

## CH: Ffracsiynau Rhannol

Itaf 2005

$$\begin{aligned} \textcircled{1} \quad \text{(a)} \quad \frac{8x^2+x-5}{(2x-1)^2(x+2)} &= \frac{A}{2x-1} + \frac{B}{(2x-1)^2} + \frac{C}{x+2} \\ \frac{8x^2+x-5}{(2x-1)^2(x+2)} &= \frac{A(2x-1)(x+2) + B(x+2) + C(2x-1)^2}{(2x-1)^2(x+2)} \\ 8x^2+x-5 &= A(2x-1)(x+2) + B(x+2) + C(2x-1)^2 \end{aligned}$$

Yn amnewid  $x = \frac{1}{2}$

$$8\left(\frac{1}{2}\right)^2 + \frac{1}{2} - 5 = A(0) + B\left(\frac{1}{2} + 2\right) + C(0)^2$$

$$2 + \frac{1}{2} - 5 = \frac{5}{2}B$$

$$-\frac{5}{2} = \frac{5}{2}B$$

$$B = -1$$

Yn amnewid  $x = -2$

$$8(-2)^2 + -2 - 5 = A(0) + B(0) + C(2(-2) - 1)^2$$

$$32 - 2 - 5 = C(-5)^2$$

$$25 = 25C$$

$$C = 1$$

Yn cymharu cyfernodau  $x^2$

$$8 = 2A + 4C$$

$$8 = 2A + 4$$

$$4 = 2A$$

$$A = 2$$

$$\text{Felly } \frac{8x^2+x-5}{(2x-1)^2(x+2)} = \frac{2}{2x-1} - \frac{1}{(2x-1)^2} + \frac{1}{x+2}$$

$$\begin{aligned}
 \text{(b)} \int \frac{8x^2+x-5}{(2x-1)^2(x+2)} dx &= \int \frac{2}{2x-1} - \frac{1}{(2x-1)^2} + \frac{1}{x+2} dx \\
 &= \int \frac{2}{2x-1} dx - \int (2x-1)^{-2} dx + \int \frac{1}{x+2} dx \\
 &= \ln|2x-1| + \frac{1}{2} (2x-1)^{-1} + \ln|x+2| + K \\
 &= \ln|2x-1| + \frac{1}{2(2x-1)} + \ln|x+2| + K
 \end{aligned}$$

Haf 2006

$$\begin{aligned}
 \text{(1) (a)} \frac{2x^2+4}{(x-2)^2(x+4)} &\equiv \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+4} \\
 \frac{2x^2+4}{(x-2)^2(x+4)} &\equiv \frac{A(x-2)(x+4) + B(x+4) + C(x-2)^2}{(x-2)^2(x+4)} \\
 2x^2+4 &\equiv A(x-2)(x+4) + B(x+4) + C(x-2)^2
 \end{aligned}$$

Yn amnewid  $x=2$ :

$$2(2^2)+4 \equiv A(0) + B(2+4) + C(0)$$

$$8+4 \equiv 6B$$

$$12 \equiv 6B$$

$$B = 2$$

Yn amnewid  $x=-4$ :

$$2(-4)^2+4 \equiv A(0) + B(0) + C(-4-2)^2$$

$$32+4 \equiv 36C$$

$$36 \equiv 36C$$

$$C = 1$$

Yn cymharu cyfernodau  $x^2$ :

$$2 \equiv A + C$$

$$2 \equiv A + 1$$

$$A = 1$$

$$\text{Felly } \frac{2x^2+4}{(x-2)^2(x+4)} = \frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x+4}$$

$$\begin{aligned} \text{(b) } f'(x) &= \frac{d}{dx} \left( \frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x+4} \right) \\ &= \frac{d}{dx} \left( (x-2)^{-1} + 2(x-2)^{-2} + (x+4)^{-1} \right) \\ &= (-1)(x-2)^{-2} - 4(x-2)^{-3} + (-1)(x+4)^{-2} \\ &= \frac{-1}{(x-2)^2} - \frac{4}{(x-2)^3} - \frac{1}{(x+4)^2} \end{aligned}$$

$$\begin{aligned} \text{Felly } f'(0) &= \frac{-1}{(0-2)^2} - \frac{4}{(0-2)^3} - \frac{1}{(0+4)^2} \\ &= \frac{-1}{4} - \frac{4}{-8} - \frac{1}{16} \\ &= \frac{3}{16} \end{aligned}$$

