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**Proving Hypotheses – Two Normal Distributions**

(Haf 2005)

6. There are 8 girls and 6 boys in a class. The teacher gives them all a problem to solve and she records the time taken by each child to complete the solution. She produces the following table.

Time taken by girls (minutes)	12.4	11.2	14.3	12.7	15.1	13.6	11.9	13.9
Time taken by boys (minutes)	14.2	15.8	13.8	14.9	12.9	15.1		

She now wishes to test whether or not there is a difference between the mean times taken by boys and girls to solve this problem. You may assume that the times taken are independent random samples from normal distributions with a common standard deviation of 1.5.

- (a) State suitable hypotheses. [1]
- (b) Calculate the  $p$ -value of these results. [7]
- (c) State your conclusion in context. [1]

(Haf 2006)

7. A Motoring Organisation wished to determine whether or not the fuel consumptions of two car models, A and B, are the same. To do this, six cars of each model were given 10 litres of petrol and driven around a track until they ran out of petrol. The distances travelled (in miles) by the cars were as follows.

Model A	83.1	84.1	84.6	85.1	81.2	82.9
Model B	81.0	82.2	79.9	83.4	81.8	80.7

You may assume that these are random samples from normal distributions with common standard deviation 1.5 miles.

- (a) State suitable hypotheses. [1]
- (b) Calculate the  $p$ -value and state your conclusion when the significance level is
- (i) 1%,
- (ii) 5%. [10]

(Haf 2007)

7. A scientist wishes to determine whether or not there is a difference in the acidity levels of two different liquids. He therefore makes five independent measurements of the acidity level of each liquid with the following results.

Liquid 1	6.31	6.38	6.33	6.34	6.35
Liquid 2	6.28	6.31	6.29	6.35	6.30

You may assume that these are random samples from normal distributions with common standard deviation 0.025.

- (a) (i) State suitable hypotheses.  
(ii) Calculate the  $p$ -value of the above measurements and interpret your value in context. [10]
- (b) Find a 95% confidence interval for the difference in the acidity levels of the two liquids. [3]

(Haf 2009)

4. A teacher wishes to investigate whether or not boys and girls take the same time, on average, to solve jigsaw puzzles. She therefore gives the same jigsaw puzzle to the 6 girls and the 5 boys in her class. She records the time taken by each girl,  $x$  minutes, and the time taken by each boy,  $y$  minutes, to complete the puzzle. She finds that

$$\sum x = 94.8, \sum y = 81.0$$

You may assume that the times are random samples from normal distributions with common standard deviation 0.5 minutes.

- (a) State suitable hypotheses for the investigation. [1]
- (b) Determine the  $p$ -value of these results and state your conclusion in context. [7]

(Haf 2010)

3. When a weighing machine is used to weigh an object, the reading obtained, in grams, is a normally distributed random variable with mean equal to the actual weight of the object and standard deviation 0.2. Successive weighings are independent.
- (a) When an object A was weighed three times, the readings obtained were 11.5, 11.7 and 11.6. Calculate a 95% confidence interval for the weight of object A. [5]
- (b) Before an object B was weighed, Graham believed that it would weigh 12 grams but Jim believed that it would weigh more than that.
- (i) State suitable hypotheses to test their beliefs.  
(ii) When the object B was weighed four times, the readings obtained were 12.1, 12.2, 12.4 and 12.1. Calculate the  $p$ -value of the four readings and state your conclusion. [7]
- (c) Calculate a 90% confidence interval for the difference between the weights of objects A and B. [5]

(Haf 2011)

4. A zoologist believes that the mean weights of the adult males and females of a certain species of animal are equal. In order to test this belief, she weighs random samples of males and females with the following results.

Weights of males (kg)	14.3	15.8	13.9	13.4	14.5	15.1	13.6	14.2
Weights of females (kg)	13.2	14.8	13.7	14.7	15.0	13.1	13.5	

You may assume that these are random samples from normal populations with a common standard deviation of 0.5.

- (a) State suitable hypotheses for carrying out a two-sided test. [1]
- (b) Determine the  $p$ -value of these results and state whether or not the zoologist's belief is supported at the 5% level of significance. [9]

(Haf 2012)

5. David and Frank are golfers and they wish to determine whether or not there is a difference between the mean distances that they can hit a golf ball. They decide that they should each hit six balls and measure the distances travelled in yards by these balls. The results are shown below.

Distances hit by David	152.1	148.3	150.6	145.4	144.7	149.3
Distances hit by Frank	143.4	147.9	150.8	144.1	145.6	147.2

You may assume that these are random samples from normal populations with a common standard deviation of 1.5.

