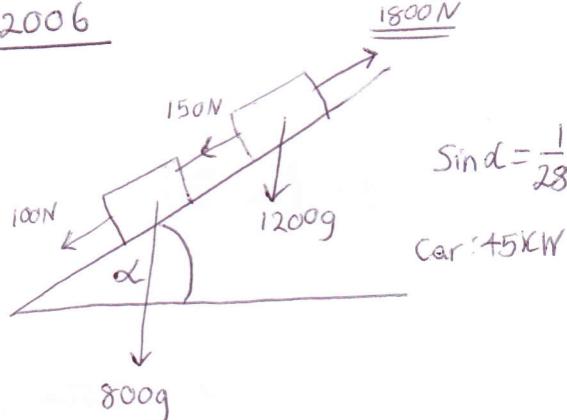


Work, Energy and Power

Haf 2006



$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$\text{so } 45 \times 1000 = \text{Force} \times 25$$

$$\text{Force} = 1800\text{ N}$$

(a) $F=ma$, applied to the whole system:

$$1800 - 150 - 100 - \text{weight component} = ma_{\text{down slope}}$$

$$1550 - 2000g \sin \alpha = 2000a$$

$$1550 - 2000g \left(\frac{1}{28}\right) = 2000a$$

$$a = 0.425 \text{ ms}^{-2}$$

(b) $F=ma$, applied to the trailer:

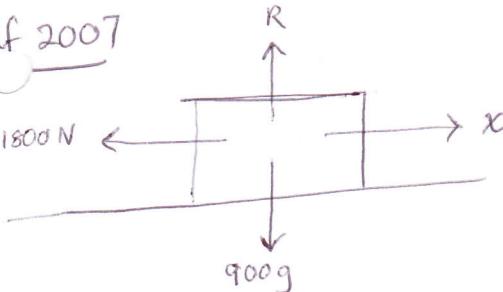
$$\text{Tension} - 100 - \text{weight component} = ma_{\text{down slope}}$$

$$T - 100 - 800g \sin \alpha = 800a$$

$$T - 100 - 800g \left(\frac{1}{28}\right) = 800 \times 0.425$$

$$T = 720\text{ N}$$

Haf 2007



(a)

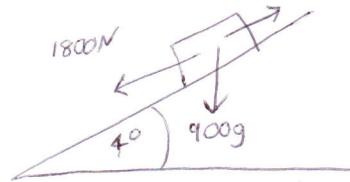
At maximum velocity the acceleration is zero, so there must be no resultant force ($F=ma$) and so we must have $x = 1800\text{ N}$.

But $\text{Power} = \text{Force} \times \text{Velocity}$

$$45 \times 1000 = 1800 \times \text{Velocity}$$

$$\text{Max. Velocity} = 25 \text{ ms}^{-1}$$

(b)



$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$\text{so } 45 \times 1000 = \text{Force} \times 15$$

$$\text{Force} = 3000\text{ N}$$

$F=ma$, applied parallel to the slope:
 $3000 - 1800 - \text{weight component} = ma_{\text{down slope}}$

$$1200 - 900g \sin 4^\circ = 900a$$

($a = 0.6497 \text{ ms}^{-2}$ to 4d.p.)

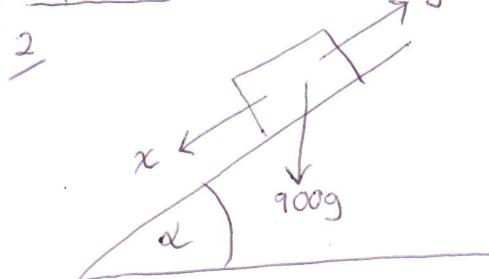
$a = 0.65 \text{ ms}^{-2}$ to 2d.p.)

(c) $\text{Work} = \text{Force} \times \text{distance}$

$$= 1800 \times 800$$

$$= 1440000 \text{ J}$$

Haf 2008



$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$32 \times 1000 = \text{Force} \times 16$$

$$\text{Force} = 2000\text{ N}$$

$$\text{So } y = 2000\text{ N}.$$

Constant speed so $a = 0$.