Haf 2007

$$\begin{aligned} \text{(1) (a) } \frac{x+3}{x^2(x-1)} &\equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \\ \frac{x+3}{x^2(x-1)} &\equiv \frac{Ax(x-1) + B(x-1) + Cx^2}{x^2(x-1)} \\ x+3 &\equiv Ax(x-1) + B(x-1) + Cx^2 \end{aligned}$$

In answerid  $x=0$ :

$$0+3 \equiv A(0) + B(0-1) + C(0)$$

$$3 \equiv -B$$

$$B = -3$$

Yn amnewid  $x = 1$ :

$$1 + 3 \equiv A(0) + B(0) + C(1^2)$$

$$4 \equiv 1C$$

$$C = 4$$

Yn cymharu cyfernodau  $x^2$ :

$$0 \equiv A + C$$

$$0 \equiv A + 4$$

$$A = -4$$

$$\text{Felly } \frac{x+3}{x^2(x-1)} = \frac{-4}{x} - \frac{3}{x^2} + \frac{4}{x-1}$$

$$\begin{aligned} \text{(b)} \int \frac{x+3}{x^2(x-1)} dx &= \int \frac{-4}{x} - \frac{3}{x^2} + \frac{4}{x-1} dx \\ &= \int \frac{-4 dx}{x} - \int 3x^{-2} dx + \int \frac{4}{x-1} dx \\ &= -4 \ln|x| + 3x^{-1} + 4 \ln|x-1| + K \\ &= -4 \ln|x| + \frac{3}{x} + 4 \ln|x-1| + K \end{aligned}$$

Itaf 2008

$$\textcircled{1} \quad (a) \quad \frac{1}{x^2(2x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$$

$$\frac{1}{x^2(2x-1)} \equiv \frac{Ax(2x-1) + B(2x-1) + Cx^2}{x^2(2x-1)}$$

$$1 \equiv Ax(2x-1) + B(2x-1) + Cx^2$$

Yn amnewid  $x=0$ :

$$1 \equiv A(0) + B(2 \times 0 - 1) + C(0)$$

$$1 \equiv B(-1)$$

$$B = -1$$

Yn amnewid  $x = \frac{1}{2}$ :

$$1 \equiv A(0) + B(0) + C\left(\frac{1}{2}\right)^2$$

$$1 \equiv \frac{1}{4}C$$

$$C = 4$$

Yn cymharu cyfernodau  $x^2$ :

$$0 \equiv 2A + C$$

$$0 \equiv 2A + 4$$

$$-4 \equiv 2A$$

$$A = -2$$

$$\text{Felly } f(x) = \frac{-2}{x} - \frac{1}{x^2} + \frac{4}{2x-1}$$

$$(b) \quad \int f(x) dx = \int \frac{-2}{x} - \frac{1}{x^2} + \frac{4}{2x-1} dx$$

$$= \int \frac{-2}{x} dx - \int x^{-2} dx + \int \frac{4}{2x-1} dx$$

$$= -2 \ln|x| + x^{-1} + 2 \ln|2x-1| + K$$

$$= -2 \ln|x| + \frac{1}{x} + 2 \ln|2x-1| + K$$

Haf 2009

$$\begin{aligned} \textcircled{1} \text{ (a)} \quad \frac{3x}{(1+x)^2(2+x)} &\equiv \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{2+x} \\ \frac{3x}{(1+x)^2(2+x)} &\equiv \frac{A(1+x)(2+x) + B(2+x) + C(1+x)^2}{(1+x)^2(2+x)} \\ 3x &\equiv A(1+x)(2+x) + B(2+x) + C(1+x)^2 \end{aligned}$$

Yn amnewid  $x = -1$ :

$$3(-1) \equiv A(0) + B(2+(-1)) + C(0)$$

$$-3 \equiv B$$

$$B = -3$$

Yn amnewid  $x = -2$ :

$$3(-2) \equiv A(0) + B(0) + C(1+(-2))^2$$

$$-6 \equiv 1C$$

$$C = -6$$

Yn cymharu cyfernodau  $x^2$ :

