{2}\left(\frac{x^2}{2}\right) + K$$

$$= \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + K$$

Haf 2006

(7) (a) $\int x \ln x \, dx$

Gadewch i $u = \ln x \qquad \frac{dv}{dx} = x$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{x^2}{2}$

$$\int x \ln x \, dx = \ln x \left(\frac{x^2}{2}\right) - \int \left(\frac{1}{x}\right)\left(\frac{x^2}{2}\right) dx$$

$$= \frac{x^2 \ln x}{2} - \int \frac{x}{2} \, dx$$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + K$$

(b) $\int_0^{\frac{\pi}{6}} \frac{\cos x}{(2\sin x + 3)^2} dx$

$u = 2\sin x + 3$

$\frac{du}{dx} = 2\cos x$

$du = 2\cos x \, dx$

$\frac{du}{2\cos x} = dx$

Terfannau:

[0] $u = 2\sin(0) + 3$

$u = 3$

$[\frac{\pi}{6}]$ $u = 2\sin(\frac{\pi}{6}) + 3$

$u = 4$

Felly $\int_0^{\frac{\pi}{6}} \frac{\cos x}{(2\sin x + 3)^2} dx = \int_3^4 \frac{\cancel{\cos x}}{u^2} \frac{du}{2\cancel{\cos x}}$

$$= \int_3^4 \frac{1}{2u^2} du$$

$$= \int_3^4 \frac{1}{2} u^{-2} du$$

$$= \left[\frac{-u^{-1}}{2}\right]_3^4$$

$$= \left[-\frac{1}{2u}\right]_3^4$$

$$= -\frac{1}{2\times4} - -\frac{1}{2\times3}$$

$$= -\frac{1}{8} + \frac{1}{6}$$

$$= \frac{1}{24}$$

Haf 2007

⑦ (a) $\int x^2 \ln x \, dx$

Gadewch i $u = \ln x$ $\frac{dv}{dx} = x^2$

$\frac{du}{dx} = \frac{1}{x}$ $v = \frac{x^3}{3}$

$$\int x^2 \ln x \, dx = \ln x \left(\frac{x^3}{3}\right) - \int \left(\frac{1}{x}\right)\left(\frac{x^3}{3}\right) dx$$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + K$$

(b) $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$ $x = 2\sin\theta$ $x^2 = 4\sin^2\theta$

$\frac{dx}{d\theta} = 2\cos\theta$

Terfannau: $dx = 2\cos\theta \, d\theta$

[0] $0 = 2\sin\theta$

$0 = \sin\theta$

$\theta = \sin^{-1}(0)$

$\theta = 0$

[$\sqrt{2}$] $\sqrt{2} = 2\sin\theta$

$\frac{\sqrt{2}}{2} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = \frac{\pi}{4}$

Felly $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx = \int_0^{\frac{\pi}{4}} \frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}} \times 2\cos\theta \, d\theta$

$$= \int_0^{\frac{\pi}{4}} \frac{8\sin^2\theta\cos\theta}{\sqrt{4(1-\sin^2\theta)}} d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{8\sin^2\theta\cos\theta}{\sqrt{4\cos^2\theta}} d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{8\sin^2\theta \cancel{\cos\theta}}{2\cancel{\cos\theta}}\,d\theta$$

$$= \int_0^{\frac{\pi}{4}} 4\sin^2\theta\,d\theta$$

$\left[\text{Felly } a = \frac{\pi}{4},\ K = 4.\right]$

$\left[\begin{array}{l} \cos 2\theta = 1 - 2\sin^2\theta \\ 2\sin^2\theta = 1 - \cos 2\theta \\ \sin^2\theta = \frac{1-\cos 2\theta}{2} \end{array}\right]$

$$= \int_0^{\frac{\pi}{4}} 4\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2(1-\cos 2\theta)\,d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2 - 2\cos 2\theta\,d\theta$$

