

Hen Gwestiynau Arholiad
Proving Hypotheses – Binomial

(Haf 2005)

5. A garden centre claims that, on average, 45% of seeds of a certain type will germinate. A gardener believes that the percentage germinating is actually less than 45%.
- (a) To test his belief, the gardener sows 50 of these seeds and he counts the number which germinate.
- (i) State appropriate hypotheses.
 - (ii) He finds that 18 of the seeds germinate. Find the p -value of this result.
 - (iii) Determine the maximum number of germinating seeds which would lead to the rejection of the claim at the 1% significance level. [6]
- (b) The gardener now sows 500 of these seeds. He finds that 202 of the seeds germinate.
- (i) Use a distributional approximation to find the p -value of this result.
 - (ii) Interpret your p -value in context using a 5% significance level. [7]

(Haf 2006)

6. Mr. Jones, a candidate in an election, believes that 40% of the electors intend to vote for him. His agent, however, believes that the support for him is less than this. To resolve this difference in opinion, they set up the hypotheses

$$H_0 : p = 0.4 \text{ versus } H_1 : p < 0.4$$

where p denotes the proportion of the electorate intending to vote for Mr. Jones.

- (a) They decide to question a random sample of 50 electors. They define the critical region to be $X \leq 14$, where X denotes the number in the sample intending to vote for Mr. Jones.
- (i) Determine the significance level of this procedure.
 - (ii) Given that p is actually 0.3, find the probability that they draw the wrong conclusion. [5]
- (b) They now decide to carry out a larger survey in which they question 500 randomly chosen electors. They find that 185 of these intend to vote for Mr. Jones. Find the p -value of this result and interpret it in context. [7]

(Haf 2007)

5. Jim is a darts player. When he throws a dart, he claims to be able to hit the 'bull' with probability 0.75. His friends believe that the probability is less than this.
- (a) State suitable hypotheses to test Jim's claim. [1]
 - (b) They decide to set up a trial in which Jim throws 20 darts and they define X to be the number of darts hitting the 'bull'.
 - (i) Taking the critical region to be $X \leq k$, find the value of k for which the significance level is nearest to 10%.
 - (ii) The actual value of the probability of Jim hitting the 'bull' is 0.5. With the value of k found in (i), find the probability of reaching an incorrect conclusion. [7]

(Haf 2008)

5. The manager of a Garden Centre claims that each seed of a particular variety of flower has probability 0.6 of germinating, independently of all other seeds. Malcolm, however, thinks that the probability is less than 0.6 so he decides to plant a number of seeds and to count how many germinate.
- (a) State suitable hypotheses. [1]
 - (b) Initially he plants 20 seeds and finds that 9 germinate. Calculate the p -value of this result. [4]
 - (c) He now decides to plant 200 of these seeds. He finds that 101 germinate. Find an approximate p -value of this result and state your conclusion in context. [7]

(Haf 2009)

7. A drug is known to cure 70% of patients suffering from a certain disease. A pharmaceutical company has developed a new drug which is claimed to cure a higher percentage than this. To test this claim, the new drug is given to 50 patients.
- (a) State suitable hypotheses. [1]
 - (b) It is found that 40 of these patients are cured of the disease. Find the p -value of this result and state your conclusion in context. [5]
 - (c) The company decides to carry out a larger trial in which the new drug is to be given to 250 patients. Let x denote the number of patients cured. Given that the critical region is $x \geq 190$, find
 - (i) the significance level,
 - (ii) the probability of concluding that the new drug does not increase the percentage of patients cured when in fact the percentage cured has increased to 80%. [10]

(Haf 2010)

6. Ann and Brenda buy a packet of seeds which states that, on average, 75% of the seeds will germinate. They believe, however, that the germination rate is less than this so they plant a certain number of seeds and count how many germinate.
- (a) State suitable hypotheses. [1]
- (b) Ann plants 50 seeds and decides to reject the statement on the packet if less than 30 germinate.
- (i) Find the significance level of this procedure.
- (ii) Find the probability of accepting the statement on the packet if the actual germination rate is 50%. [6]
- (c) Brenda plants 200 seeds and finds that 140 germinate. Find the approximate p -value of this result and state your conclusion in context. [6]

(Haf 2011)

6. David is given a biased coin and is told that the probability of obtaining a head when the coin is tossed is either 0.3 or 0.6. To determine which, he defines the following hypotheses.

$$H_0 : p = 0.3; \quad H_1 : p = 0.6.$$

- (a) He tosses the coin 20 times and denotes the number of heads obtained by x .
He will accept H_1 if $x \geq 9$ and he will accept H_0 if $x \leq 8$.
Calculate the probability of
- (i) accepting H_1 when H_0 is true,
- (ii) accepting H_0 when H_1 is true. [8]
- (b) He now tosses the coin 80 times and denotes the number of heads obtained by y .
He will accept H_1 if $y \geq 36$ and he will accept H_0 if $y \leq 35$.
Using a normal approximation, calculate the probability of
- (i) accepting H_1 when H_0 is true,
- (ii) accepting H_0 when H_1 is true. [8]

(Haf 2012)

7. A garden centre sells large bags of wallflower seeds. Type A bags contain a mixture of seeds of which, on average, 50% will produce white flowers and 50% red flowers. Type B bags contain a mixture of seeds of which, on average, 70% will produce white flowers and 30% red flowers. The manager finds an unlabelled bag of these seeds and she wants to know if it is Type A or Type B.