$$0 \equiv A + C$$

$$0 \equiv A - 6$$

$$A = 6$$

$$\text{Felly } f(x) = \frac{6}{1+x} - \frac{3}{(1+x)^2} - \frac{6}{2+x}$$

$$\begin{aligned} \text{(b)} \quad \int_0^1 f(x) dx &= \int_0^1 \frac{6}{1+x} - \frac{3}{(1+x)^2} - \frac{6}{2+x} dx \\ &= \int_0^1 \frac{6}{1+x} - 3(1+x)^{-2} - \frac{6}{2+x} dx \\ &= \left[ 6 \ln|1+x| + 3(1+x)^{-1} - 6 \ln|2+x| \right]_0^1 \end{aligned}$$

$$\begin{aligned}
&= \left[ \left( 6 \ln(2) + 3(2)^{-1} - 6 \ln(3) \right) \right. \\
&\quad \left. - \left( 6 \ln(1) + 3(1)^{-1} - 6 \ln(2) \right) \right] \\
&= 6 \ln(2) + \frac{3}{2} - 6 \ln(3) - 6 \ln(1) - 3 + 6 \ln(2) \\
&= 0.226 \text{ i } 3 \text{ ike degol}
\end{aligned}$$

Haf 2010

① (a) 
$$\frac{8-x-x^2}{x(x-2)^2} \equiv \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{8-x-x^2}{x(x-2)^2} \equiv \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2}$$

$$8-x-x^2 \equiv A(x-2)^2 + Bx(x-2) + Cx$$

Amnewid  $x=2$ :

$$\begin{aligned}
8-2-2^2 &\equiv A(0) + B(0) + C(2) \\
2 &\equiv 2C \\
C &= 1
\end{aligned}$$

Amnewid  $x=0$ :

$$\begin{aligned}
8-0-0^2 &\equiv A(0-2)^2 + B(0) + C(0) \\
8 &\equiv 4A \\
A &= 2
\end{aligned}$$

Yn cymharu cyfernodau  $x^2$ :

$$\begin{aligned}
-1 &\equiv A + B \\
-1 &\equiv 2 + B \\
B &= -3
\end{aligned}$$

$$\text{Felly } f(x) = \frac{2}{x} - \frac{3}{x-2} + \frac{1}{(x-2)^2}$$

$$(b) f(x) = 2x^{-1} - 3(x-2)^{-1} + (x-2)^{-2}$$

$$f'(x) = 2(-1)x^{-2} - 3(-1)(x-2)^{-2} + (-2)(x-2)^{-3}$$

$$f'(x) = \frac{-2}{x^2} + \frac{3}{(x-2)^2} - \frac{2}{(x-2)^3}$$

$$f'(1) = \frac{-2}{1^2} + \frac{3}{(1-2)^2} - \frac{2}{(1-2)^3}$$

$$f'(1) = -2 + 3 - \frac{2}{-1}$$

$$f'(1) = -2 + 3 + 2$$

$$f'(1) = 3$$

Haf 2011

$$(1) (a) \frac{x^2+x+13}{(x+2)^2(x-3)} \equiv \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-3}$$

$$\frac{x^2+x+13}{(x+2)^2(x-3)} \equiv \frac{A(x+2)(x-3) + B(x-3) + C(x+2)^2}{(x+2)^2(x-3)}$$

$$x^2+x+13 \equiv A(x+2)(x-3) + B(x-3) + C(x+2)^2$$

Amnewid  $x = 3$ :

$$3^2+3+13 \equiv A(0) + B(0) + C(3+2)^2$$

$$25 \equiv 25C$$

$$C = 1$$

Amnewid  $x = -2$ :

$$(-2)^2 + -2 + 13 \equiv A(0) + B(-2-3) + C(0)$$

$$15 \equiv -5B$$

$$B = -3$$

Yn symharu cyfernodau  $x^2$ :

$$1 \equiv A + C$$

$$1 \equiv A + 1$$

$$A = 0$$



$$\text{Felly } f(x) = \frac{-3}{(x+2)^2} + \frac{1}{x-3}$$

$$\begin{aligned} \text{(b) } \int_6^7 f(x) dx &= \int_6^7 \frac{-3}{(x+2)^2} + \frac{1}{x-3} dx \\ &= \int_6^7 -3(x+2)^{-2} + \frac{1}{x-3} dx \\ &= \left[ 3(x+2)^{-1} + \ln|x-3| \right]_6^7 \\ &= \left[ \left( 3(7+2)^{-1} + \ln(4) \right) - \left( 3(6+2)^{-1} + \ln(3) \right) \right] \\ &= \frac{3}{9} + \ln(4) - \frac{3}{8} - \ln(3) \\ &= 0.246 \text{ i } 3 \text{ lle degol.} \end{aligned}$$

