

S 2: Y Dosraniad Normal

Itaf 2006

③

$$\text{Cân gwryw } X \sim N(30, 2^2)$$

$$Z \sim N(0, 1)$$

$$\text{Cân bengw } Y \sim N(25, 1 \cdot 8^2)$$

$$(a) (i) P(28 \leq X \leq 34) = P(X \leq 34) - P(X \leq 28)$$

$$= P\left(Z \leq \frac{34-30}{2}\right) - P\left(Z \leq \frac{28-30}{2}\right)$$

$$= P(Z \leq 2) - P(Z \leq -1)$$

$$= 0.97725 - P(Z > 1)$$

$$= 0.97725 - (1 - P(Z \leq 1))$$

$$= 0.97725 - (1 - 0.84134)$$

$$= 0.81859$$

$$(ii) \text{ Rydym angen } P(X \leq x) = 0.99$$

$$P\left(Z \leq \frac{x-25}{1.8}\right) = 0.99.$$

$$\text{O'r tablau, } P(Z \leq 2.326) = 0.99.$$

$$\text{Felly } \frac{x-25}{1.8} = 2.326$$

$$x-25 = 2.326 \times 1.8$$

$$x = 2.326 \times 1.8 + 25$$

$$\underline{x = 29.126 \text{ Kg}}$$

$$(b) P(X > Y) = P(X - Y > 0).$$

$$E(X - Y) = E(X) - E(Y)$$

$$= 30 - 25$$

$$= 5$$

$$\text{Var}(X - Y) = 1^2 \text{Var}(X) + (-1)^2 \text{Var}(Y)$$

$$= 1 \times 2^2 + 1 \times 1.8^2$$

$$= 7.24.$$

$$\text{Felly } X - Y \sim N(5, 7.24)$$

$$\begin{aligned}
 P(X-Y>0) &= P\left(Z > \frac{0-5}{\sqrt{7.24}}\right) \\
 &= P(Z > -1.858235366) \\
 &= P(Z \leq 1.858235366) \\
 &= P(Z \leq 1.86) \\
 &= 0.96856.
 \end{aligned}$$

ltaf 2007

③ Pwysau afabau $X \sim N(75, 5^2)$ $Z \sim N(0,1)$

$$\begin{aligned}
 \text{(a) (i)} \quad P(X < 80) &= P\left(Z < \frac{80-75}{5}\right) \\
 &= P(Z < 1) \\
 &= P(Z \leq 1) \\
 &= 0.84134
 \end{aligned}$$

(ii) Rydym angen $P(X \leq x) = 0.75$

$$P\left(Z \leq \frac{x-75}{5}\right) = 0.75.$$

o'r efta blaen, $P(Z \leq 0.674) = 0.75$

Felly $\frac{x-75}{5} = 0.674$

$$x-75 = 0.674 \times 5$$

$$x = 0.674 \times 5 + 75$$

$$x = 78.37 \text{ g}$$

(b) Pwysau einin $Y \sim N(56, 4^2)$

$$P(Y+Y+Y > X+X) = P(Y+Y+Y-X-X > 0).$$

$$\begin{aligned} E(Y+Y+Y-X-X) &= E(Y) + E(Y) + E(Y) - E(X) - E(X) \\ &= 56 + 56 + 56 - 75 - 75 \\ &= 18 \end{aligned}$$

$$\begin{aligned} \text{Var}(Y+Y+Y-X-X) &= \text{Var}(Y) + \text{Var}(Y) + \text{Var}(Y) \\ &\quad + \text{Var}(X) + \text{Var}(X) \\ &= 3 \times 4^2 + 2 \times 5^2 \\ &= 98 \end{aligned}$$

Felly $Y+Y+Y-X-X \sim N(18, 98)$

$$\begin{aligned} P(Y+Y+Y-X-X > 0) &= P\left(Z > \frac{0-18}{\sqrt{98}}\right) \\ &= P(Z > -1.81827458) \\ &= P(Z \leq 1.81827458) \\ &= P(Z \leq 1.82) \\ &= 0.96562 \end{aligned}$$