$$= \Big[2\theta - \sin 2\theta\Big]_0^{\frac{\pi}{4}}$$

$$= \left[\left(2\times\frac{\pi}{4} - \sin\left(2\times\frac{\pi}{4}\right)\right) - \left(2\times 0 - \sin(2\times 0)\right)\right]$$

$$= \left[\left(\frac{\pi}{2} - 1\right) - (0 - 0)\right]$$

$$= \frac{\pi}{2} - 1$$

$(\approx 0.57$ i 2 le dégol$)$

Haf 2008

6 (a) $\int (3x+1)e^{2x}\,dx$

Gadewch i $u = 3x+1$ $\quad \frac{dv}{dx} = e^{2x}$

$\frac{du}{dx} = 3 \quad v = \frac{1}{2}e^{2x}$

$$\int (3x+1)e^{2x}\,dx = (3x+1)\tfrac{1}{2}e^{2x} - \int 3\left(\tfrac{1}{2}e^{2x}\right)dx$$

$$= \tfrac{1}{2}(3x+1)e^{2x} - \tfrac{3}{2}\int e^{2x}\,dx$$

$$= \tfrac{1}{2}(3x+1)e^{2x} - \tfrac{3}{2}\left(\tfrac{1}{2}e^{2x}\right) + K$$

$$= \frac{(3x+1)e^{2x}}{2} - \frac{3}{4}e^{2x} + K$$

(b) $\int_{1.5}^{3} \sqrt{9-x^2}\,dx$

$x = 3\sin\theta$ $\quad x^2 = 9\sin^2\theta$

$\frac{dx}{d\theta} = 3\cos\theta$

$dx = 3\cos\theta\, d\theta$

Terfannau:

[1.5] $1.5 = 3\sin\theta$

$\frac{1.5}{3} = \sin\theta$

$\theta = \sin^{-1}(\frac{1}{2})$

$\theta = \frac{\pi}{6}$

[3] $3 = 3\sin\theta$

$1 = \sin\theta$

$\theta = \sin^{-1}(1)$

$\theta = \frac{\pi}{2}$

Felly $\int_{1.5}^{3} \sqrt{9-x^2}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{9-9\sin^2\theta}\; 3\cos\theta\, d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{9(1-\sin^2\theta)}\, 3\cos\theta\, d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{9\cos^2\theta}\; 3\cos\theta\, d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 3\cos\theta\,(3\cos\theta)\, d\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 9\cos^2\theta\, d\theta$ $\quad$ [Felly $a = \frac{\pi}{6}$, $b = \frac{\pi}{3}$, $K = 9$.]

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta + 1 = 2\cos^2\theta$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 9\left(\frac{\cos 2\theta + 1}{2}\right) d\theta$

$= \frac{9}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos 2\theta + 1\, d\theta$

$= \frac{9}{2}\left[\frac{1}{2}\sin 2\theta + \theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \frac{9}{2}\left[\left(\frac{1}{2}\sin(2\times\frac{\pi}{2}) + \frac{\pi}{2}\right) - \left(\frac{1}{2}\sin(2\times\frac{\pi}{6}) + \frac{\pi}{6}\right)\right]$

$= \frac{9}{2}\left[\left(0 + \frac{\pi}{2}\right) - \left(\frac{\sqrt{3}}{4} + \frac{\pi}{6}\right)\right]$

$= \frac{9}{2}\left(\frac{\pi}{2} - \frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

$= \frac{3}{2}\pi - \frac{9\sqrt{3}}{8}$ $\quad$ ($\simeq 2.764$ i 3 lle degol)

Haf 2009

(6) (a) $\int (x+3)e^{2x}\,dx$

Gadewch i $u = x+3$ $\quad \frac{dv}{dx} = e^{2x}$

$\frac{du}{dx} = 1$ $\quad v = \frac{1}{2}e^{2x}$

$$\int (x+3)e^{2x} = (x+3)\tfrac{1}{2}e^{2x} - \int \tfrac{1}{2}e^{2x}\,dx$$
$$= \tfrac{1}{2}(x+3)e^{2x} - \tfrac{1}{4}e^{2x} + K$$