Haf 2012

$$\begin{aligned} \text{(a) } \frac{11+x-x^2}{(x+1)(x-2)^2} &\equiv \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \\ \frac{11+x-x^2}{(x+1)(x-2)^2} &\equiv \frac{A(x-2)^2 + B(x+1)(x-2) + C(x+1)}{(x+1)(x-2)^2} \\ 11+x-x^2 &\equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1) \end{aligned}$$

Yn amnewid  $x=2$ :

$$11+2-2^2 \equiv A(0) + B(0) + C(2+1)$$

$$9 \equiv 3C$$

$$C = 3$$

Yn amnewid  $x=-1$ :

$$11+(-1)-(-1)^2 \equiv A(-1-2)^2 + B(0) + C(0)$$

$$9 \equiv 9A$$

$$A = 1$$

Yn cymharu cyfernodau  $x^2$ :

$$-1 \equiv A + B$$

$$-1 \equiv 1 + B$$

$$B = -2$$

$$\text{Felly } f(x) = \frac{1}{x+1} - \frac{2}{x-2} + \frac{3}{(x-2)^2}$$

$$(b) \quad f(x) = (x+1)^{-1} - 2(x-2)^{-1} + 3(x-2)^{-2}$$

$$f'(x) = (-1)(x+1)^{-2} + 2(x-2)^{-2} + 3(-2)(x-2)^{-3}$$

$$f'(x) = \frac{-1}{(x+1)^2} + \frac{2}{(x-2)^2} - \frac{6}{(x-2)^3}$$

$$f'(0) = \frac{-1}{(0+1)^2} + \frac{2}{(0-2)^2} - \frac{6}{(0-2)^3}$$

$$f'(0) = \frac{-1}{1} + \frac{2}{4} - \frac{6}{-8}$$

$$f'(0) = \frac{1}{4}$$

Haf 2013

$$(i) \quad (a) \quad \frac{6+x-9x^2}{x^2(x+2)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$$

$$\frac{6+x-9x^2}{x^2(x+2)} \equiv \frac{Ax(x+2) + B(x+2) + Cx^2}{x^2(x+2)}$$

$$6+x-9x^2 \equiv Ax(x+2) + B(x+2) + Cx^2$$

Amnewid  $x=0$ :

$$6+0-9(0^2) \equiv A(0) + B(0+2) + C(0)$$

$$6 \equiv 2B$$

$$B = 3$$

Amnewid  $x = -2$ :

$$6 + -2 - 9(-2)^2 \equiv A(0) + B(0) + C(-2)^2$$

$$6 - 2 - 36 \equiv 4C$$

$$-32 \equiv 4C$$

$$C = -8$$

Yn cymharu cyfernodau  $x^2$ :

$$-9 \equiv A + C$$

$$-9 \equiv A + -8$$

$$A = -1$$

$$\text{Felly } f(x) = \frac{-1}{x} + \frac{3}{x^2} - \frac{8}{x+2}$$

$$(b) \quad f(x) = -1x^{-1} + 3x^{-2} - 8(x+2)^{-1}$$

$$f'(x) = (-1)(-1)x^{-2} + 3(-2)x^{-3} - 8(-1)(x+2)^{-2}$$

$$f'(x) = \frac{1}{x^2} - \frac{6}{x^3} + \frac{8}{(x+2)^2}$$

$$(ii) \quad f'(2) = \frac{1}{2^2} - \frac{6}{2^3} + \frac{8}{(2+2)^2}$$

$$= \frac{1}{4} - \frac{6}{8} + \frac{8}{16}$$

$$= 0$$

Felly mae gan  $f(x)$  werth arhosol pan fydd  $x=2$ .