Haf 2008

(3) Pwysau Wyaun cywiar $X \sim N(50, 2^2)$ $Z \sim N(0, 1)$

$$\begin{aligned} (\text{a}) \text{ (i)} \quad P(X < 49) &= P\left(Z < \frac{49-50}{2}\right) \\ &= P(Z < -0.5) \\ &= P(Z > 0.5) \\ &= 1 - P(Z \leq 0.5) \\ &= 1 - 0.69146 \\ &= 0.30854 \end{aligned}$$

$$\begin{aligned} (\text{ii}) \quad \text{Nifer yn pwysau llai na } 49g: \quad Y &\sim \text{Bin}(6, 0.30854) \\ P(Y=3) &= {}^6C_3 \times 0.30854^3 \times (1-0.30854)^{6-3} \\ &= 0.1942 \quad ; \quad \text{4 lle degol}. \end{aligned}$$

(b) Pwysau wya u safliar: $S \sim N(18, 1.2^2)$

$$P(X > 3S) = P(X - 3S > 0).$$

$$\begin{aligned}E(X - 3S) &= E(X) - 3E(S) \\&= 50 - 3 \times 18 \\&= -4\end{aligned}$$

$$\begin{aligned}\text{Var}(X - 3S) &= 1^2 \text{Var}(X) + (-3)^2 \text{Var}(S) \\&= 1 \times 2^2 + 9 \times 1.2^2 \\&= 16.96\end{aligned}$$

Felly $X - 3S \sim N(-4, 16.96)$

$$\begin{aligned}P(X - 3S > 0) &= P\left(Z > \frac{0 - -4}{\sqrt{16.96}}\right) \\&= P(Z > 0.9712858624) \\&= 1 - P(Z \leq 0.9712858624) \\&= 1 - 0.83398 \\&= 0.16602\end{aligned}$$

Haf 2009

(2)

Cyflymder serf Rhys $X \sim N(140, 8^2)$

$Z \sim N(0, 1)$

$$\begin{aligned}
 \text{(a) (i)} \quad P(X > 150) &= P\left(Z > \frac{150 - 140}{8}\right) \\
 &= P(Z > 1.25) \\
 &= 1 - P(Z \leq 1.25) \\
 &= 1 - 0.89435 \\
 &= 0.10565
 \end{aligned}$$

DULL1

(ii) Nifer o serfiau mwy na 150 Km/awr: $S \sim \text{Bin}(3, 0.10565)$

$$\begin{aligned}
 P(S=3) &= {}^3C_3 \times 0.10565^3 (1 - 0.10565)^{3-3} \\
 &= 0.0012 \text{ i 4 lle degol.}
 \end{aligned}$$

DULL2

$$\begin{aligned}
 \text{Pob serf mwy na 150 Km/awr} &= 1^{\text{st}} \text{ yn fwy, } 2^{\text{nd}} \text{ yn} \\
 \text{fyng, } 3^{\text{ydd}} \text{ yn fwy} &= 0.10565 \times 0.10565 \times 0.10565 \\
 &= 0.10565^3 \\
 &= 0.0012 \text{ i 4 lle degol.}
 \end{aligned}$$

(b) cyflymder serf Alun $Y \sim N(145, 6^2)$

$$P(X > Y) = P(X - Y > 0).$$

$$\begin{aligned}
 E(X - Y) &= E(X) - E(Y) & \text{Var}(X - Y) &= 1^2 \text{Var}(X) + (-1)^2 \text{Var}(Y) \\
 &= 140 - 145 & &= 1 \times 8^2 + 1 \times 6^2 \\
 &= -5 & &= 100
 \end{aligned}$$

Felly $X - Y \sim N(-5, 100)$

$$\begin{aligned}
 P(X-Y > 0) &= P\left(Z > \frac{0 - (-5)}{10}\right) \\
 &= P(Z > 0.5) \\
 &= 1 - P(Z \leq 0.5) \\
 &= 1 - 0.69146 \\
 &= 0.30854
 \end{aligned}$$

Haf 2010

① Pawysau tomatos $X \sim N(106, 8^2)$ $Z \sim N(0, 1)$

$$\begin{aligned}
 \text{(a)} \quad P(X > 120) &= P\left(Z > \frac{120 - 106}{8}\right) \\
 &= P(Z > 1.75) \\
 &= 1 - P(Z \leq 1.75) \\
 &= 1 - 0.95994 \\
 &= 0.04006
 \end{aligned}$$

(b) $P(X+X+X+\dots+X < 1000)$

$$\begin{aligned}
 E(X+X+\dots+X) &= E(X)+E(X)+\dots+E(X) \\
 &= 10 \times 106 \\
 &= 1060
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}(X+X+\dots+X) &= 1^2 \text{Var}(X) + 1^2 \text{Var}(X) + \dots + 1^2 \text{Var}(X) \\
 &= 10 \times 1 \times 64 \\
 &= 640
 \end{aligned}$$