(b) $\int_0^{\frac{\pi}{3}} \frac{\sin x}{\sqrt{(2\cos x+1)}}\,dx$ $\quad u = 2\cos x + 1$

$\frac{du}{dx} = -2\sin x$

Terfannau:

$du = -2\sin x\,dx$

[0] $u = 2\cos(0)+1$

$u = 3$

$\frac{du}{-2\sin x} = dx$

$[\frac{\pi}{3}]$ $u = 2\cos(\frac{\pi}{3})+1$

$u = 2$

Felly $\int_0^{\frac{\pi}{3}} \frac{\sin x}{\sqrt{(2\cos x+1)}}\,dx = \int_3^2 \frac{\cancel{\sin x}}{\sqrt{u}}\,\frac{du}{-2\cancel{\sin x}}$

$$= \int_3^2 -\frac{1}{2}u^{-\frac{1}{2}}\,du$$
$$= \left[\left(-\frac{1}{2}\right)\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_3^2$$
$$= \left[-u^{\frac{1}{2}}\right]_3^2$$
$$= -\sqrt{2} - -\sqrt{3}$$
$$= -\sqrt{2} + \sqrt{3}$$
$$\approx 0.318 \text{ i 3 lle degol}$$

Haf 2010

7 (a) $\int x^3 \ln x \, dx$

Gadewch i $u = \ln x$ $\qquad \frac{dv}{dx} = x^3$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad v = \frac{x^4}{4}$

$$\int x^3 \ln x \, dx = \ln x \left(\frac{x^4}{4}\right) - \int \left(\frac{1}{x}\right)\left(\frac{x^4}{4}\right) dx$$
$$= \frac{x^4 \ln x}{4} - \int \frac{x^3}{4} dx$$
$$= \frac{x^4 \ln x}{4} - \frac{x^4}{16} + K$$

(b) $\int_1^2 x(2x-3)^4 \, dx$

$u = 2x-3$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{du}{2} = dx$

$u = 2x-3$

$u+3 = 2x$

$\frac{u+3}{2} = x$

Terfannau:

[1] $u = 2\times1-3$

$u = -1$

[2] $u = 2\times2-3$

$u = 1$

Felly $\int_1^2 x(2x-3)^4 \, dx = \int_{-1}^{1} \left(\frac{u+3}{2}\right) u^4 \frac{du}{2}$

$$= \frac{1}{4}\int_{-1}^{1} (u+3)u^4 \, du$$
$$= \frac{1}{4}\int_{-1}^{1} u^5 + 3u^4 \, du$$
$$= \frac{1}{4}\left[\frac{u^6}{6} + \frac{3u^5}{5}\right]_{-1}^{1}$$
$$= \frac{1}{4}\left[\left(\frac{1^6}{6} + \frac{3(1^5)}{5}\right) - \left(\frac{(-1)^6}{6} + \frac{3(-1)^5}{5}\right)\right]$$
$$= \frac{1}{4}\left[\left(\frac{1}{6} + \frac{3}{5}\right) - \left(\frac{1}{6} - \frac{3}{5}\right)\right]$$

$$= \frac{1}{4}\left(\frac{3}{5} + \frac{3}{5}\right)$$

$$= \frac{3}{10}$$

Haf 2011

7 (a) $\int x \sin 2x \, dx$

Gadewch i $u = x$ $\quad \frac{dv}{dx} = \sin 2x$

$\frac{du}{dx} = 1$ $\quad v = -\frac{1}{2}\cos 2x$

Felly $\int x \sin 2x \, dx = x\left(-\frac{1}{2}\cos 2x\right) - \int -\frac{1}{2}\cos 2x \, dx$