- (a) State suitable hypotheses for testing whether or not there is a difference between the mean distances. [1]
- (b) Determine the  $p$ -value of these results and state your conclusion in context. [10]

(Haf 2013)

3. A teacher in a large college wishes to investigate whether or not boys and girls perform equally well in examinations in practical mathematics. She therefore selects a random sample of 8 boys and 8 girls and gives them an examination. The marks obtained were as follows.

Boys 52, 47, 62, 75, 51, 69, 56, 70  
Girls 48, 39, 56, 69, 71, 45, 43, 59

You may assume that these are random samples from normal populations with a common standard deviation of 7.5.

- (a) State suitable hypotheses for this investigation. [1]
- (b) Determine the  $p$ -value of these results and state your conclusion in context. [10]

(Haf 2014)

3. A new species of animal has been found on an uninhabited island. A zoologist wishes to investigate whether or not there is a difference in the mean weights of males and females of the species. She traps some of the animals and weighs them with the following results.

Males (kg)	5.3, 4.6, 5.2, 4.5, 4.3, 5.5, 5.0, 4.8
Females (kg)	4.9, 5.0, 4.1, 4.6, 4.3, 5.3, 4.2, 4.5, 4.8, 4.9

You may assume that these are random samples from normal populations with a common standard deviation of 0.5 kg.

- (a) State suitable hypotheses for this investigation. [1]
- (b) Determine the  $p$ -value of these results and state your conclusion in context. [9]

(Haf 2015)

3. When the sugar content of a jar of jam is measured using a certain machine, the reading obtained, in grams, is a normally distributed random variable with mean equal to the actual sugar content and standard deviation 1.5 grams. Successive readings are independent. A shopkeeper sells two varieties of strawberry jam and he wishes to investigate whether or not there is a difference between the sugar contents of the two varieties. He therefore selects 8 jars of Variety A and 8 jars of the same size of Variety B and measures the sugar content of the 16 jars with the following results.

Variety A	203.1	201.8	199.8	200.7	200.6	202.5	200.9	202.6
Variety B	198.2	197.5	196.4	199.1	199.7	197.1	198.6	197.4

- (a) Calculate a 99% confidence interval for the difference between the sugar contents of Variety A and Variety B. [7]
- (b) The shopkeeper's assistant uses the same data to determine another confidence interval for this difference and he obtains [2.19, 4.81]. Determine the confidence level of this interval. [4]

(Haf 2016)

4. The independent random variables  $X$ ,  $Y$  are such that  $X$  is  $N(\mu_x, 1.5^2)$  and  $Y$  is  $N(\mu_y, 2.5^2)$ . In order to test the hypotheses

$$H_0 : \mu_x = \mu_y \quad ; \quad H_1 : \mu_x \neq \mu_y$$

a random sample of size 8 is taken from the distribution of  $X$  and a random sample of size 12 is taken from the distribution of  $Y$ . The means of these two samples are denoted by  $\bar{x}$  and  $\bar{y}$  respectively. The significance level is to be 10%.

- (a) Determine the critical region in the form  $|\bar{x} - \bar{y}| > k$ , where the value of  $k$  is to be found. [5]
- (b) (i) If, in fact,  $\mu_x - \mu_y = 0.5$ , find the probability of incorrectly accepting  $H_0$ .
- (ii) Comment on your result in (i). [9]

(Haf 2017)

4. A motorbike club wished to compare the fuel consumptions of two motorbike models, A and B. To do this, eight motorbikes of each model were given 15 litres of petrol and driven around a track until they ran out of petrol. The distances travelled (in miles) by the motorbikes were as follows.

Model A	168.2	170.5	164.2	169.2	165.8	166.6	162.2	168.5
Model B	161.7	166.3	167.4	164.1	162.7	160.3	165.6	163.1

You may assume that these are random samples from normal distributions with means  $\mu_A, \mu_B$  respectively and common standard deviation 2.5.

- (a) Determine a 95% confidence interval for  $\mu_A - \mu_B$ . [7]
- (b) Find the smallest confidence level for which the corresponding confidence interval includes zero. Give your answer as a percentage correct to three significant figures. [4]

(Haf 2018)

5. A machine is used to measure the refractive index of glass fragments. The reading obtained is a normally distributed random variable with mean equal to the true refractive index and standard deviation 0.02. Measurements were made on two samples of glass fragments believed to have come from two windows having the same refractive index. These measurements are shown in the table below.

Window 1	1.51	1.54	1.53	1.49	1.52	1.53	
Window 2	1.54	1.56	1.57	1.53	1.52	1.55	1.58

- (a) State suitable hypotheses to test the above belief using a two-sided test. [1]
- (b) (i) Calculate the  $p$ -value for this test.
- (ii) Interpret the  $p$ -value in context. [9]