Hen Gwestiynau Arholiad

Further Integration

(Haf 2005)

7. *(a)* Use the substitution $u = 2x - 1$ to evaluate

$$\int_0^1 x(2x-1)^9 \, \mathrm{d}x \, .$$ [5]

(b) (i) Find $\int x \cos 2x \, \mathrm{d}x$. [4]

(ii) Use the result of *(b)*(i) to find

$$\int x \cos^2 x \, \mathrm{d}x \, .$$ [3]

(Haf 2006)

7. *(a)* Find $\int x \ln x \, \mathrm{d}x$. [5]

(b) Use the substitution $u = 2\sin x + 3$ to evaluate

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{(2\sin x + 3)^2} \, \mathrm{d}x \, .$$ [4]

(Haf 2007)

7. *(a)* Find $\int x^2 \ln x \, \mathrm{d}x$. [4]

(b) Use the substitution $x = 2\sin\theta$ to show that

$$\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4 - x^2}} \, \mathrm{d}x = \int_0^a k \sin^2 \theta \, \mathrm{d}\theta \, ,$$

where the values of a and k are to be determined.

Hence, or otherwise, evaluate $\displaystyle\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4 - x^2}} \, \mathrm{d}x$. [8]

(Haf 2008)

6. *(a)* Find $\int (3x+1)\, e^{2x} dx$. [4]

(b) Use the substitution $x = 3\sin\theta$ to show that

$$\int_{1\cdot 5}^{3} \sqrt{9-x^2}\, dx = \int_a^b k\cos^2\theta\, d\theta \quad ,$$

where the values of the constants a, b and k are to be found.

Hence evaluate $\int_{1\cdot 5}^{3} \sqrt{9-x^2}\, dx$. [8]

(Haf 2009)

6. *(a)* Find $\int (x+3)e^{2x}\, dx$. [4]

(b) Use the substitution $u = 2\cos x + 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \frac{\sin x}{\sqrt{(2\cos x + 1)}}\, dx.$$

[5]

(Haf 2010)

7. *(a)* Find $\int x^3 \ln x\, dx$.

[4]

(b) Use the substitution $u = 2x - 3$ to evaluate $\int_1^2 x(2x-3)^4\, dx$. [5]

(Haf 2011)

7. *(a)* Find $\int x\sin 2x\, dx$. [4]

(b) Use the substitution $u = 5 - x^2$ to evaluate

$$\int_0^2 \frac{x}{(5-x^2)^3}\, dx \ .$$

[4]

(Haf 2012)

7. *(a)* Find $\int xe^{-2x}\, dx$. [4]

(b) Use the substitution $u = 1 + 3\ln x$ to evaluate

$$\int_1^{e} \frac{1}{x(1+3\ln x)} dx.$$

Give your answer correct to four decimal places. [4]

(Haf 2013)

7. *(a)* Find $\int (3x-1)\cos 2x \, dx$. [4]

(b) Use the substitution $u = 2x + 1$ to evaluate

$$\int_0^1 \frac{x}{(2x+1)^3} \, dx.$$ [5]

(Haf 2014)

7. *(a)* Find $\int x^4 \ln 2x \, dx$. [4]

(b) Use the substitution $u = 10\cos x - 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \sqrt{(10\cos x - 1)} \sin x \, dx.$$ [4]

(Haf 2015)

7. *(a)* Use the substitution $u = 12 - x^3$ to evaluate

$$\int_0^2 \frac{x^2}{(12-x^3)^2} \, dx .$$ [4]

(b) (i) Find $\int x\cos 2x \, dx$.

(ii) Use the result of *(b)*(i) to find

$$\int x \sin^2 x \, dx.$$ [7]

(Haf 2016)

6. *(a)* Find $\int (2x+1)e^{-3x} dx$. [4]

(b) Use the substitution $u = 4 + 5\tan x$ to evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5\tan x}}{\cos^2 x} \, dx.$$ [4]

(Haf 2017)

7. *(a)* Find $\int \frac{\ln x}{x^4} \, dx$. [4]

(b) Use the substitution $u = x^2 + 1$ to evaluate

$$\int_0^1 x^3(x^2+1)^4 \, dx .$$ [5]

(Haf 2018)

7. *(a)* Find $\int (4x+1)e^{4x-5}\,dx$. Simplify your answer. [4]

(b) (i) Use the substitution $x = 4\sin\theta$ to show that

$$\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}}\,dx = \int_0^a b\sin^2\theta\,d\theta,$$

where a and b are constants whose values are to be determined.

(ii) **Hence**, evaluate

$$\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}}\,dx.$$

Give your answer in the form $c\pi + d$, where c and d are integers whose values are to be determined. [8]