Hen Gwestiynau Arholiad

Fectorau (Vectors)

(Haf 2006)

2. Particle A is moving with constant velocity $-2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$, and at time $t = 0$ s it has position vector $\mathbf{i} - 10\mathbf{k}$. Particle B is moving with constant velocity $\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}$, and at time $t = 0$ s it has position vector $7\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$.

(a) Write down the position vectors of A and B at time t s. [2]

(b) Find the distance between A and B when $t = 2$ s. [3]

6. A particle P moves such that its position vector $\mathbf{r}$ with respect to the origin O at time t is given by

$$\mathbf{r} = \cos 3t\mathbf{i} + \sin 3t\mathbf{j}.$$

(a) Find an expression for $\mathbf{v}$, the velocity of P at time t. [3]

(b) Show that the direction of $\mathbf{v}$ is perpendicular to that of $\mathbf{r}$ for all values of t. [3]

(c) Find the speed of P. [3]

(Haf 2007)

5. Vectors $\mathbf{a}$ and $\mathbf{b}$ are given by

$$\mathbf{a} = 2\mathbf{i} + 13\mathbf{j} - 10\mathbf{k},$$
$$\mathbf{b} = -\mathbf{i} + y\mathbf{j} + 5\mathbf{k}.$$

(a) If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, find the value of y. [4]

(b) If $\mathbf{a}$ and $\mathbf{b}$ are parallel, find the value of y. [2]

8. A toy plane A is moving with constant velocity $(3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})\,\text{ms}^{-1}$ and at time $t = 0$, its position vector is $(3\mathbf{j} - 140\mathbf{k})$ m. Another toy plane B is moving with constant velocity $(-2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})\,\text{ms}^{-1}$ and at time $t = 0$, its position vector is $(-9\mathbf{i} - 4\mathbf{j} - 6\mathbf{k})$ m.

(a) Write down the position vectors of A and B at time t s. [3]

(b) Find an expression for the square of the distance between A and B at time t s. [3]

(c) Determine the time when A and B are closest together. [4]

(Haf 2008)

6. A constant force $\mathbf{F} = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$ acts on a bead as it moves along a straight smooth wire from point A to point B. Point A has position vector $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and point B has position vector $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find

(a) the vector **AB**, [2]

(b) the work done by the force **F**. [3]

7. *(a)* A vehicle moves with velocity $\mathbf{v} = \sin(3t)\mathbf{i} + 2\cos(5t)\mathbf{j} + 3t^3\mathbf{k}$ at time t. Find an expression for the acceleration of the vehicle at time t. [3]

(b) Two vehicles A and B move on the same horizontal plane. At time t, A is at position $(-8t - 2)\mathbf{i} + (3t + 3)\mathbf{j}$ and B is at position $(-16t + 11)\mathbf{i} + (9t - 8)\mathbf{j}$. Determine the value of t when the distance between A and B is least, and calculate this distance. [7]

(Haf 2009)

6. A particle, of mass 2 kg, moves in a horizontal plane such that its position vector **r** m at time t s is given by

$$\mathbf{r} = (1 - 4t^2)\,\mathbf{i} + (3t^2 - 5t)\mathbf{j}.$$

(a) Find, in terms of t, an expression for the momentum of the particle at time t s. [3]

(b) Show that the acceleration of the particle is constant and find its magnitude. [4]

(c) Find the time when the velocity of the particle is perpendicular to its acceleration. [4]

(Haf 2010)

2. At time t s, the position vector **r** m of a particle P is given by

$$\mathbf{r} = (3t^2 + 1)\,\mathbf{i} + (13t - 2t^2)\,\mathbf{j}.$$

(a) Find the speed of P when $t = 2$. [4]

(b) Calculate the value of t when the velocity of P is perpendicular to the vector $2\mathbf{i} - \mathbf{j}$. [3]

(c) Show that the acceleration of P is constant and find its magnitude. [3]

(d) Find the angle between the direction of the acceleration of P and the direction of the velocity of P when $t = 2$. [3]

(Haf 2011)

3. A particle P, of mass 2 kg, is moving under the action of a force **F** N so that its velocity **v** ms^{-1} at time t s is given by

$$\mathbf{v} = 2\mathbf{i} + 6t\,\mathbf{j} + 4t^3\mathbf{k}.$$

(a) Find an expression for **F** at time t s. [3]

(b) Determine the value of **F.v** when $t = 1$ and state the units of your answer. [4]

7. At time t, the position vectors relative to a fixed origin O, of two particles A and B are given by $\mathbf{OA} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(2\mathbf{i} - 6\mathbf{j} + 9\mathbf{k})$ and $\mathbf{OB} = 5\mathbf{i} - 8\mathbf{j} + 10\mathbf{k} + t(3\mathbf{i} - 6\mathbf{j} + 7\mathbf{k})$.

(a) Find the speed of particle A. [3]

(b) Show that the distance AB at time t is given by $AB^2 = 5t^2 - 30t + 211$. Determine the time at which the particles A and B are closest together. [7]

(Haf 2012)

3. A particle moves on a horizontal plane so that at time t seconds its position vector $\mathbf{r}$ metres relative to a fixed origin O is given by

$$\mathbf{r} = (t + 2t^2)\mathbf{i} + (1{\cdot}5t^2 - 2t)\mathbf{j}.$$

(a) Determine the time when the velocity of the particle is perpendicular to the vector $(-\mathbf{i} + 2\mathbf{j})$. [5]

(b) Show that the acceleration of the particle is constant and find its magnitude. [3]

8. A ship S is moving in a straight line with constant velocity. At time $t = 0$, its position vector relative to a fixed origin O is $(8\mathbf{i} + 7\mathbf{j})$. At time $t = 3$, its position vector is $(14\mathbf{i} - 5\mathbf{j})$.