$= -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x \, dx$

$= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + K$

(b) $\int_0^2 \frac{x}{(5-x^2)^3} \, dx$ $\quad u = 5 - x^2$

$\frac{du}{dx} = -2x$

Terfannau: $\quad du = -2x \, dx$

$[0]$ $u = 5 - 0^2$ $\quad \frac{du}{-2x} = dx$

$u = 5$

$[2]$ $u = 5 - 2^2$

$u = 1$

Felly $\int_0^2 \frac{x}{(5-x^2)^3} \, dx = \int_5^1 \frac{x}{u^3} \frac{du}{-2x}$

$= \int_5^1 \frac{-1}{2u^3} \, du$

$= \int_5^1 -\frac{1}{2} u^{-3} \, du$

$= \left[\frac{u^{-2}}{4}\right]_5^1$

$$= \left[\frac{1}{4u^2} \right]_5^1$$

$$= \frac{1}{4\times1^2} - \frac{1}{4\times5^2}$$

$$= \frac{1}{4} - \frac{1}{100}$$

$$= \frac{6}{25}$$

<u>Haf 2012</u>

7 (a) $\int x e^{-2x} \, dx$

Gadewch i $u = x$ $\qquad \frac{dv}{dx} = e^{-2x}$

$\frac{du}{dx} = 1$ $\qquad v = -\frac{1}{2} e^{-2x}$

$$\int x e^{-2x} \, dx = x\left(-\tfrac{1}{2} e^{-2x}\right) - \int -\tfrac{1}{2} e^{-2x} \, dx$$

$$= -\tfrac{1}{2} x e^{-2x} + \tfrac{1}{2} \int e^{-2x} \, dx$$

$$= -\tfrac{1}{2} x e^{-2x} - \tfrac{1}{4} e^{-2x} + K$$

(b) $\int_1^e \frac{1}{x(1+3\ln x)} \, dx$ $\qquad u = 1 + 3\ln x$

$\frac{du}{dx} = \frac{3}{x}$

$x \, du = 3 \, dx$

$\frac{x \, du}{3} = dx$

Terfannau:

[1] $u = 1 + 3\ln(1)$

$u = 1$

[e] $u = 1 + 3\ln(e)$

$u = 1 + 3$

$u = 4$

Felly $\int_1^e \frac{1}{x(1+3\ln x)} \, dx = \int_1^4 \frac{1}{xu} \frac{x \, du}{3}$

$$= \int_1^4 \frac{1}{3u} \, du$$

$$= \left[\frac{1}{3} \ln|u| \right]_1^4$$

$$= \frac{1}{3}\ln|4| - \frac{1}{3}\ln|1|$$

$$= \frac{1}{3}\ln(4)$$

$$\approx 0.4621 \text{ i 4 lle degol}$$

<u>Haf 2013</u>

7 (a) $\int (3x-1)\cos 2x \, dx$

Gadewch i $u = 3x-1$ $\quad \frac{dv}{dx} = \cos 2x$

$\frac{du}{dx} = 3$ $\quad v = \frac{1}{2}\sin 2x$

$$\int (3x-1)\cos 2x \, dx = (3x-1)\tfrac{1}{2}\sin 2x - \int \tfrac{3}{2}\sin 2x \, dx$$

$$= \frac{(3x-1)}{2}\sin 2x + \frac{3}{4}\cos 2x + K$$

(b) $\int_0^1 \frac{x}{(2x+1)^3} \, dx$

$u = 2x+1$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{du}{2} = dx$

$u = 2x+1$

$u-1 = 2x$

$\frac{u-1}{2} = x$

Terfannau:

[0] $u = 2\times 0 + 1$

$u = 1$

[1] $u = 2\times 1 + 1$

$u = 3$

Felly $\int_0^1 \frac{x}{(2x+1)^3} \, dx = \int_1^3 \frac{\left(\frac{u-1}{2}\right)}{u^3} \frac{du}{2}$

$$= \int_1^3 \frac{1}{4}\left(\frac{u-1}{u^3}\right) du$$

$$= \frac{1}{4}\int_1^3 (u-1)u^{-3} \, du$$

$$= \frac{1}{4}\int_1^3 u^{-2} - u^{-3}\, du$$

$$= \frac{1}{4}\left[-u^{-1} + \frac{u^{-2}}{2}\right]_1^3$$

$$= \frac{1}{4}\left[-\frac{1}{u} + \frac{1}{2u^2}\right]_1^3$$

$$= \frac{1}{4}\left[\left(-\frac{1}{3} + \frac{1}{2\times 3^2}\right) - \left(-\frac{1}{1} + \frac{1}{2\times 1^2}\right)\right]$$

$$= \frac{1}{4}\left[\left(-\frac{1}{3} + \frac{1}{18}\right) - \left(-1 + \frac{1}{2}\right)\right]$$

$$= \frac{1}{4}\left(-\frac{1}{3} + \frac{1}{18} + 1 - \frac{1}{2}\right)$$

$$= \frac{1}{4} \times \frac{2}{9}$$

$$= \frac{1}{18}$$

C4 Haf 2014

7 (a) $\int x^4 \ln 2x \, dx$

Gadewch i $u = \ln 2x$ $\quad \frac{dv}{dx} = x^4$

$\frac{du}{dx} = \frac{2}{2x}$ $\quad v = \frac{x^5}{5}$

$$\int x^4 \ln 2x \, dx = \ln(2x)\left(\frac{x^5}{5}\right) - \int \frac{\cancel{2}}{\cancel{2}x}\left(\frac{x^5}{5}\right) dx$$

$$= \frac{x^5 \ln 2x}{5} - \int \frac{x^4}{5} dx$$

$$= \frac{x^5 \ln 2x}{5} - \frac{x^5}{25} + K$$

(b) $\int_0^{\frac{\pi}{3}} \sqrt{(10\cos x - 1)} \sin x \, dx$

Terfannau: $\quad u = 10\cos x - 1$

$[0]$ $\quad u = 10\cos(0) - 1$ $\quad \frac{du}{dx} = -10\sin x$

$u = 10 \times 1 - 1$

$u = 9$ $\quad du = -10\sin x \, dx$

$[\frac{\pi}{3}]$ $\quad u = 10\cos(\frac{\pi}{3}) - 1$ $\quad \frac{du}{-10\sin x} = dx$

$u = 10 \times \frac{1}{2} - 1$

$u = 4$

Felly $\int_0^{\frac{\pi}{3}} \sqrt{(10\cos x - 1)} \sin x \, dx = \int_9^4 \sqrt{u} \cancel{\sin x} \left(\frac{du}{-10\cancel{\sin x}}\right)$

$$= \int_9^4 -\frac{1}{10}\sqrt{u} \, du$$

$$= -\frac{1}{10}\int_9^4 u^{\frac{1}{2}} du$$

$$= \frac{-1}{10}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_9^4$$

$$= -\frac{1}{10} \times \frac{2}{3}\left[u^{\frac{3}{2}}\right]_9^4$$

$$= -\frac{2}{30}\left[4^{\frac{3}{2}} - 9^{\frac{3}{2}}\right]$$

$$= -\frac{1}{15}[8 - 27]$$

$$= \frac{19}{15}$$

C4 Haf 2015

7) a) $\int_0^2 \frac{x^2}{(12-x^3)^2}\,dx$

$u = 12 - x^3$

$\frac{du}{dx} = -3x^2$

$du = -3x^2\,dx$

$\frac{du}{-3x^2} = dx$

Terfannau:

