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Proving Hypotheses – Poisson

(Haf 2005)

4. The number of machine breakdowns per day in a certain factory may be assumed to have a Poisson distribution with mean μ . The value of μ is known, from past experience, to be 1.5. In an attempt to reduce the value of μ , all the machines are fitted with new control units. To investigate whether or not this succeeds in reducing the value of μ , the number of breakdowns, x , during a 30-day period following the fitting of these new units is recorded.
- (a) State suitable hypotheses. [1]
- (b) It is decided to conclude that the value of μ has been reduced if $x \leq 35$.
- (i) What name is given to the region $x \leq 35$?
- (ii) Define the term ‘significance level’ and find its approximate value for this procedure. [7]

(Haf 2006)

5. On any weekday, the number of passengers, X , using an early morning bus service may be assumed to follow a Poisson distribution. In the past, the mean value of X has been 2.4. The local council wishes to increase this mean so they decide to offer this service free of charge.
- (a) During the 5-day week following this offer, a total of 18 passengers used this service. Clearly stating your hypotheses, calculate the p -value of this result. Interpret your value in context. [5]
- (b) During the 100 weekdays following this offer, a total of 280 passengers used this service. Determine, at the 1% significance level, whether or not the offer has resulted in an increase in the mean number of passengers using this early morning bus service. [7]

(Haf 2007)

6. A plumber knows that the number of emergency calls received per day follows a Poisson distribution with mean $\mu = 2$.
- (a) Calculate the probability that, in a 7-day period, he receives
- (i) exactly 10 calls,
- (ii) more than 12 calls. [5]
- (b) Wishing to increase the value of μ , he increases his advertising budget.
- (i) State suitable hypotheses for investigating whether or not this achieves the desired result.
- (ii) In the first 7-day period after increasing the budget, he receives 20 emergency calls. Calculate and interpret the p -value of this result.
- (iii) In the next 100-day period, he receives 230 emergency calls. Calculate an approximate p -value of this result and interpret it. [12]

(Haf 2008)

7. The random variable X has a Poisson distribution with unknown mean μ . It is required to test whether $\mu = 2.5$ against a two-sided alternative.
- (a) State suitable hypotheses. [1]
- (b) Let S denote the sum of six randomly chosen values of X . It is decided to reject the null hypothesis if either $S \leq 8$ or $S \geq 23$.
- (i) Calculate the significance level of this test.
- (ii) Given that the value of μ is actually 2, find the probability of reaching an incorrect conclusion. [8]
- (c) It is now decided to obtain a random sample of 100 values of X . It is found that the sum of these 100 values is 270. Find an approximate p -value of this sum and interpret your result. [8]

(Haf 2009)

1. The number of telephone enquiries received per hour at a certain office may be assumed to follow a Poisson distribution with mean μ . Office records indicate that $\mu = 2$ but the office manager believes that the value of μ has increased.
- (a) To test this belief, he counts the number of enquiries received during a 6-hour period. Given that 18 enquiries are received, calculate the p -value. [3]
- (b) He now decides to count the number of enquiries received during a 50-hour period. Given that 125 enquiries are received, calculate the p -value and state your conclusion. [5]

(Haf 2010)

2. The number of computer breakdowns per day at a large office may be assumed to follow a Poisson distribution with mean μ . The IT Manager believes that the value of μ should be 1.5 but he decides to check this. He therefore defines the following hypotheses.

$$H_0: \mu = 1.5; \quad H_1: \mu \neq 1.5$$

- (a) For one test, he decides to count the number of breakdowns, x , in a 10-day period and to define the critical region as $x \leq 9$ or $x \geq 22$. Find the significance level of this test. [5]
- (b) For another test, he decides to count the number of breakdowns occurring during a 100-day period. Given that 170 breakdowns occur, calculate the approximate p -value and state your conclusion. [7]

(Haf 2011)

3. A factory manufactures screws and packs them in large bags. The number of defective screws in a bag can be modelled by a Poisson distribution whose mean is known to have been 0.5. However, new equipment has been installed which, it is hoped, will decrease this mean. The Quality Controller plans to take samples of bags to investigate whether or not there is a reduction in the mean.
- (a) State suitable hypotheses. [1]
 - (b) He takes a random sample of 30 bags and finds that they contain a total of 12 defective screws. Calculate the p -value and state your conclusion. [4]
 - (c) He then takes a random sample of 200 bags and finds that they contain a total of 80 defective screws. Calculate an approximate p -value and state your conclusion. [6]