(a) Show that the velocity of S is $(2\mathbf{i} - 4\mathbf{j})$. [2]

(b) Find an expression, in terms of t, for the position vector of S at time t. [2]

At time $t = 10$, a boat B leaves O and travels with constant velocity $x\mathbf{i} + y\mathbf{j}$, intercepting S at time $t = 50$.

(c) Calculate the value of x and the value of y. [6]

(Haf 2013)

2. A particle P, of mass 2 kg, is moving so that at time t s its velocity $\mathbf{v}$ ms^{-1} is given by $\mathbf{v} = (13t - 3)\mathbf{i} + (2 + 3t^2)\mathbf{j}$. At time $t = 0$ s, the position vector of the particle is $(2\mathbf{i} + 7\mathbf{j})$ m.

(a) Find the position vector $\mathbf{r}$ of P at time t s. [5]

(b) Determine the acceleration $\mathbf{a}$ of P at time t s. [2]

(c) Calculate the values of t when the velocity of P is perpendicular to the vector $\mathbf{i} - 2\mathbf{j}$. [5]

(Haf 2014)

4. At time $t = 0$, an aeroplane A has position vector $(3\mathbf{i} + 5\mathbf{j} + 20\mathbf{k})$ m and is flying with constant velocity $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})\,\text{ms}^{-1}$.
At time $t = 0$, another aeroplane B has position vector $(-2\mathbf{i} + x\mathbf{j} + 15\mathbf{k})$ m, and is flying with constant velocity $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})\,\text{ms}^{-1}$.

(a) Find expressions for the position vector of A and the position vector of B at time t s. [3]

(b) Determine an expression for AB^2, where AB is the distance between A and B at time t s. [4]

(c) Given that the shortest distance between A and B occurs at $t = 5$, calculate the value of x. [3]

6. A particle of mass 3 kg moves on a horizontal plane. At time $t = 0$, the particle has position vector $-2\mathbf{i} + 3\mathbf{j}$ m, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the x-axis and y-axis respectively. At time t s, the particle moves with velocity $\mathbf{v}\,\text{ms}^{-1}$ given by

$$\mathbf{v} = 4\sin 2t\,\mathbf{i} + 15\cos 5t\,\mathbf{j}.$$

(a) Find the magnitude of the force acting on the particle at time $t = \frac{3\pi}{2}$ s. [5]

(b) Determine the position vector of the particle at time t s. [4]

(c) Calculate the time and the distance of the particle from the origin when it crosses the y-axis for the first time. [4]

(Haf 2015)

1. The vectors $\mathbf{x}$ and $\mathbf{y}$ are given by

$$\mathbf{x} = \sin\theta\,\mathbf{i} + 2\cos 2\theta\,\mathbf{j},$$
$$\mathbf{y} = 2\mathbf{i} - \mathbf{j}.$$

Find the values of θ between 0 and 2π such that $\mathbf{x}$ is perpendicular to $\mathbf{y}$. [6]

4. A particle of mass 0·5 kg is moving under the action of a single force $\mathbf{F}$ N, where $\mathbf{F} = (4t - 3)\mathbf{i} + (3t^2 - 5t)\mathbf{j}$.

(a) The velocity of the particle at time t s is $\mathbf{v}\,\text{ms}^{-1}$. When $t = 0$, $\mathbf{v} = 8\mathbf{i} - 7\mathbf{j}$.
Find an expression for $\mathbf{v}$ in terms of t. [5]

(b) When $t = 3$, the particle receives an impulse of $2\mathbf{i} - 9\mathbf{j}$ Ns. Find the speed of the particle immediately after the impulse. [5]

(Haf 2016)

3. At time $t = 0$ s, the position vector of an object A is $\mathbf{i}$ m and the position vector of another object B is $3\mathbf{i}$ m. The constant velocity vector of A is $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}\,\text{ms}^{-1}$ and the constant velocity vector of B is $\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}\,\text{ms}^{-1}$. Determine the value of t when A and B are closest together and find the least distance between A and B. [9]

6. A particle moves on a horizontal plane such that its velocity vector $\mathbf{v}$ ms^{-1} at time t s is given by

$$\mathbf{v} = 7\sin 2t\,\mathbf{i} + 6\cos 3t\,\mathbf{j}.$$

(a) Find the acceleration vector of the particle at time t s. [2]

(b) Given that when $t = 0$, the particle has position vector $(0{\cdot}5\mathbf{i} + 3\mathbf{j})$ m, find the position vector of the particle when $t = \frac{\pi}{2}$. [5]

(Haf 2017)

1. The position vector of a particle P at time t seconds is given by

$$\mathbf{r} = t\sin t\,\mathbf{i} + t\cos t\,\mathbf{j}.$$

(a) (i) Find the velocity vector of P and an expression for the speed of P at time t seconds in its simplest form.

(ii) Given that the mass of P is 3 kg, write down the momentum vector of P at time t seconds. [6]

(b) At time $t = \frac{\pi}{6}$, the vector $b\mathbf{i} + \sqrt{3}\mathbf{j}$ is perpendicular to $\mathbf{r}$. Find the value of b. [5]

(Haf 2018)

2. A particle P, of mass 8 kg, moves in a plane such that its position vector at time t seconds is given by

$$\mathbf{r} = (3t^2 + 1)\mathbf{i} + (t\cos 4t)\mathbf{j} \text{ metres.}$$

(a) Find the momentum vector of P when $t = 0$. [3]

(b) Calculate the kinetic energy of P when $t = \pi$. [3]

(c) Determine the force acting on P when $t = \pi$ and find a vector that is perpendicular to this force. [4]

(d) Find the rate of work of the force acting on P when $t = \pi$. [3]