$[0]$ $u = 12 - 0^3$

$u = 12$

$[2]$ $u = 12 - 2^3$

$u = 4$

$$\int_{12}^{4} \frac{\cancel{x^2}}{(u)^2}\,\frac{du}{-3\cancel{x^2}}$$

$$= \int_{12}^{4} -\frac{1}{3} u^{-2}\,du$$

$$= -\frac{1}{3}\left[-u^{-1}\right]_{12}^{4}$$

$$= -\frac{1}{3}\left[-\frac{1}{4} - -\frac{1}{12}\right]$$

$$= -\frac{1}{3} \times -\frac{1}{6}$$

$$= \underline{\underline{\frac{1}{18}}}$$

b) 1) $\int x \cos 2x\,dx$

Gadewch i $u = x$ $\quad \frac{dv}{dx} = \cos 2x$

$\frac{du}{dx} = 1$ $\quad v = \frac{1}{2}\sin 2x$

$$\int x\cos 2x\,dx = x\left(\tfrac{1}{2}\sin 2x\right) - \int 1\left(\tfrac{1}{2}\sin 2x\right)dx$$

$$= \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + K$$

(ii) $\int x \sin^2 x \, dx$.

Nawr $\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

Felly $\int x \sin^2 x \, dx = \int x \left(\dfrac{1 - \cos 2x}{2} \right) dx$

$= \int \dfrac{x}{2} - \dfrac{x \cos 2x}{2} \, dx$

$= \dfrac{1}{2} \int x \, dx - \dfrac{1}{2} \int x \cos 2x \, dx$

$= \dfrac{1}{2} \left(\dfrac{x^2}{2} \right) - \dfrac{1}{2} \left(\dfrac{x \sin 2x}{2} + \dfrac{\cos 2x}{4} \right) + K$

$= \dfrac{x^2}{4} - \dfrac{x \sin 2x}{4} - \dfrac{\cos 2x}{8} + K$

C4 Haf 2016

Gadewch i

6 a) $\int (2x+1)e^{-3x}\,dx$

$u = 2x+1$ $\quad \frac{dv}{dx} = e^{-3x}$

$\frac{du}{dx} = 2$ $\quad v = -\frac{1}{3}e^{-3x}$

Yna $\int (2x+1)e^{-3x}\,dx = (2x+1)\left(-\frac{1}{3}e^{-3x}\right) - \int -\frac{2}{3}e^{-3x}\,dx$

$= -\frac{1}{3}(2x+1)e^{-3x} + \frac{2}{3}\int e^{-3x}\,dx$

$= -\frac{1}{3}(2x+1)e^{-3x} + \frac{2}{3}\left(-\frac{1}{3}e^{-3x}\right) + K$

$= -\frac{1}{3}(2x+1)e^{-3x} - \frac{2}{9}e^{-3x} + K$

$= e^{-3x}\left(-\frac{1}{3}(2x+1) - \frac{2}{9}\right) + K$

$= e^{-3x}\left(-\frac{2}{3}x - \frac{1}{3} - \frac{2}{9}\right) + K$

$= e^{-3x}\left(-\frac{2}{3}x - \frac{5}{9}\right) + K$

$= \frac{e^{-3x}}{9}\left(-6x-5\right) + K$

b) $\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5\tan x}}{\cos^2 x}\,dx$

$u = 4 + 5\tan x$

$\frac{du}{dx} = 5\sec^2 x$

$du = 5\sec^2 x\,dx$

$\frac{du}{5\sec^2 x} = dx$

$\frac{1}{5}\cos^2 x\,du = dx$

Terfannau:

$[0]$ $u = 4 + 5\tan(0)$

$u = 4$

$\left[\frac{\pi}{4}\right]$ $u = 4 + 5\tan\left(\frac{\pi}{4}\right)$

$u = 9$

@mathemateg

Felly $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5\tan x}}{\cos^2 x}\,dx$

$$= \int_4^9 \frac{\sqrt{4+5\tan x}}{\cancel{\cos^2 x}}\left(\frac{1}{5}\cancel{\cos^2 x}\right)du$$