$F=ma$, applied parallel to the slope:

$$2000 - \text{resistive} - \text{weight component} = ma_{\text{down slope}}$$

Force

$$2000 - x - 900g \sin \alpha = 0$$

$$2000 - x - 900g \left(\frac{8}{49}\right) = 0$$

$$x = 560\text{ N}$$

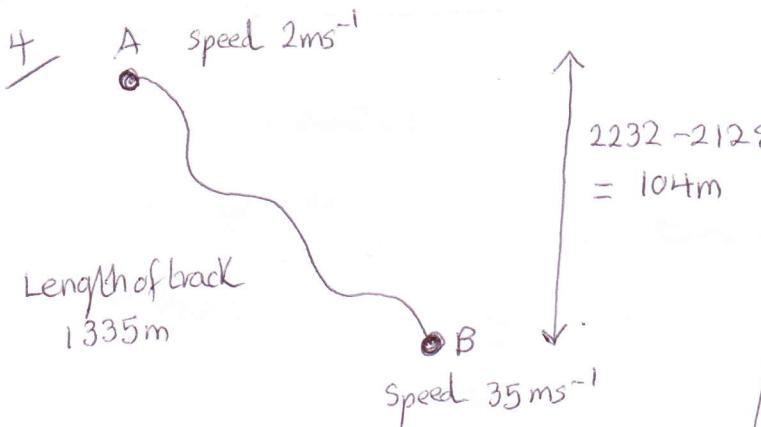
Formulae:

$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$\text{Work} = \text{Force} \times \text{Distance}$$

$$F = ma$$

Haf 2008



$$(a) \text{Loss in potential energy} = mgh$$

$$= (50 + 40) \times 9.8 \times 104$$

$$= 91728 \text{ J}$$

Gain in Kinetic energy

$$= \frac{1}{2} m V_B^2 - \frac{1}{2} m V_A^2$$

$$= \frac{1}{2} \times 90 \times 35^2 - \frac{1}{2} \times 90 \times 2^2$$

$$= 54945 \text{ J}$$

The difference between the gain in K.E. and the loss in P.E. is the work done against resistance.

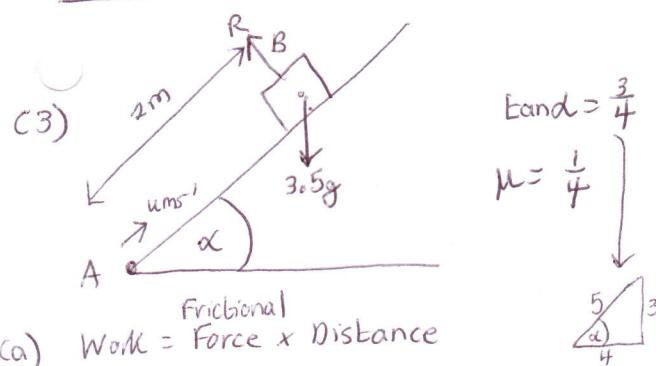
$$91728 - 54945 = \underline{\underline{36783 \text{ J}}}$$

$$(b) \text{Work} = \text{Force} \times \text{Distance}$$

$$36783 = \text{Force} \times 1335$$

$$\text{Force} = 27.5528 \text{ N, to 4d.p.}$$

Haf 2009



$$(a) \text{Work} = \text{Force} \times \text{Distance}$$

$$\text{Frictional Force} = \mu R$$

Resolve perpendicular to the plane:

$$R = 3.5g \cos \alpha$$

$$R = 3.5 \times 9.8 \times \frac{4}{5}$$

$$R = 27.44 \text{ N}$$

$$\text{So Frictional Force} = 6.86 \text{ N}$$

Work done against friction

$$= 6.86 \times 2$$

$$= 13.72 \text{ J}$$

$$(b) \text{KE at } A = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 3.5 \times u^2$$

$$= 1.75u^2$$

At B, this has been turned into PE.

$$\text{PE at } B = mgh$$

$$= 3.5 \times 9.8 \times 2 \sin \alpha$$

$$= 3.5 \times 9.8 \times 2 \times \frac{3}{5}$$

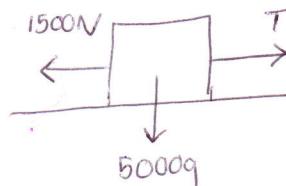
$$= 41.16 \text{ J}$$

$$\text{so } 1.75u^2 = 41.16 + 13.72$$

$$u^2 = 31.36$$

$$u = 5.6 \cdot \text{ms}^{-1}$$

(4)



$$a) \text{Power} = \text{Force} \times \text{Velocity}$$

$$\text{Power} = \text{Force} \times 12$$

$$\text{Force using } F = ma$$

$$T - 1500 = 5000 \times 0.2$$

$$T = 2500 \text{ N}$$

$$\text{so Power} = 2500 \times 12$$

$$= 30000 \text{ W}$$

$$b) \text{Power} = \text{Force} \times \text{Velocity}$$

$$45000 = \text{Force} \times \text{Velocity}$$

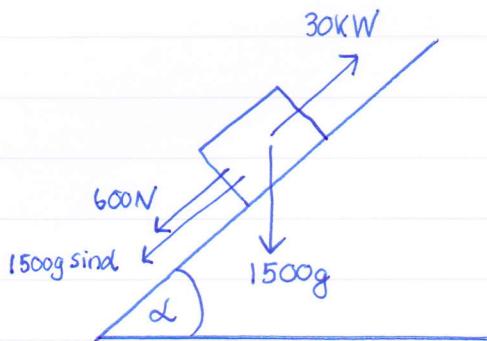
At maximum speed the acceleration is zero.