C4 Haf 2014

$$\textcircled{2} \text{ (a) } \frac{5x^2+7x+17}{(x+1)^2(x-4)} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-4}$$
$$\frac{5x^2+7x+17}{(x+1)^2(x-4)} \equiv \frac{A(x+1)(x-4) + B(x-4) + C(x+1)^2}{(x+1)^2(x-4)}$$
$$5x^2+7x+17 \equiv A(x+1)(x-4) + B(x-4) + C(x+1)^2$$

Yn amnewid  $x=4$

$$5(4^2)+7 \times 4+17 = A(0) + B(0) + C(5^2)$$

$$80 + 28 + 17 \equiv 25C$$

$$125 = 25C$$

$$C = 5$$

Yn amnewid  $x=-1$

$$5(-1^2)+7 \times -1+17 = A(0) + B(-5) + C(0)$$

$$5 - 7 + 17 = -5B$$

$$15 = -5B$$

$$B = -3$$

Yn cymharu cyfernodau  $x^2$

$$5 = A + C$$

$$5 = A + 5$$

$$A = 0$$

$$\text{Felly } \frac{5x^2+7x+17}{(x+1)^2(x-4)} = \frac{-3}{(x+1)^2} + \frac{5}{x-4}$$

$$\begin{aligned}
 \text{(b)} \quad \frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} &= \frac{5x^2 + 7x + 17 + 2x - 8}{(x+1)^2(x-4)} \\
 &= \frac{5x^2 + 7x + 17}{(x+1)^2(x-4)} + \frac{2x - 8}{(x+1)^2(x-4)} \\
 &= \frac{-3}{(x+1)^2} + \frac{5}{x-4} + \frac{2(x-4)}{(x+1)^2(x-4)} \\
 &= \frac{-3}{(x+1)^2} + \frac{5}{x-4} + \frac{2}{(x+1)^2} \\
 &= \frac{5}{x-4} - \frac{1}{(x+1)^2}
 \end{aligned}$$

## C4 Haf 2015

$$\begin{aligned} 1) \quad (a) \quad \frac{2x^2+5x+25}{(x+3)^2(x-1)} &= \frac{A}{(x+3)} + \frac{B}{(x+3)^2} + \frac{C}{(x-1)} \\ \frac{2x^2+5x+25}{(x+3)^2(x-1)} &= \frac{A(x+3)(x-1) + B(x-1) + C(x+3)^2}{(x+3)^2(x-1)} \\ 2x^2+5x+25 &= A(x+3)(x-1) + B(x-1) + C(x+3)^2 \end{aligned}$$

Yn amnewid  $x=1$

$$2(1)^2 + 5(1) + 25 = A(0) + B(0) + C(4)^2$$

$$2 + 5 + 25 = 16C$$

$$32 = 16C$$

$$\underline{C=2}$$

Yn amnewid  $x=-3$

$$2(-3)^2 + 5(-3) + 25 = A(0) + B(-4) + C(0)^2$$

$$18 - 15 + 25 = -4B$$

$$28 = -4B$$

$$\underline{B=-7}$$

Yn cymharu cyfernodau  $x^2$

$$2 = A + C$$

$$2 = A + 2$$

$$\underline{A=0}$$

$$\text{Felly } \frac{2x^2+5x+25}{(x+3)^2(x-1)} = \frac{-7}{(x+3)^2} + \frac{2}{(x-1)}$$

$$\begin{aligned}
b) \int_3^{10} f(x) dx &= \int_3^{10} \frac{2x^2 + 5x + 25}{(x+3)^2(x-1)} dx \\
&= \int_3^{10} \frac{-7}{(x+3)^2} + \frac{2}{x-1} dx \\
&= -7 \int_3^{10} (x+3)^{-2} dx + 2 \int_3^{10} \frac{1}{x-1} dx \\
&= -7 \left[ -(x+3)^{-1} \right]_3^{10} + 2 \left[ \ln|x-1| \right]_3^{10} \\
&= -7 \left[ \frac{-1}{(x+3)} \right]_3^{10} + 2 \left[ \ln(10-1) - \ln(3-1) \right] \\
&= -7 \left[ \frac{-1}{10+3} - \frac{-1}{3+3} \right] + 2 \left[ \ln(9) - \ln(2) \right] \\
&= -7 \left[ -\frac{1}{13} + \frac{1}{6} \right] + 2 \ln(9) - 2 \ln(2) \\
&= \frac{-49}{78} + 2 \ln(9) - 2 \ln(2) \\
&= 2.379949665 \\
&= 2.38 \text{ ; 2 le dego}
\end{aligned}$$