$$= \int_4^9 \frac{\sqrt{u}}{5}\,du$$

$$= \frac{1}{5}\int_4^9 u^{\frac{1}{2}}\,du$$

$$= \frac{1}{5}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^9$$

$$= \frac{1}{5}\times\frac{2}{3}\left[u^{\frac{3}{2}}\right]_4^9$$

$$= \frac{2}{15}\left[9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$$

$$= \frac{2}{15}(27-8)$$

$$= \frac{38}{15}$$

C4 Haf 2014

7) a) $\int \frac{\ln x}{x^4}\,dx = \int \ln x\,(x^{-4})\,dx$

Gadewch i $u = \ln x$ $\qquad \frac{dv}{dx} = x^{-4}$

$\frac{du}{dx} = \frac{1}{x} \qquad v = -\frac{1}{3}x^{-3}$

$$\int \ln x\,(x^{-4})\,dx = \ln x\left(-\frac{1}{3}x^{-3}\right) - \int \frac{1}{x}\left(-\frac{1}{3}x^{-3}\right)dx$$
$$= \ln x\left(-\frac{1}{3}x^{-3}\right) - \int -\frac{1}{3}x^{-4}\,dx$$
$$= -\frac{\ln x}{3x^3} + \frac{1}{3}\int x^{-4}\,dx$$
$$= -\frac{\ln x}{3x^3} + \frac{1}{3}\left(-\frac{1}{3}x^{-3}\right) + K$$
$$= -\frac{\ln x}{3x^3} - \frac{1}{9x^3} + K$$
$$= -\frac{1}{3x^3}\left(\ln x + \frac{1}{3}\right) + K$$

b) $\int_0^1 x^3(x^2+1)^4\,dx$ $\qquad u = x^2+1$

$\frac{du}{dx} = 2x$

Terfannau:

[0] $u = 0^2 + 1$

$u = 1$

[1] $u = 1^2 + 1$

$u = 2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

Felly $\int_0^1 x^3(x^2+1)^4\,dx = \int_1^2 x^3(u)^4\,\frac{du}{2x}$

$$= \int_1^2 \frac{x^2u^4}{2}\,du$$

$$= \int_1^2 \frac{(u-1)u^4}{2}\,du$$

$$= \frac{1}{2}\int_1^2 (u-1)u^4\,du$$

$$= \frac{1}{2}\int_1^2 u^5 - u^4\,du$$

$$= \frac{1}{2}\left[\frac{u^6}{6} - \frac{u^5}{5}\right]_1^2$$

$$= \frac{1}{2}\left[\left(\frac{2^6}{6} - \frac{2^5}{5}\right) - \left(\frac{1^6}{6} - \frac{1^5}{5}\right)\right]$$

$$= \frac{1}{2}\left[\frac{64}{15} - -\frac{1}{30}\right]$$

$$= \underline{\underline{2.15}} \qquad \left(\text{neu } \frac{43}{20}\right)$$

C4 Haf 2018

7) a) $\int (4x+1)e^{4x-5}\,dx.$

Gadewch i $u = 4x+1$ $\quad \frac{dv}{dx} = e^{4x-5}$

$\frac{du}{dx} = 4 \quad v = \frac{1}{4}e^{4x-5}$

$$\int (4x+1)e^{4x-5}\,dx = (4x+1)\left(\tfrac{1}{4}e^{4x-5}\right) - \int 4\left(\frac{1}{4}e^{4x-5}\right)dx$$

$$= (4x+1)\left(\tfrac{1}{4}e^{4x-5}\right) - \int e^{4x-5}\,dx$$

$$= (4x+1)\left(\tfrac{1}{4}e^{4x-5}\right) - \tfrac{1}{4}e^{4x-5} + K$$

$$= 4x\left(\frac{1}{4}e^{4x-5}\right) + \frac{1}{4}e^{4x-5} - \frac{1}{4}e^{4x-5} + K$$

$$= \underline{\underline{xe^{4x-5} + K}}$$

b) $\displaystyle\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}}\,dx$

$x = 4\sin\theta$

$x^2 = 16\sin^2\theta$

$\frac{dx}{d\theta} = 4\cos\theta$

$dx = 4\cos\theta\, d\theta$

Terfannau:

[0] $0 = 4\sin\theta$

$0 = \sin\theta$

$\theta = \sin^{-1}(0)$

$\theta = 0^\circ$ neu 0 radian

[$2\sqrt{2}$] $2\sqrt{2} = 4\sin\theta$

$\frac{\sqrt{2}}{2} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = 45^\circ$ neu $\frac{\pi}{4}$ radian.