So in $F = ma$, we must have $T = 1500$.

$$\text{so } 45000 = 1500 \times v$$

$$v = 30 \text{ ms}^{-1}$$

(4)



$$\sin \alpha = \frac{6}{49}$$

a) Yn defnyddio $F = ma$ ar y gwrtiwrch, yn barael i'r hethr

$$T - 600 - 1500g \sin \alpha = 1500a$$

↓

$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$30000 \text{ W} = \text{Force} \times 8 \text{ ms}^{-1}$$

$$\text{Force} = 3750 \text{ N}$$

$$3750 - 600 - 1500 \times 9.8 \times \frac{6}{49} = 1500a$$

$$1350 = 1500a$$

$$\underline{a = 0.9 \text{ ms}^{-2}}$$

b) Buannedd mawsum ym awgrymu $a = 0 \text{ ms}^{-2}$.

$$T - 600 - 1500g \sin \alpha = 1500 \times 0$$

$$T = 600 + 1500 \times 9.8 \times \frac{6}{49}$$

$$T = 2400 \text{ N}$$

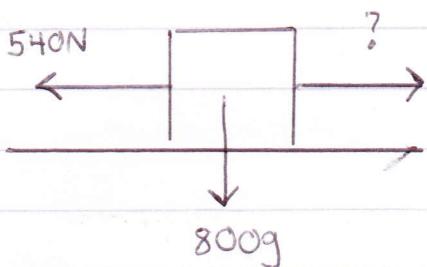
$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$30000 \text{ W} = 2400 \times v$$

$$\underline{v = 12.5 \text{ ms}^{-1}}$$

Itay '11

4)



a) Constant speed 60 ms^{-1} so the acceleration is 0 ms^{-1} .

Using $F=ma$

$$? - 540 = 800 \times 0$$

$$? - 540 = 0$$

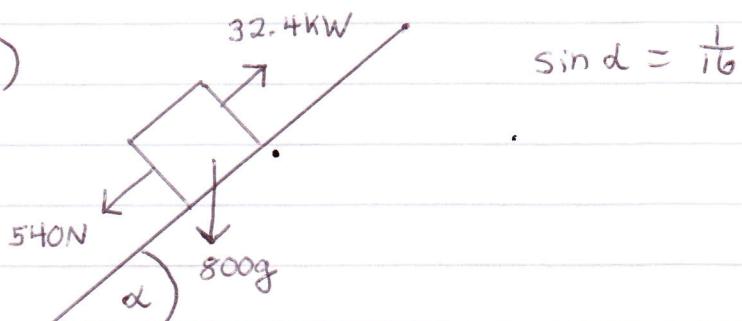
$$? = 540\text{ N}$$

Using Power = Force \times Velocity

$$\text{Power} = 540 \times 60$$

$$\text{Power} = \underline{\underline{32400\text{ W}}}$$

b)



Using Force = Power \div Velocity

$$\text{Force} = (32.4 \times 1000) \div 15$$

$$\text{Force} = 2160\text{ N}$$

Using $F=ma$, parallel to the slope,

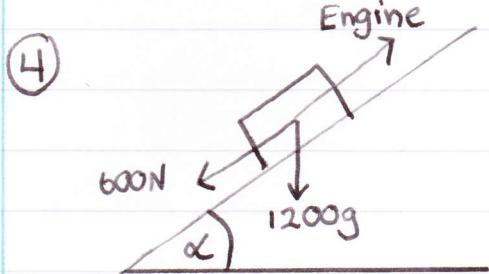
$$2160 - 540 - 800g \sin \alpha = 800 \times a$$

$$2160 - 540 - 800 \times 9.8 \times \frac{1}{16} = 800 \times a$$

$$1130 = 800a$$

$$a = \underline{\underline{1.4125\text{ ms}^{-2}}}$$

M2 Haf 2012



$$\sin \alpha = 0.1$$

(a) Using Power = Force × Velocity

$$75000 = F \times 25$$

$$F = 75000 \div 25$$

$$F = 3000 \text{ N}$$

Using $F=ma$ on the vehicle, parallel to the slope ($\uparrow = +ve$)

$$3000 - 600 - \text{weight component down slope} = 1200a$$

$$3000 - 600 - mgsin\alpha = 1200a$$

$$2400 - 1200 \times 9.8 \times 0.1 = 1200a$$

$$2400 - 1176 = 1200a$$

$$1224 = 1200a$$

$$\therefore a = \underline{\underline{1.02 \text{ ms}^{-2}}}$$

(b) Using $F=ma$ on the vehicle, parallel to the slope ($\uparrow = +ve$)