(Haf 2012)

4. (a) When Jack types a page of a document, the number of errors made may be modelled by a Poisson distribution with mean 0.8. He types a 10-page document. Determine the probability that the total number of errors is less than 5. [3]
- (b) When Mary types a page of a document, the number of errors made may be modelled by a Poisson distribution with mean μ . Mary claims that the value of μ is less than 0.8 but Jack claims that μ is equal to 0.8.
- (i) State suitable hypotheses for testing these claims.
 - (ii) Mary types an 80-page document and makes 60 errors. Find the approximate p -value of this result and state your conclusion. [7]

(Haf 2013)

5. The number of accidents occurring per day along a certain stretch of road can be modelled by a Poisson distribution. The value of the mean μ has been 1.2 in the past but the local council has recently introduced a lower speed limit in the hope of reducing the value of μ .
- (a) State suitable hypotheses for testing whether or not lowering the speed limit has had the desired effect. [1]
 - (b) It is decided to count the number of accidents, x , occurring in a 60-day period and to define the critical region as $x \leq 58$.
 - (i) Determine the significance level.
 - (ii) Given that the value of μ has actually fallen to 0.8, determine the probability of concluding that there has been no reduction in the value of μ . [12]

(Haf 2014)

6. When John types a page of a document, the number of errors can be modelled by a Poisson distribution with mean μ . He claims that the value of μ is 1.5 but his employer wants to test this claim so they define the following hypotheses.

$$H_0 : \mu = 1.5; \quad H_1 : \mu \neq 1.5$$

- (a) John is asked to type a 10-page document and the critical region is taken as $x \leq 10$ or $x \geq 20$, where x denotes the total number of errors in the document.
- (i) Find the significance level of this test.
- (ii) Find the probability of incorrectly accepting H_0 when the value of μ is actually 1.0. [7]
- (b) John now types a 50-page document and makes 92 errors. Find the p -value and state your conclusion. [6]

(Haf 2015)

6. In a certain factory, the number of breakdowns occurring during a day is a Poisson random variable with mean 1.2. Successive breakdowns are independent. In an attempt to reduce the number of breakdowns, the machines are modified.

- (a) In the 10 days following the modifications, the total number of breakdowns was 9.
- (i) State suitable hypotheses for testing whether the mean number of breakdowns per day has decreased.
- (ii) Calculate the p -value and state your conclusion in context. [5]
- (b) In the 100 days following the modifications, the total number of breakdowns was 101. Calculate an approximate p -value and interpret it in context. [6]

(Haf 2017)

2. The number of computer breakdowns per day in a large IT Department may be assumed to follow a Poisson distribution with mean 0.8. In an attempt to reduce the number of breakdowns, the IT Manager moves the department to a new purpose-built office. He defines the following hypotheses

$$H_0 : \mu = 0.8; \quad H_1 : \mu < 0.8$$

where μ denotes the mean number of breakdowns per day after the move.

He finds that in the first 100 days after the move, there was a total of 64 computer breakdowns. You may assume that the numbers of breakdowns on successive days are independent. Calculate the approximate p -value of this result and interpret it in context. [7]

(Haf 2018)

4. When Alun types a page of a document, the number of errors made may be modelled by a Poisson distribution with mean μ . The numbers of errors on different pages are independent.

(a) He types a 20-page document. Assuming that $\mu = 0.5$, determine the probability that the total number of errors in the document is equal to 12. [3]

(b) Alun believes that his typing has improved and the value of μ is now less than 0.5. He therefore defines the following hypotheses.

$$H_0 : \mu = 0.5 \quad ; \quad H_1 : \mu < 0.5$$

He types a 30-page document and he makes 9 errors.

(i) Calculate the p -value of this result.

(ii) Interpret this p -value using a 5% significance level. Justify your answer. [4]

(c) He now types a 100-page document and he makes 36 errors.

(i) Calculate the approximate p -value of this result.

(ii) What evidence does this p -value give regarding whether or not Alun's typing has improved? [5]