Felly $\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}} dx$

$$= \int_0^{\frac{\pi}{4}} \frac{16\sin^2\theta}{\sqrt{16-16\sin^2\theta}} 4\cos\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{16\sin^2\theta}{\sqrt{16(1-\sin^2\theta)}} 4\cos\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{16\sin^2\theta}{\sqrt{16\cos^2\theta}} 4\cos\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{16\sin^2\theta}{\cancel{4\cos\theta}} \cancel{4\cos\theta} \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} 16\sin^2\theta \, d\theta$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

(Felly $a = \frac{\pi}{4}$, $b = 16$.)

ii) $\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}} dx = \int_0^{\frac{\pi}{4}} 16\sin^2\theta \, d\theta$

$$= \int_0^{\frac{\pi}{4}} 16\left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \int_0^{\frac{\pi}{4}} 8(1-\cos 2\theta) d\theta$$

$$= 8\int_0^{\frac{\pi}{4}} 1-\cos 2\theta \, d\theta$$

$$= 8\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 8\left[\left(\frac{\pi}{4} - \frac{\sin(2\times\frac{\pi}{4})}{2}\right) - \left(0 - \frac{\sin(2\times 0)}{2}\right)\right]$$

$$= 8\left[\left(\frac{\pi}{4} - \frac{1}{2}\right) - (0 - 0)\right]$$

$$= \underline{\underline{2\pi - 4}}$$ (Felly $c = 2$, $d = -4$.)

C4 May 2019

7) a) $\int (5x-1)\sin 3x \, dx$ $\left[\begin{array}{l} u = 5x - 1 \\ \frac{du}{dx} = 5 \end{array} \right]$

$$= (5x-1)\left(-\frac{\cos(3x)}{3}\right) - \int \left(-\frac{\cos(3x)}{3}\right)(5)\, dx$$

$$= \frac{-1}{3}(5x-1)\cos(3x) + \frac{5}{9}\sin(3x) + c$$

b) $\int_0^1 \frac{x}{(3x+1)^4}\, dx$

$u = 3x + 1$

$\frac{du}{dx} = 3$

$du = 3\, dx$

$dx = \frac{du}{3}$

$x = \frac{u-1}{3}$

$\Rightarrow$ Terjemahan:

$[0]$ $u = 3(0) + 1$

$= 1$

$[1]$ $u = 3(1) + 1$

$= 4$

Fang $\int_0^1 \frac{x}{(3x+1)^4}\, dx = \int_1^4 \frac{\frac{u-1}{3}}{(u)^4} \frac{du}{3}$

$$= \frac{1}{9}\int_1^4 \frac{u-1}{u^4}\, du$$

$$= \frac{1}{9}\int_1^4 u^{-3} - u^{-4}\, du$$

$$= \frac{1}{9}\left[\frac{u^{-2}}{-2} - \frac{u^{-3}}{-3}\right]_1^4$$

$$= \frac{1}{9}\left[\left(\frac{4^{-2}}{-2} + \frac{4^{-3}}{3}\right) - \left(\frac{1^{-2}}{-2} + \frac{1^{-3}}{3}\right)\right]$$

$$= \frac{1}{9}\left[\left(\frac{-1}{2(4)^2} + \frac{1}{3(4)^3}\right) - \left(\frac{-1}{2(1)^2} + \frac{1}{3(1)^3}\right)\right]$$

$$= \frac{1}{9}\left[\frac{-1}{32} + \frac{1}{192} + \frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{1}{9} \times \frac{9}{64}$$

$$= \frac{1}{64}$$

$$[n = 64]$$