$$\text{Engine Force} - 600 - 1176 = 1200 \times 0$$

$$\text{Engine Force} - 1776 = 0$$

$$\text{Engine Force} = 1776 \text{ N}$$

constant speed
so zero acceleration

Using Power = Force × Velocity

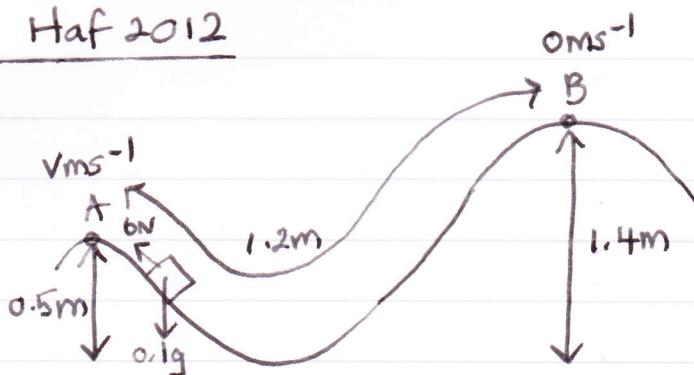
$$90000 = 1776 \times v$$

$$v = 90000 \div 1776$$

$$v = \underline{\underline{50.7 \text{ ms}^{-1}}} \text{ correct to 3 s.f.}$$

M2 Haf 2012

⑤



By Energy Considerations

Kinetic Energy lost = Potential Energy gained + Work done
against friction

$$\frac{1}{2}mv^2 = mg(1.4 - 0.5) + \text{Force} \times \text{Distance}$$

$$\frac{1}{2} \times 0.1 \times v^2 = 0.1 \times 9.8 \times 0.9 + 6 \times 1.2$$

$$0.05v^2 = 0.882 + 7.2$$

$$0.05v^2 = 8.082$$

$$v^2 = 161.64$$

$$v = 12.7 \text{ ms}^{-1}, \text{ correct to 3 s.f.}$$

M2 Haf 2013

i)



a) Kinetic Energy at A = $\frac{1}{2}mv^2$
 $= 0.5 \times 8 \times 7^2$
 $= 196\text{J}$

Kinetic Energy at B = 0J (at rest)

So 196J of Kinetic Energy is lost in going from A to B.

b) All the 196J of Kinetic Energy is lost to work done against friction.

Work done against Friction = 196

Fictional Force \times Distance Travelled = 196

$\mu R \times 15 = 196$

$\mu R = \frac{196}{15}$

But $R = 8g$

$R = 78.4\text{N}$

so

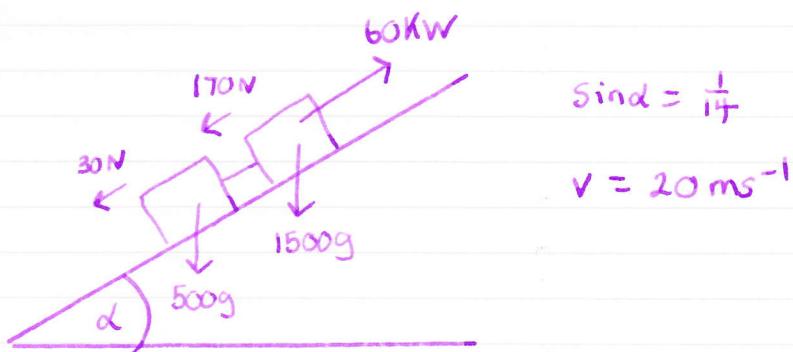
$$\mu(78.4) = \frac{196}{15}$$

$$\mu = \frac{196}{15 \times 78.4}$$

$$\mu = \frac{1}{6}$$

M2 Haf 2013

6)



(a) Power = Force \times Velocity

Car Power = Tractive Force \times Velocity

$$60\text{kW} = F \times 20$$

$$60000 = 20F$$

$$F = 60000 \div 20$$

$$\underline{F = 3000 \text{ N}}$$

(b) $F = ma$ on the trailer and car, parallel to the slope,
 $\nearrow = +$ ive

Tractive Force = $170\text{N} - 30\text{N} - \text{weight component down slope}$
 $= (500 + 1500)a$