Felly $X+X+X+\dots+X \sim N(1060, 640)$

$$\begin{aligned}
 \text{Ateb} &= P\left(Z < \frac{1000 - 1060}{\sqrt{640}}\right) \quad \rightarrow = 1 - P(Z \leq 2.37) \\
 &= P(Z < -2.371708245) \quad = 1 - 0.99111 \\
 &= P(Z > 2.371708245) \quad = 0.00889.
 \end{aligned}$$

Haf 2011

① Amser Atun $X \sim N(28, 2^2)$

$Z \sim N(0, 1)$

$$\begin{aligned}
 \text{(a) (i)} \quad P(X > 30) &= P\left(Z > \frac{30-28}{2}\right) \\
 &= P(Z > 1) \\
 &= 1 - P(Z \leq 1) \\
 &= 1 - 0.84134 \\
 &= 0.15866
 \end{aligned}$$

(ii) Amser cymedrig yn llai na 30 munud yr un peth
 â chyfanswm yr amser am y 5 diwrnod yn llai
 na $5 \times 30 = 150$ munud.

$$P(X + X + X + X + X < 150).$$

$$\begin{aligned}
 E(X + X + X + X + X) &= E(X) + E(X) + E(X) + E(X) + E(X) \\
 &= 5 \times 28 \\
 &= 140
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}(X + X + X + X + X) &= 1^2 \text{Var}(X) + 1^2 \text{Var}(X) + \dots + 1^2 \text{Var}(X) \\
 &= 5 \times 1 \times 2^2 \\
 &= 20
 \end{aligned}$$

Felly $X + X + X + X + X \sim N(140, 20)$.

$$\begin{aligned}
 P(X + X + X + X + X < 150) &= P\left(Z < \frac{150-140}{\sqrt{20}}\right) \\
 &= P(Z < \sqrt{5}) \\
 &= P(Z \leq 2.24) \\
 &= 0.98745
 \end{aligned}$$

(b) Amser Beti $Y \sim N(25, 3^2)$

$$P(Y > X) = P(Y - X > 0).$$

$$\begin{aligned} E(Y - X) &= E(Y) - E(X) & \text{Var}(Y - X) &= 1^2 \text{Var}(Y) + (-1)^2 \text{Var}(X) \\ &= 25 - 28 & &= 1 \times 3^2 + 1 \times 2^2 \\ &= -3 & &= 13 \end{aligned}$$

Felly $Y - X \sim N(-3, 13)$

$$\begin{aligned} P(Y - X > 0) &= P\left(Z > \frac{0 - -3}{\sqrt{13}}\right) \\ &= P(Z > 0.8320502943) \\ &= 1 - P(Z \leq 0.83) \\ &= 1 - 0.79673 \\ &= 0.20327 \end{aligned}$$

Haf 2012

②

$$X \sim N(4.4, 0.2^2)$$

$$Z \sim N(0, 1)$$

$$\begin{aligned} (\text{a}) \quad (\text{i}) \quad P(X > 4.5) &= P\left(Z > \frac{4.5 - 4.4}{0.2}\right) \\ &= P(Z > 0.5) \\ &= 1 - P(Z \leq 0.5) \\ &= 1 - 0.69146 \\ &= 0.30854 \end{aligned}$$

(ii) Rydym angen $P(X \leq x) = 0.95$

$$P\left(Z \leq \frac{x - 4.4}{0.2}\right) = 0.95$$

Fr tablau, $P(Z \leq 1.645) = 0.95$

$$\text{Finally } \frac{x-4.4}{0.2} = 1.645$$

$$x-4.4 = 1.645 \times 0.2$$

$$x = 1.645 \times 0.2 + 4.4$$

$$\underline{x = 4.729}$$

$$(b) Y \sim N(2.6, 0.15^2)$$

$$\begin{aligned}\text{(i)} \quad E(2Y-X) &= 2E(Y) - E(X) \\ &= 2 \times 2.6 - 4.4 \\ &= 0.8\end{aligned}$$

$$\begin{aligned}\text{Var}(2Y-X) &= 2^2 \text{Var}(Y) + (-1)^2 \text{Var}(X) \\ &= 4 \times 0.15^2 + 1 \times 0.2^2 \\ &= 0.13\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad P(X > 2Y) &= P(X-2Y > 0) \\ &= P(2Y-X < 0)\end{aligned}$$