C4 Haf 2016

$$\textcircled{1} \text{ a) } \frac{17+4x-x^2}{(2x-1)(x-3)^2} \equiv \frac{A}{2x-1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$\frac{17+4x-x^2}{(2x-1)(x-3)^2} \equiv \frac{A(x-3)^2 + B(2x-1)(x-3) + C(2x-1)}{(2x-1)(x-3)^2}$$

$$17+4x-x^2 \equiv A(x-3)^2 + B(2x-1)(x-3) + C(2x-1)$$

Yn amnewid  $x = 3$

$$17+4(3)-3^2 = A(0) + B(6-1)(0) + C(6-1)$$

$$17+12-9 = 5C$$

$$20 = 5C$$

$$C = 4$$

Yn amnewid  $x = \frac{1}{2}$

$$17+4\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)^2 = A\left(\frac{1}{2}-3\right)^2 + B(0)\left(\frac{1}{2}-3\right) + C(0)$$

$$17+2-0.25 = 6.25A$$

$$18.75 = 6.25A$$

$$A = 3$$

Yn cymharu cyfernodau  $x^2$

$$-1 = A + 2B$$

$$-1 = 3 + 2B$$

$$-4 = 2B$$

$$B = -2$$

Felly 
$$\frac{17+4x-x^2}{(2x-1)(x-3)^2} = \frac{3}{2x-1} - \frac{2}{x-3} + \frac{4}{(x-3)^2}$$



$$b) f(x) = \frac{17 + 4x - x^2}{(2x-1)(x-3)^2}$$

$$f(x) = \frac{3}{2x-1} - \frac{2}{x-3} + \frac{4}{(x-3)^2}$$

$$f(x) = 3(2x-1)^{-1} - 2(x-3)^{-1} + 4(x-3)^{-2}$$

$$f'(x) = (-1)3(2x-1)^{-2}(2) - 2(-1)(x-3)^{-2}(1) - 2(4)(x-3)^{-3}(1)$$

$$f'(x) = -6(2x-1)^{-2} + 2(x-3)^{-2} - 8(x-3)^{-3}$$

$$f'(x) = \frac{-6}{(2x-1)^2} + \frac{2}{(x-3)^2} - \frac{8}{(x-3)^3}$$

## Atebion C4 Haf 2017

$$\begin{aligned} 1) \quad \frac{8x^2 + 7x - 25}{(x-1)^2(x+4)} &= \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+4)} \\ \frac{8x^2 + 7x - 25}{(x-1)^2(x+4)} &= \frac{A(x-1)(x+4) + B(x+4) + C(x-1)^2}{(x-1)^2(x+4)} \\ 8x^2 + 7x - 25 &= A(x-1)(x+4) + B(x+4) + C(x-1)^2 \end{aligned}$$

Yn amnewid  $x = 1$

$$8(1)^2 + 7(1) - 25 = A(0) + B(1+4) + C(0)$$

$$8 + 7 - 25 = 5B$$

$$-10 = 5B$$

$$\underline{B = -2}$$

Yn amnewid  $x = -4$

$$8(-4)^2 + 7(-4) - 25 = A(0) + B(0) + C(-4-1)^2$$

$$8(16) - 28 - 25 = 25C$$

$$75 = 25C$$

$$\underline{C = 3}$$

Yn cymharu cyfernodau  $x^2$

$$8 = A + C$$

$$8 = A + 3$$

$$\underline{A = 5}$$

$$\text{Felly } \frac{8x^2 + 7x - 25}{(x-1)^2(x+4)} = \frac{5}{x-1} - \frac{2}{(x-1)^2} + \frac{3}{x+4}$$

$$\begin{aligned}
 \text{b) } \frac{9x^2 + 5x - 24}{(x-1)^2(x+4)} &= \frac{8x^2 + 7x - 25}{(x-1)^2(x+4)} + \frac{x^2 - 2x + 1}{(x-1)^2(x+4)} \\
 &= \frac{5}{x-1} - \frac{2}{(x-1)^2} + \frac{3}{x+4} + \frac{\cancel{(x-1)^2}}{\cancel{(x-1)^2}(x+4)} \\
 &= \frac{5}{x-1} - \frac{2}{(x-1)^2} + \frac{3}{x+4} + \frac{1}{x+4} \\
 &= \frac{5}{x-1} - \frac{2}{(x-1)^2} + \frac{4}{x+4}
 \end{aligned}$$