$$3000 - 170 - 30 - (500 + 1500) \times 9.8 \times \sin \alpha = 2000a$$

$$2800 - 2000 \times 9.8 \times \frac{1}{14} = 2000a$$

$$2800 - 1400 = 2000a$$

$$\frac{1400}{2000} = a$$

$$\underline{a = 0.7 \text{ ms}^{-2}} \quad \checkmark$$

(c) $F = ma$ on the trailer, parallel to the slope, $\nearrow = +$ ive

Tension in tow bar = $30 - \text{weight component down slope}$
 $= 500a$

$$T - 30 - 500a \sin \alpha = 500a$$

$$T - 30 - 500 \times 9.8 \times \frac{1}{14} = 500 \times 0.7$$

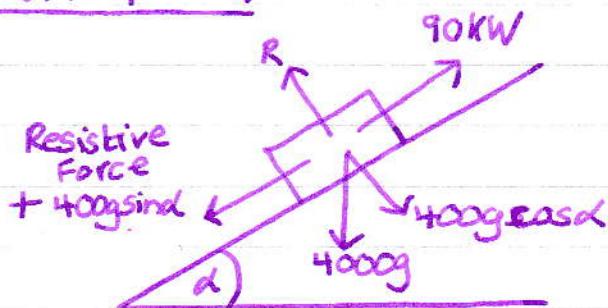
$$T - 30 - 350 = 350$$

$$T = 350 + 350 + 30$$

$$\underline{T = 730 \text{ N}}$$

M2 Haf 2014

③



$$\sin \alpha = \frac{2}{49}$$

(a) When the Speed is 4.8 ms^{-1} , $\text{Power} = \text{Force} \times \text{Velocity}$
 $90000 = \text{Force} \times 4.8$
 $\text{Force} = 90000 \div 4.8$
 $\underline{\text{Force} = 18750 \text{ N}}$
 (Engine Force)

$F=ma$, applied parallel to the slope, $\nearrow = +$
 Engine Force - weight component - Resistive = $4000a$
 down slope Force

$$18750 - 4000g \sin \alpha - \text{Resistive Force} = 4000 \times 1.2$$

$$18750 - 4000 \times 9.8 \times \frac{2}{49} - \text{Resistive Force} = 4800$$

$$17150 - \text{Resistive Force} = 4800$$

$$17150 - 4800 = \text{Resistive Force}$$

$$\underline{\text{Resistive Force} = 12350 \text{ N}}$$

(b) Maximum velocity $\Rightarrow a = 0 \text{ ms}^{-2}$.

$F=ma$, applied parallel to the slope, $\nearrow = +$

Engine Force - weight component - Resistive = $4000a$
 down slope Force

$$\text{Engine Force} - 4000g \sin \alpha - 12800 = 4000 \times 0$$

$$\text{Engine Force} = 4000 \times 9.8 \times \frac{2}{49} + 12800$$

$$\text{Engine Force} = 14400 \text{ N}$$

So, using $\text{Power} = \text{Force} \times \text{Velocity}$

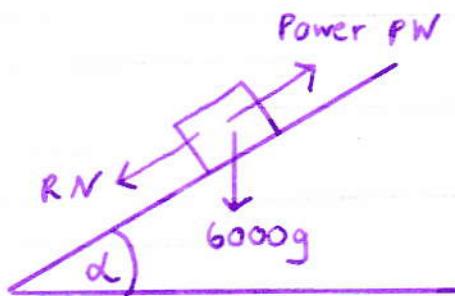
$$90000 = 14400 \times \text{Velocity}$$

$$\text{Velocity} = 90000 \div 14400$$

The maximum
velocity is
 6.25 ms^{-1}

M& Haf 2015

3)



$$\sin \alpha = \frac{6}{49}$$

$$\text{When } v = \frac{16}{5} \text{ ms}^{-1}, a = 2 \text{ ms}^{-2}$$

$$\text{Power} = \text{Force} \times \text{Velocity} \quad \text{so} \quad \text{Force} = \frac{\text{Power}}{\text{Velocity}}.$$

$F = ma$ on the vehicle, parallel to the slope, $\nearrow = +$ ive.

$$\frac{\text{Vehicle's Power}}{\text{Vehicle's Velocity}} - \text{Resistance} - \text{weight component down slope} = ma$$

$$\frac{P}{v} - R - mg \sin \alpha = ma$$

$$\frac{P}{v} - R - 6000g \times \frac{6}{49} = 6000a$$

$$\frac{P}{v} - R - 7200 = 6000a$$

$$\text{When } v = \frac{16}{5}, a = 2, \text{ so}$$

$$\frac{P}{\frac{16}{5}} - R - 7200 = 6000 \times 2$$

$$\frac{5}{16}P - R - 7200 = 12000$$

$$\frac{5}{16}P - 7200 - 12000 = R$$

$$\frac{5}{16}P - 19200 = R \quad \text{--- (1)}$$

$$\text{When } v = \frac{16}{3}, a = 0 \quad (\text{maximum velocity; no acceleration}) \text{ so}$$

$$\frac{P}{\frac{16}{3}} - R - 7200 = 6000 \times 0$$

$$\frac{3}{16}P - R - 7200 = 0$$

$$\frac{3}{16}P - 7200 = R \quad \text{--- (2)}$$

Equating ① and ②:

$$\frac{5}{16}P - 19200 = \frac{3}{16}P - 7200$$

$$\frac{5}{16}P - \frac{3}{16}P = -7200 + 19200$$

$$\frac{2}{16}P = 12000$$

$$P = \frac{12000 \times 16}{2}$$

$$\underline{P = 96000 \text{ W}}$$

Substituting back into equation ①

$$\frac{5}{16}P - 19200 = R$$

$$R = \frac{5}{16} \times 96000 - 19200$$

$$R = 30000 - 19200$$

$$\underline{R = 10800 \text{ N}}$$

M2 Itaf 2016

- (4) On leaving the cannon, the cannon ball has momentum mass \times velocity
 $= 12 \times 600$
 $= 7200 \text{ Ns}$

Using the conservation of momentum, the cannon, on recoiling, will have momentum 7200 Ns.