$$\text{Hence } 2Y-X \sim N(0.8, 0.13)$$

$$\begin{aligned}P(2Y-X < 0) &= P\left(Z < \frac{0-0.8}{\sqrt{0.13}}\right) \\ &= P(Z < -2.218800785) \\ &= P(Z > 2.218800785) \\ &= 1 - P(Z \leq 2.22) \\ &= 1 - 0.98679 \\ &= 0.01321\end{aligned}$$

$$\text{(iii)} \quad P(X+X+Y+Y+Y > 16)$$

$$\begin{aligned}E(X+X+Y+Y+Y) &= E(X) + E(X) + E(Y) + E(Y) + E(Y) \\ &= 2 \times 4.4 + 3 \times 2.6 \\ &= 16.6\end{aligned}$$

$$\begin{aligned}\text{Var}(X+X+Y+Y+Y) &= 1^2 \text{Var}(X) + 1^2 \text{Var}(X) + \dots + 1^2 \text{Var}(Y) \\ &= 2 \times 0.2^2 + 3 \times 0.15^2 \\ &= 0.1475\end{aligned}$$

Felly $X+X+Y+Y+Y \sim N(16.6, 0.1475)$

$$\begin{aligned} P(X+X+Y+Y+Y > 16) &= P\left(Z > \frac{16-16.6}{\sqrt{0.1475}}\right) \\ &= P(Z > -1.562266932) \\ &= P(Z \leq 1.56) \\ &= 0.94062 \end{aligned}$$

Haf 2013

① $X \sim N(10, 2^2)$ $Z \sim N(0, 1)$

(a) (i) $P(X \leq 10.5) = P\left(Z \leq \frac{10.5-10}{2}\right)$
 $= P(Z \leq 0.25)$
 $= 0.59871$

(ii) $P(X \geq x) = 0.1$

Felly $P(X \leq x) = 0.9$

$$P\left(Z \leq \frac{x-10}{2}\right) = 0.9$$

ór tablau, $P(Z \leq 1.282) = 0.9$

Felly $\frac{x-10}{2} = 1.282$

$$x-10 = 1.282 \times 2$$

$$x = 1.282 \times 2 + 10$$

$$x = 12.564$$

$$(b) Y \sim N(12, 3^2)$$

$$(i) P(X+2Y < 36).$$

$$\begin{aligned} E(X+2Y) &= E(X) + 2E(Y) \\ &= 10 + 2 \times 12 \\ &= 34 \end{aligned}$$

$$\begin{aligned} \text{Var}(X+2Y) &= 1^2 \text{Var}(X) + 2^2 \text{Var}(Y) \\ &= 1 \times 2^2 + 4 \times 3^2 \\ &= 40 \end{aligned}$$

$$\text{Felly } X+2Y \sim N(34, 40)$$

$$\begin{aligned} \text{Felly } P(X+2Y < 36) &= P\left(Z < \frac{36-34}{\sqrt{40}}\right) \\ &= P(Z < 0.316227766) \\ &= 0.62552 \end{aligned}$$

$$(ii) P(X_1 + X_2 + X_3 - Y_1 - Y_2 < 0).$$

$$\begin{aligned} &= P(X_1 + X_2 + X_3 - Y_1 - Y_2 < 0) \\ &= E(X_1) + E(X_2) + E(X_3) - E(Y_1) - E(Y_2) \\ &= 3 \times 10 - 2 \times 12 \\ &= 6 \end{aligned}$$

$$\begin{aligned} &= \text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + (-1)^2 \text{Var}(Y_1) \\ &\quad + (-1)^2 \text{Var}(Y_2) \\ &= 3 \times 1 \times 2^2 + 2 \times 1 \times 3^2 \\ &= 30 \end{aligned}$$

$$\text{Felly } X_1 + X_2 + X_3 - Y_1 - Y_2 \sim N(6, 30)$$

$$Akeeb = P\left(Z < \frac{0-6}{\sqrt{30}}\right)$$

$$\begin{aligned} &= P(Z < -1.095445115) \rightarrow = 1 - 0.86433 \\ &= P(Z > 1.095445115) \quad = 0.13567 \\ &= 1 - P(Z \leq 1.10) \end{aligned}$$