## Atebion C4 Haf 2018

$$1) \quad f(x) = \frac{3x^2 - 3x - 8}{x(x-2)^2}$$

$$a) \quad \frac{3x^2 - 3x - 8}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{3x^2 - 3x - 8}{x(x-2)^2} = \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2}$$

$$3x^2 - 3x - 8 = A(x-2)^2 + Bx(x-2) + Cx$$

Yn amnewid  $x = 2$ :

$$3(2)^2 - 3(2) - 8 = A(0) + B(2)(0) + C(2)$$

$$12 - 6 - 8 = 2C$$

$$-2 = 2C$$

$$\underline{C = -1}$$

Yn amnewid  $x = 0$ :

$$3(0)^2 - 3(0) - 8 = A(-2)^2 + B(0)(-2) + C(0)$$

$$-8 = 4A$$

$$\underline{A = -2}$$

Yn cymharu cyfernodau  $x^2$ :

$$3 = A + B$$

$$3 = -2 + B$$

$$\underline{B = 5}$$

$$\text{Felly } \frac{3x^2 - 3x - 8}{x(x-2)^2} = \frac{-2}{x} + \frac{5}{x-2} - \frac{1}{(x-2)^2}$$

$$\begin{aligned}
b) \int_6^9 f(x) dx &= \int_6^9 \frac{3x^2 - 3x - 8}{x(x-2)^2} dx \\
&= \int_6^9 \left( -\frac{2}{x} + \frac{5}{x-2} - \frac{1}{(x-2)^2} \right) dx \\
&= \int_6^9 \left( -\frac{2}{x} + \frac{5}{x-2} - (x-2)^{-2} \right) dx \\
&= \left[ -2 \ln|x| + 5 \ln|x-2| + (x-2)^{-1} \right]_6^9 \\
&= \left( -2 \ln(9) + 5 \ln(7) + (9-2)^{-1} \right) - \left( -2 \ln(6) + 5 \ln(4) + (6-2)^{-1} \right) \\
&= -2 \ln(9) + 5 \ln(7) + \frac{1}{7} + 2 \ln(6) - 5 \ln(4) - \frac{1}{4} \\
&= 1.880005866 \\
&= \underline{\underline{1.88}} \text{ ; 2 le dego!}
\end{aligned}$$

C4 Haf 2019

$$1) f(x) = \frac{2 + 17x + 9x^2}{(x+1)^2(x+3)}$$

$$a) \frac{2 + 17x + 9x^2}{(x+1)^2(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$$

$$\frac{2 + 17x + 9x^2}{(x+1)^2(x+3)} = \frac{A(x+1)(x+3) + B(x+3) + C(x+1)^2}{(x+1)^2(x+3)}$$

$$2 + 17x + 9x^2 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$$

Yn amrywid  $x = -1$ :

$$2 + 17(-1) + 9(-1)^2 = A(0) + B(2) + C(0)$$

$$2 - 17 + 9 = 2B$$

$$-6 = 2B$$

$$\underline{B = -3}$$

Yn amrywid  $x = -3$ :

$$2 + 17(-3) + 9(-3)^2 = A(0) + B(0) + C(4)$$

$$2 - 51 + 81 = 4C$$

$$32 = 4C$$

$$\underline{C = 8}$$

Yn cyntaru cyfernodau  $x^2$ :

$$9 = A + C$$

$$A = 9 - C$$

$$= 9 - 8$$

$$\underline{A = 1}$$

$$\text{Felly } \frac{2 + 17x + 9x^2}{(x+1)^2(x+3)} = \frac{1}{(x+1)} - \frac{3}{(x+1)^2} + \frac{8}{(x+3)}$$

$$\begin{aligned}
 b) \text{ ii) } f(x) &= \frac{1}{(x+1)} - \frac{3}{(x+1)^2} + \frac{8}{(x+3)} \\
 &= (x+1)^{-1} - 3(x+1)^{-2} + 8(x+3)^{-1} \\
 f'(x) &= -1(x+1)^{-2}(1) + 6(x+1)^{-3}(1) - 8(x+3)^{-2}(1)
 \end{aligned}$$

$$f'(x) = \frac{-1}{(x+1)^2} + \frac{6}{(x+1)^3} - \frac{8}{(x+3)^2}$$

$$\begin{aligned}
 \text{ii) } f'(1) &= \frac{-1}{(1+1)^2} + \frac{6}{(1+1)^3} - \frac{8}{(1+3)^2} \\
 &= \frac{-1}{4} + \frac{6}{8} - \frac{8}{16} \\
 &= \frac{-4}{16} + \frac{12}{16} - \frac{8}{16} \\
 &= 0
 \end{aligned}$$

Gan fod  $f'(x)$  yn 0, mae ganddo swynk arhosol pan fo  $x=1$ .