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

$$7200 = 1600 \times v$$

$$7200 \div 1600 = v$$

$$v = 4.5 \text{ ms}^{-1}$$

- b) On separation, the total energy is given by

$$\begin{aligned} & \text{Kinetic Energy of Cannon} + \text{Kinetic Energy of Cannon Ball} \\ &= \frac{1}{2} m v^2 + \frac{1}{2} m v^2 \\ &= 0.5 \times 1600 \times 4.5^2 + 0.5 \times 12 \times 600^2 \\ &= 16200 + 2160000 \\ &= 2,176,200 \text{ J.} \end{aligned}$$

By the principle of conservation of energy and, assuming that no energy is lost to sound etc; the energy created by the burning of the charge is 2,176,200 J

- c) Work Done = Force \times Distance

$$16,200 = \text{Force} \times 1.2$$

$$\text{Force} = 16,200 \div 1.2$$

$$\text{Force} = 13500 \text{ N}$$

M2 Haf 2016

⑦ Kinetic Energy at A = $\frac{1}{2}mv^2$
= $\frac{1}{2}(70)v^2$
= $35v^2$

Work done against resistance in going from A to B
= Force × Distance
= 50×16
= 800 J

Change in gravitational potential energy going from A to B
= mgh
= $70 \times 9.8 \times (22 - 20)$
= $70 \times 9.8 \times 2$
= 1372 J

For the biker to reach B without pedalling, we have
KE at A = W.D. against resistance + change in GPE

$$35v^2 = 800 + 1372$$

$$35v^2 = 2172$$

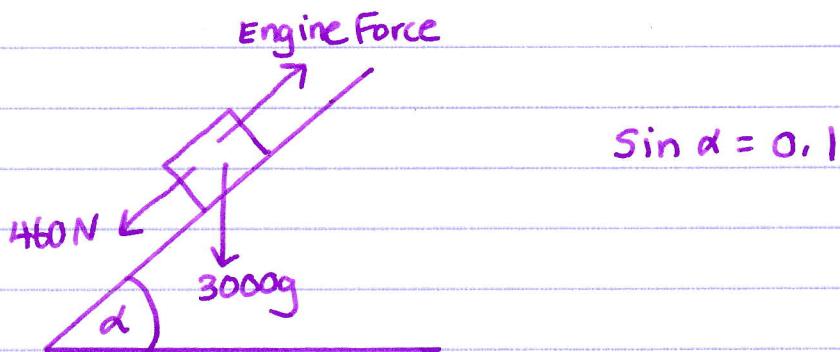
$$v = \sqrt{\frac{2172}{35}}$$

$$v = 7.877635613$$

$$\underline{v = 7.88 \text{ ms}^{-1} \text{ to 2 d.p.}}$$

M1 Hat 2017

3)



- a) Maximum acceleration occurs when the engine's power is operating at its maximum, 12000 W.

Using $\text{Power} = \text{Force} \times \text{Velocity}$

$$12000 = \text{Engine Force} \times 3$$

$$\text{Engine Force} = 4000 \text{ N}$$

Using $F=ma$, parallel to the slope, $\nearrow = +$ ive

Engine Force - Resistance to motion

- weight component down slope = ma

$$4000 - 460 - 3000g \sin \alpha = 3000a$$

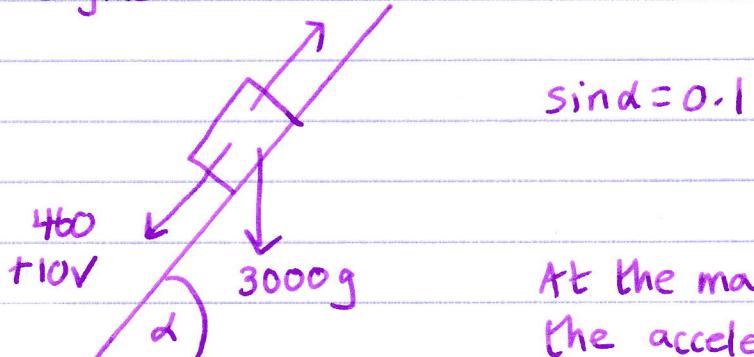
$$4000 - 460 - 3000 \times 9.8 \times 0.1 = 3000a$$

$$600 = 3000a$$

$$\underline{\underline{a = 0.2 \text{ ms}^{-2}}}$$

Engine Force

b)