S2 Haf 2014

(2)

$$\text{Orennau } X \sim N(248, 8^2)$$

$$Z \sim N(0, 1)$$

$$\text{Lemonnau } Y \sim N(85, 1.5^2)$$

$$(a) \text{ Rydym angen } P(Y \leq y) = 0.75$$

$$P\left(Z \leq \frac{y-85}{1.5}\right) = 0.75$$

$$\text{örf tablau, } P(Z \leq 0.674) = 0.75$$

$$\text{Felly } \frac{y-85}{1.5} = 0.674$$

$$y - 85 = 0.674 \times 1.5$$

$$y = 1.011 + 85$$

$$\underline{\underline{y = 86.011 \text{ gram}}}$$

$$(b) P(X + X + X + X + X + X + X + X < 2000)$$

$$\begin{aligned} E(X + X + X + X + X + X + X + X) &= E(X) + E(X) + \dots + E(X) \\ &= 8 \times 248 \\ &= 1984 \end{aligned}$$

$$\begin{aligned} \text{Var}(X + X + X + X + X + X + X + X) &= 1^2 \text{Var}(X) + 1^2 \text{Var}(X) + \dots + 1^2 \text{Var}(X) \\ &= 8 \times 1 \times 64 \\ &= 512 \end{aligned}$$

$$\text{Felly } X + X + X + X + X + X + X + X + X \sim N(1984, 512)$$

$$\text{Ateb} = P\left(Z < \frac{2000 - 1984}{\sqrt{512}}\right)$$

$$= P(Z < 0.7071067812)$$

$$= P(Z < 0.71) \text{ örf tablau}$$

$$= \underline{\underline{0.76115}}$$

$$(c) P(X > 3Y) = P(X - 3Y > 0)$$

$$\begin{aligned}E(X - 3Y) &= E(X) - 3E(Y) \\&= 248 - 3 \times 85 \\&= -7\end{aligned}$$

$$\begin{aligned}\text{Var}(X - 3Y) &= 1^2 \text{Var}(X) + (-3)^2 \text{Var}(Y) \\&= 1 \times 8^2 + 9 \times 1.5^2 \\&= 84.25\end{aligned}$$

Then $X - 3Y \sim N(-7, 84.25)$

$$\begin{aligned}P(X - 3Y > 0) &= P\left(Z > \frac{0 - -7}{\sqrt{84.25}}\right) \\&= P(Z > 0.762628595) \\&= 1 - P(Z \leq 0.762628595) \\&= 1 - P(Z \leq 0.76) \\&= 1 - 0.77637 \\&= \underline{\underline{0.22363}}\end{aligned}$$

S2 Haf 2015

I) Gadewch i X ddynodi serf siôn ar ôl cael rased newydd.

$$X \sim N(\mu, 1 \cdot 2^2)$$

(a) $H_0: \mu = 120$ yn erbyn

$$H_1: \mu \neq 120$$

(b) Gadewch i \bar{X} ddynodi'r sampl o 10 serf.

Gan dybio bod H_0 yn wir, mae

$$\bar{X} \sim N(120, \frac{1 \cdot 2^2}{10} = 0.144)$$

$$\sum \text{serfiau} = 1192$$

$$\text{Cymedr} = \frac{1192}{10}$$

$$\text{Cymedr} = 119.2$$

Mae'r gwerth arsylwedig $\bar{x} = 119.2$ yn lhai na $E(\bar{X}) = 120$.

Gan fod H_1 yn ddwy-ochrog, y gwerth-p yw

$$2P(\bar{X} \leq 119.2)$$

$$= 2P\left(Z < \frac{119.2 - 120}{\sqrt{0.144}}\right)$$

$$= 2P(Z < -2.108185107)$$

$$= 2 \times (1 - 0.98257)$$

$$= 2 \times 0.01743$$

$$= \underline{\underline{0.03486}}$$

Gan fod y gwerth-p yn lhai na 0.05 ond yn fwy na 0.01, mae yna dystiolaeth gref yn erbyn H_0 ac felly'r casgliad yw bod newid yn y cyflymder cymedrig. Gan fod y cymedr arsylwedig yn lhai na 120 gallun gasglu. Fod cyflymder serfiol rased newydd yn lhai na 120 m.y.a, ar gyfartaledd.