She therefore plants 120 seeds and she decides to label the bag Type A if the number of these seeds producing white flowers is less than 70. You may assume that all the seeds germinate and produce flowers. Determine, approximately, the probability of

- (a) labelling the bag as Type A when it is actually Type B, [6]
(b) labelling the bag as Type B when it is actually Type A. [6]

(Haf 2013)

4. A politician believes that 40% of the voters support him but his agent believes that his support is greater than this. They decide to question a random sample of voters to try to resolve the situation.

- (a) State suitable hypotheses. [1]
(b) They question a random sample of 50 voters, of whom 25 state that they support the politician. Determine the p -value and state your conclusion in context at the 5% significance level. [4]
(c) They now decide to question a random sample of 400 voters, of whom 181 state that they support the politician. Determine an approximate p -value and state your conclusion in context. [7]

(Haf 2014)

4. Gwilym buys a new computer game. He claims that he wins, on average, 60% of games played. His friend Huw believes that Gwilym wins less than 60% of games played.

- (a) To investigate these conflicting claims, Gwilym plays the game 20 times and wins 7 of them.
(i) State suitable hypotheses for testing these claims.
(ii) Determine the p -value of the above result and state your conclusion in context. [7]
(b) During the following week, Gwilym plays the game 80 times and wins 37 of them. Use a suitable approximation to determine the p -value and state your conclusion in context. [7]

(Haf 2015)

4. When Edwin shoots an arrow at a target, he hits it with probability $p = 0.4$. Successive shots are independent. He attends a course to try to improve his technique and he wishes to test whether or not the course has succeeded in doing that. He defines the following hypotheses.

$$H_0 : p = 0.4; \quad H_1 : p > 0.4$$

- (a) He decides initially to shoot 20 arrows at the target and X denotes the number of these shots which hit the target. Determine the critical region having a significance level closest to 1%. [3]
- (b) He decides to carry out a further test in which he shoots 120 arrows at the target. He finds that 55 of these shots hit the target. Calculate an approximate p -value and state your conclusion in context. [7]

(Haf 2016)

5. A seed manufacturer claims that 70% of seeds of a certain variety will germinate but the manager of a garden centre claims that the germination rate is less than this.
- (a) A trial is therefore conducted in which 50 seeds of this variety are planted. It is found that 32 of these seeds germinate.
- (i) State suitable hypotheses to test these claims.
- (ii) Calculate the p -value of this result and state your conclusion in context. [7]
- (b) A further trial is conducted in which 500 seeds of this variety are planted and it is found that 329 of these seeds germinate. Carry out a hypothesis test using a normal approximation and state your conclusion in context. [7]

(Haf 2017)

5. Charlie is given a coin and he is told that it is biased so that the probability, p , of obtaining a head when tossed is 0.75. To test this, he defines the following hypotheses.

$$H_0 : p = 0.75; \quad H_1 : p \neq 0.75$$

- (a) He decides to toss the coin 50 times and he denotes the number of heads obtained by x . He defines the following critical region.

$$(x \leq 31) \cup (x \geq 44)$$

- (i) Determine the significance level of this test.
- (ii) Find the probability of accepting H_0 if the value of p is actually 0.5. [7]
- (b) In a further attempt to test whether or not the value of p is 0.75, he decides to toss the coin 200 times. He obtains 139 heads.
- (i) Calculate the approximate p -value of this result.
- (ii) Interpret the p -value. [7]

(Haf 2019)

5. Alun has a coin which he thinks might be biased so that, when tossed, the probability p of obtaining heads may not be 0.5. He therefore defines the following hypotheses.

$$H_0 : p = 0.5; \quad H_1 : p \neq 0.5$$

- (a) He decides to toss the coin 20 times and to accept H_0 if the number of heads obtained lies in the interval $[10 - k, 10 + k]$, where k is a positive integer.
- (i) Assuming H_0 to be true, determine the probability that the number of heads obtained lies in the interval $[10 - k, 10 + k]$ for $k = 2, 3, 4$.
 - (ii) Hence state the value of k which gives a significance level closest to 10% and state this significance level. [7]
- (b) He now decides to toss the coin 120 times and he obtains 52 heads.
- (i) Calculate an approximate p -value for this result.
 - (ii) What does your p -value tell you about the bias of the coin? [8]