At the maximum velocity,
the acceleration will be zero,

$$\text{Power} = \text{Engine Force} \times \text{Velocity}$$

$$12000 = \text{Engine Force} \times v$$

$$\text{Engine Force} = \frac{12000}{v}$$

Using $F=ma$, parallel to the slope, $\uparrow = +ive$

$$\text{Engine Force} - \text{Resistance to motion} - 10v$$

$$- \text{weight component down slope} = ma$$

$$\frac{12000}{v} - 460 - 10v - 3000g \sin \alpha = 3000 \times 0$$

$$\frac{12000}{v} - 460 - 10v - 2940 = 0$$

$$\frac{12000}{v} - 3400 - 10v = 0$$

$$12000 - 3400v - 10v^2 = 0$$

$$10v^2 + 3400v - 12000 = 0$$

$$v^2 + 340v - 1200 = 0$$

$$a=1, b=340, c=-1200$$

$$v = \frac{-340 \pm \sqrt{340^2 - 4(1)(-1200)}}{2}$$

$$\frac{2 \times 1}{2}$$

$$\text{Either } v = \frac{-340 + \sqrt{340^2 - 4(1)(-1200)}}{2}$$

$$v = 3.49 \text{ ms}^{-1} \text{ to 2 d.p.}$$

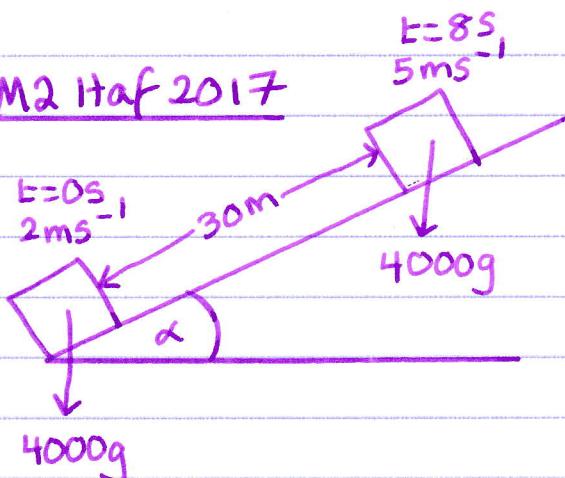
$$\text{or } v = \frac{-340 - \sqrt{340^2 - 4(1)(-1200)}}{2}$$

$$v = -343.49 \text{ ms}^{-1} \text{ to 2 d.p.}$$

↳ Nonsensical; vehicle is now travelling down slope.

M2 Itaf 2017

5)



$$\sin \alpha = \frac{1}{20}$$

Engine Power 43000W

Power = Change in Energy \Rightarrow Engine Power = Work Done by Engine
Change in Time Change in Time

$$43000 = \frac{\text{Work Done by Engine}}{8}$$

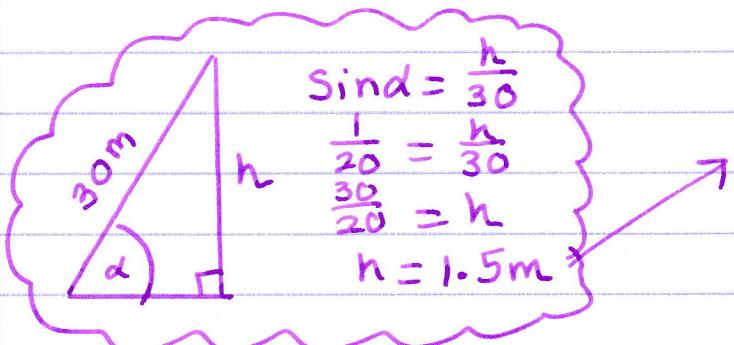
$$\begin{aligned}\text{Work Done by Engine} &= 43000 \times 8 \\ &= 344000 \text{ J}\end{aligned}$$

Some of this energy is converted into Kinetic energy; some into gravitational potential energy; and some is lost due to work done against resistive forces.

$$\begin{aligned}\text{Gain in Kinetic Energy} &= \frac{1}{2} m V_{\text{end}}^2 - \frac{1}{2} m V_{\text{beginning}}^2 \\ &= 0.5 \times 4000 \times 5^2 - 0.5 \times 4000 \times 2^2 \\ &= 42000 \text{ J}\end{aligned}$$

Gain in Gravitational Potential Energy = mgh

$$\begin{aligned}&= 4000 \times 9.8 \times 1.5 \\ &= 58800 \text{ J}\end{aligned}$$



Energy Equation:

Work Done by Engine = Gain in KE + Gain in GPE
+ work done against resistive forces.