S2. Haf 2015

2) Dymision $X \sim N(82, 2.5^2)$ $Z \sim N(0, 1)$
Merched $Y \sim N(65, 2^2)$

(a) Rydym angen $P(X \leq x) = 0.95$

$$P\left(Z \leq \frac{x-82}{2.5}\right) = 0.95$$

o'r eisblau, $P(Z \leq 1.645) = 0.95$

Felly $\frac{x-82}{2.5} = 1.645$

$$x-82 = 1.645 \times 2.5$$

$$x = 4.1125 + 82$$

$$\underline{\underline{x = 86.1125 \text{ Kg}}}$$

(b) $P(64 \leq Y \leq 68) = P(Y \leq 68) - P(Y \leq 64)$

$$= P\left(Z \leq \frac{68-65}{2}\right) - P\left(Z \leq \frac{64-65}{2}\right)$$

$$= P(Z \leq 1.5) - P(Z \leq -0.5)$$

$$= 0.93319 - P(Z > 0.5)$$

$$= 0.93319 - (1 - P(Z \leq 0.5))$$

$$= 0.93319 - (1 - 0.69146)$$

$$\underline{\underline{= 0.62465}}$$

(c) $P(X+X+X+Y+Y+Y+Y) > 500$

$$E(X+X+X+Y+Y+Y+Y) = E(X) + E(X) + E(X)$$

$$+ E(X) + E(Y) + E(Y) + E(Y)$$

$$= 3 \times 82 + 4 \times 65$$

$$= 506$$

$$\text{Var}(X+X+X+Y+Y+Y+Y) = 1^2 \text{Var}(X) + 1^2 \text{Var}(X) + 1^2 \text{Var}(X)$$

$$+ 1^2 \text{Var}(Y) + 1^2 \text{Var}(Y) + 1^2 \text{Var}(Y) + 1^2 \text{Var}(Y)$$

$$= 3 \times 2.5^2 + 4 \times 2^2$$

$$= 34.75$$

Felly $X+X+X+Y+Y+Y+Y \sim N(506, 34.75)$

$$\begin{aligned} & P(X+X+X+Y+Y+Y+Y) > 500 \\ &= P\left(Z > \frac{500 - 506}{\sqrt{34.75}}\right) \\ &= P(Z > -1.017826716) \\ &= P(Z \leq 1.02) \\ &= \underline{0.84614} \end{aligned}$$

5) Haf 2016

(3) Gwylw $G \sim N(40, 2.5^2)$
 Benyw $B \sim N(32, 1.5^2)$

a) Rydym angen $P(G \leq g) = 0.75$
 $P\left(Z \leq \frac{g - 40}{2.5}\right) = 0.75$

o'r tablau, $P(Z \leq 0.674) = 0.75$

Felly $\frac{g - 40}{2.5} = 0.674$

$$g - 40 = 0.674 \times 2.5$$

$$g = 1.685 + 40$$

$$\underline{g = 41.685 \text{ Kg}}$$

b) (i) $P(G+G+G+B+B > 185)$

$$\begin{aligned} E(G+G+G+B+B) &= E(G)+E(G)+E(G)+E(B)+E(B) \\ &= 3 \times 40 + 2 \times 32 \\ &= 184 \end{aligned}$$

$$\begin{aligned} \text{Var}(G+G+G+B+B) &= 1^2 \text{Var}(G) + 1^2 \text{Var}(G) + 1^2 \text{Var}(G) \\ &\quad + 1^2 \text{Var}(B) + 1^2 \text{Var}(B) \\ &= 3 \times 2.5^2 + 2 \times 1.5^2 \\ &= 23.25 \end{aligned}$$

Felly $G+G+G+B+B \sim N(184, 23.25)$

$$\begin{aligned} \text{Nawr } P\left(Z < \frac{185 - 184}{\sqrt{23.25}}\right) &= P(Z < 0.2073903389) \\ &= P(Z < 0.21) \text{ o'r tablau} \\ &\approx 0.58317 \end{aligned}$$

$$\begin{aligned} \text{Felly'r ateb yw } 1 - P(G+G+G+B+B \leq 185) \\ &= 1 - 0.58317 \\ &= 0.41683 \end{aligned}$$

$$\text{(ii)} \quad P(G+G+G < 2(B+B)) \\ = P(G+G+G - 2B - 2B < 0)$$

$$E(G+G+G - 2B - 2B) \\ = E(G) + E(G) + E(G) - 2E(B) - 2E(B) \\ = 40 + 40 + 40 - 2 \times 32 - 2 \times 32 \\ = -8$$

$$\text{Var}(G+G+G - 2B - 2B) \\ = 1^2 \text{Var}(G) + 1^2 \text{Var}(G) + 1^2 \text{Var}(G) + (-2^2) \text{Var}(B) + (-2^2) \text{Var}(B) \\ = 2.5^2 + 2.5^2 + 2.5^2 + 4 \times 1.5^2 + 4 \times 1.5^2 \\ = 36.75$$

Female $G+G+G - 2B - 2B \sim N(-8, 36.75)$

$$A_{\text{Feb}} = P\left(Z < \frac{0 - -8}{\sqrt{36.75}}\right) \\ = P(Z < 1.319657758) \\ = P(Z < 1.32) \text{ or tablau} \\ = 0.90658$$

Sar Haf 2017

3) Afalau $X \sim N(110, 14^2)$ $Z \sim N(0, 1)$
Gellyg $Y \sim N(160, 16^2)$

a) Rydym angen $P(X \leq x) = 0.9$

$$P\left(Z \leq \frac{x - 110}{14}\right) = 0.9$$

o'r tablau, $P(Z \leq 1.282) = 0.9$

Felly $\frac{x - 110}{14} = 1.282$

$$x - 110 = 17.948$$

$$\underline{x = 127.948 \text{ gram}}$$

b) $P(X + X + X + X + X + X + X + X + X + X < 1000)$

$$\begin{aligned} E(X + X + X + X + X + X + X + X + X) &= E(X) + E(X) + \dots + E(X) \\ &= 110 \times 10 \\ &= 1100 \end{aligned}$$

$$\begin{aligned} \text{Var}(X + X + X + X + X + X + X + X + X) &= 1^2 \text{Var}(X) + 1^2 \text{Var}(X) + \dots + 1^2 \text{Var}(X) \\ &= 1 \times 14^2 \times 10 \\ &= 1960 \end{aligned}$$

Felly $X + X + X + X + X + X + X + X + X \sim N(1100, 1960)$

$$\begin{aligned} \text{Ateb} &= P\left(Z < \frac{1000 - 1100}{\sqrt{1960}}\right) \\ &= P(Z < -2.258769757) \\ &= 1 - P(Z \leq 2.26) \\ &= 1 - 0.98809 \end{aligned}$$

$$c) P(X+X+X > Y+Y) = P(X+X+X-Y-Y > 0)$$

$$\begin{aligned} E(X+X+X-Y-Y) &= E(X)+E(X)+E(X)-E(Y)-E(Y) \\ &= 3 \times 110 - 2 \times 160 \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{Var}(X+X+X-Y-Y) &= 1^2 \text{Var}(X) + 1^2 \text{Var}(X) + 1^2 \text{Var}(X) \\ &\quad + (-1)^2 \text{Var}(Y) + (-1)^2 \text{Var}(Y) \\ &= 3\text{Var}(X) + 2\text{Var}(Y) \\ &= 3 \times 14^2 + 2 \times 16^2 \\ &= 1100 \end{aligned}$$

Felly $X+X+X-Y-Y \sim N(10, 1100)$

$$\begin{aligned} \text{Ateb} &= P\left(Z > \frac{0-10}{\sqrt{1100}}\right) \\ &= P(Z \geq -0.3015113446) \\ &= P(Z \leq 0.30) \\ &= \underline{\underline{0.61791}} \end{aligned}$$

S2 Haf 2018

- 2) Pwysau i eir $X \sim N(2.6, 0.15^2)$ $Z \sim N(0, 1)$
 Pwysau ceiliogod $Y \sim N(4.2, 0.25^2)$

$$\text{a)} P(Y > 2X) = P(Y - 2X > 0) \\ = P(2X - Y < 0)$$

$$\text{Nawr } E(2X - Y) = 2E(X) - E(Y) \\ = 2 \times 2.6 - 4.2 \\ = 1$$

$$\text{Var}(2X - Y) = 2^2 \text{Var}(X) + (-1)^2 \text{Var}(Y) \\ = 4 \times 0.15^2 + 1 \times 0.25^2 \\ = 0.1525$$