Let x denote the work done against resistive forces.

Then

$$344000 = 42000 + 58800 + x$$

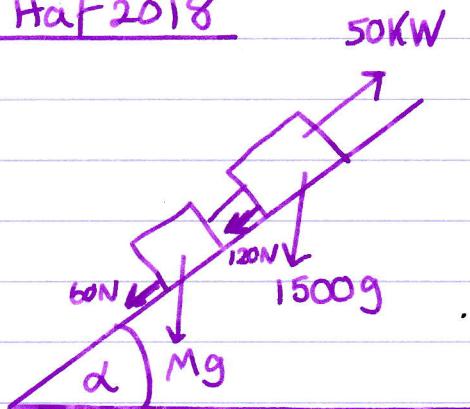
$$x = 344000 - 42000 - 58800$$

$$x = 243200 \text{ J}$$

The total work done against the resistive forces
during this 8 second period is 243200 J

M2 Haf 2018

3)



$$\sin \alpha = \frac{1}{21}$$

$$\text{ Mae } a = 0.4 \text{ ms}^{-2}$$
$$\text{ pan fo } v = 25 \text{ ms}^{-1}$$

$$P_{\text{Wer}} = G_{\text{rym}} \times (\text{cyflymder})$$

$$50,000 = G_{\text{rym}} \times 25$$

$$50,000 \div 25 = G_{\text{rym}}$$

$$G_{\text{rym}} = 2000 \text{ N}$$

$F = ma$, ar yr holl system, yn baratol ir llefthr, $\nearrow = +$ if

$$2000 - 120 - 60 - \text{pwysau lawr y llefthr} = ma$$

$$1820 - (1500 + M)g \sin \alpha = (1500 + M) \times 0.4$$

$$1820 - (1500 + M) \times 9.8 \times \frac{1}{21} = (1500 + M) \times 0.4$$

$$1820 - \frac{7}{15}(1500 + M) = 0.4(1500 + M)$$

$$1820 - 700 - \frac{7}{15}M = 600 + 0.4M$$

$$1820 - 700 - 600 = 0.4M + \frac{7}{15}M$$

$$520 = \frac{13}{15}M$$

$$M = 520 \div \frac{13}{15}$$

$$M = \underline{\underline{600 \text{ Kg}}}$$

$F = ma$, ar y trelar, yn baratol ir llefthr, $\nearrow = +$ if

$$T - 60 - \text{pwysau lawr y llefthr} = ma$$

$$T - 60 - Mg \sin \alpha = M \times 0.4$$

$$T - 60 - 600 \times 9.8 \times \frac{1}{21} = 600 \times 0.4$$

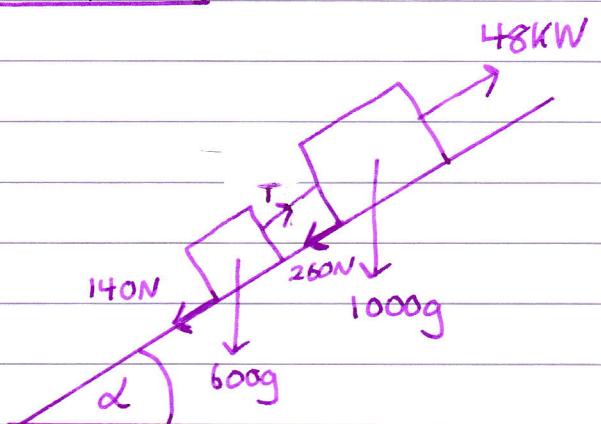
$$T - 60 - 280 = 240$$

$$T = 240 + 60 + 280$$

$$T = \underline{\underline{580 \text{ N}}}$$

M2 Haf 2019

2)



$$\sin \alpha = \frac{1}{20}$$

$$\text{Power} = \text{Grym} \times \text{Cyflymder}$$

a) $F = ma$ ar y system, yn barael ir llétrir, $\nearrow = +if$

$$\text{Grym yr inian} - 260 - 140 - (1600g) \sin \alpha = 1600a$$

$$\frac{\text{Power}}{\text{Cyflymder}} - 260 - 140 - (1600g) \left(\frac{1}{20} \right) = 1600a$$

$$\frac{48000}{24} - 1184 = 1600a$$

$$816 = 1600a$$

$$a = 0.51 \text{ ms}^{-2}$$

b) Os yw $v = 24 \text{ ms}^{-1}$ cyflymiaid yr ôl-gevbyd yw 0.51 ms^{-2} .

$F = ma$ ar yr ôl-gevbyd, yn barael ir llétrir, $\nearrow = +if$

$$T - 140 - 600g \sin \alpha = 600a$$

$$T - 140 - 600 \times 9.8 \times \frac{1}{20} = 600 \times 0.51$$

$$T - 434 = 306$$

$$T = 740 \text{ N}$$