Mae $2X - Y \sim N(1, 0.1525)$

$$\text{Felly } P(Y > 2X) = P(2X - Y < 0) \\ = P\left(Z < \frac{0 - 1}{\sqrt{0.1525}}\right) \\ = P(Z < -2.560737599) \\ = P(Z > 2.560737599) \\ = 1 - P(Z \leq 2.560737599) \\ = 1 - P(Z \leq 2.56) \\ = 1 - 0.99477 \\ = \underline{\underline{0.00523}}$$

$$\text{b)} P(Y_1 + Y_2 + X_1 + X_2 + X_3 + X_4 + X_5 > 21) \\ E(Y_1 + Y_2 + X_1 + X_2 + X_3 + X_4 + X_5) = E(Y_1) + E(Y_2) + E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5) \\ = 2 \times 4.2 + 5 \times 2.6 \\ = 21.4 \\ \text{Var}(Y_1 + Y_2 + X_1 + X_2 + X_3 + X_4 + X_5) = 1^2 \text{Var}(Y_1) + 1^2 \text{Var}(Y_2) + \dots + 1^2 \text{Var}(X_5) \\ = 2 \times 0.25^2 + 5 \times 0.15^2 \\ = 0.2375$$

Felly $Y+Y+X+X+X+X+X \sim N(21.4, 0.2375)$

$$\begin{aligned} P(Y+Y+X+X+X+X+X > 21) &= P\left(Z > \frac{21-21.4}{\sqrt{0.2375}}\right) \\ &= P(Z > -0.8207826817) \\ &= P(Z \leq 0.82) \\ &= \underline{\underline{0.79389}} \end{aligned}$$

S2 Haf 2019

- 2) Gaderchi: X gynrychioli physau'r tabws yn y siop.
 $X \sim N(60, 8^2)$.

a) Rydym angen $P(X \leq x) = 0.25$

$$P\left(Z \leq \frac{x-60}{8}\right) = 0.25$$

o'r tablau, $P(Z \leq -0.674) = 0.25$

Felly $\frac{x-60}{8} = -0.674$

$$x-60 = -5.392$$

$$x = 54.608 \text{ g}$$

b) $P(\underbrace{X+X+\dots+X}_{16 \text{ graith}} > 1000)$

$$\begin{aligned} E(X+X+\dots+X) &= E(X) + E(X) + \dots + E(X) \\ &= 16 \times 60 \\ &= 960 \end{aligned}$$

$$\begin{aligned} \text{Var}(X+X+\dots+X) &= 1^2 \text{Var}(X) + 1^2 \text{Var}(X) + \dots + 1^2 \text{Var}(X) \\ &= 16 \times 1 \times 64 \\ &= 1024 \end{aligned}$$

Felly $\underbrace{X+X+\dots+X}_{16 \text{ gwaith}} \sim N(960, 1024)$

$$\begin{aligned} \text{Ateb} &= P\left(Z > \frac{1000-960}{\sqrt{1024}}\right) \\ &= P(Z > 1.25) \\ &= 1 - P(Z \leq 1.25) \end{aligned} \quad \begin{aligned} &\rightarrow = 1 - 0.89435 \\ &= \underline{\underline{0.10565}} \end{aligned}$$

c) Rydym anger $P(\underbrace{X_1 + X_2 + \dots + X_9}_{\text{Etabls Hmw}} - \underbrace{Y_1 - Y_2 - \dots - Y_{10}}_{\text{Etabls Anne}} > 0)$

$$\begin{aligned} & E(X_1 + X_2 + \dots + X_9 - Y_1 - Y_2 - \dots - Y_{10}) \\ &= E(X_1) + E(X_2) + \dots + E(X_9) - E(Y_1) - E(Y_2) - \dots - E(Y_{10}) \\ &= 9 \times 60 - 10 \times 60 \\ &= -60 \end{aligned}$$

$$\begin{aligned} & \text{Var}(X_1 + X_2 + \dots + X_9 - Y_1 - Y_2 - \dots - Y_{10}) \\ &= 1^2 \text{Var}(X_1) + 1^2 \text{Var}(X_2) + \dots + 1^2 \text{Var}(X_9) + (-1)^2 \text{Var}(Y_1) \\ &\quad + (-1)^2 \text{Var}(Y_2) + \dots + (-1)^2 \text{Var}(Y_{10}) \\ &= 19 \times 1 \times 64 \\ &= 1216 \end{aligned}$$

Felly $X_1 + X_2 + \dots + X_9 - Y_1 - Y_2 - \dots - Y_{10} \sim N(-60, 1216)$

$$\begin{aligned} \text{Ateb} &= P(Z > \frac{0 - -60}{\sqrt{1216}}) \\ &= P(Z > 1.720618004) \\ &= 1 - P(Z \leq 1.720618004) \\ &= 1 - P(Z \leq 1.72) \quad \text{oir tablau} \\ &= 1 - 0.95728 \\ &= \underline{\underline{0.04272}} \end{aligned}$$