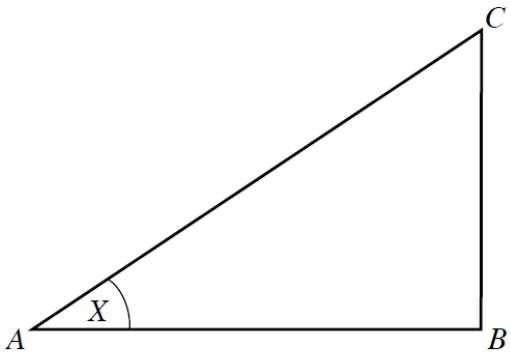
Hen Gwestiynau Arholiad

The Uniform Distribution

(Haf 2005)

7.



The above diagram shows a right-angled triangle ABC in which the length of the hypotenuse AC is 2 cm and $B\hat{A}C$ equals X radians, where X is a uniformly distributed random variable defined on the interval $\left[0, \frac{\pi}{3}\right]$.

(a) Write down, in full, the probability density function of X. [2]

(b) The length of BC is denoted by H cm. Calculate

(i) $P(H \leqslant \sqrt{2})$,

(ii) $E(H)$. [8]

(Haf 2006)

2. The random variable X is uniformly distributed on the interval $[a, b]$. Given that the variance of X is 3, show that

$$b - a = 6.$$

Given further that the mean of X is 10, find the values of a and b. [7]

(Haf 2007)

4. A circle has radius R cm, where R is a continuous random variable that is uniformly distributed on the interval [0, 5].

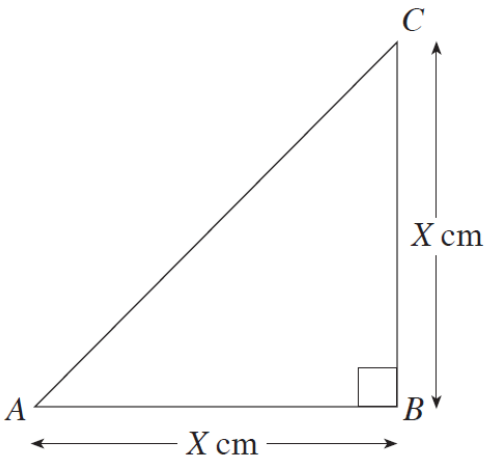
(a) State the probability density function of R. [1]

(b) Find the expected area of the circle. [4]

(c) Find the probability that the area of the circle is greater than 25 cm^2, giving your answer correct to three decimal places. [4]

(Haf 2008)

6.



The above diagram shows an isosceles right-angled triangle ABC in which $AB = BC = X$ cm where X is a continuous random variable uniformly distributed on the interval [4, 6].

(a) Find the probability that

(i) the length of AC exceeds 8 cm,

(ii) the area of the triangle ABC is less than 10 cm^2. [8]

(b) Find the expected value of the area of the triangle ABC. [4]

(Haf 2009)

6. The continuous random variable X is uniformly distributed on the interval [9, 16].

(a) (i) Write down an expression for $f(x)$, valid for $9 \leqslant x \leqslant 16$, where f denotes the probability density function of X.

(ii) Obtain an expression for $F(x)$, valid for $9 \leqslant x \leqslant 16$, where F denotes the cumulative distribution function of X. [4]

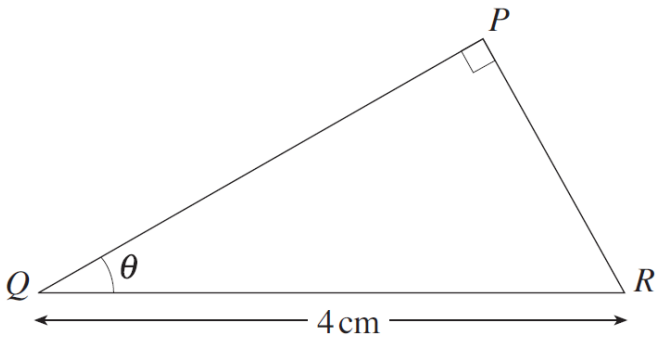
(b) The random variable Y is defined by $Y = \sqrt{X}$. Giving your answers correct to 2 decimal places, determine

(i) $E(Y)$,

(ii) the median of Y. [8]

(Haf 2010)

5.



The above diagram shows a right-angled triangle in which the hypotenuse $QR = 4$ cm and $P\hat{Q}R = \theta$ radians, where θ is a continuous random variable uniformly distributed between 0 and $\frac{\pi}{4}$.

(a) Show that the area, A cm^2, of the triangle PQR is given by

$$A = 4 \sin 2\theta.$$ [1]

(b) Calculate $P(A \leqslant 2)$. [5]

(c) Determine $E(A)$. [4]

(Haf 2011)

5. *(a)* The continuous random variable U is uniformly distributed on $[a, b]$.
Write down the probability density function of U and **hence** show that

$$E\left(U^2\right) = \frac{a^2 + ab + b^2}{3}.$$ [5]

(b) A piece of string of length 12 cm is cut at a random point. The length of the resulting shorter piece is denoted by X cm and the length of the longer piece by Y cm. You may assume that X is uniformly distributed on [0, 6].

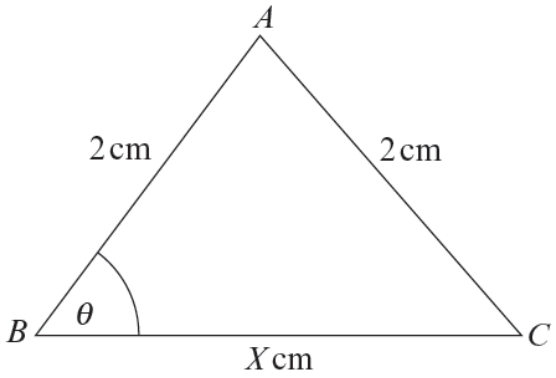
(i) Find the mean and variance of X.

(ii) Express Y in terms of X and hence find the mean of XY.

(iii) Suppose now that 100 pieces of string of length 12 cm are each cut at a random point. Use the Central Limit Theorem to find, approximately, the probability that the total length of the 100 shorter pieces is greater than 280 cm. [11]

(Haf 2012)

6.



The diagram shows an isosceles triangle ABC in which $AB = AC = 2\,\text{cm}$.

The angle $A\hat{B}C$, denoted by θ, is a random variable that is uniformly distributed on the interval $(0, \frac{\pi}{2})$. The length BC is denoted by $X\,\text{cm}$.

(a) Show that $X = 4\cos\theta$. [2]

(b) Evaluate

(i) $E(X)$,

(ii) $P(X \leqslant 3)$. [8]

(Haf 2013)

6. The radius R of a circle is a continuous random variable that is uniformly distributed on the interval $[6, 8]$.

(a) Let C denote the circumference of the circle. Determine

(i) the mean and the variance of C,

(ii) $P(C \leqslant 45)$. [7]

(b) Let A denote the area of the circle. Determine

(i) $P(A \geqslant 150)$,

(ii) $E(A)$. [7]

(Haf 2014)

7. The sides of a square are of length $L\,\text{cm}$ and its area is $A\,\text{cm}^2$. Given that A is uniformly distributed on the interval $[15, 20]$, find

(a) $P(L \leqslant 4)$, [3]

(b) $E(L)$, [4]

(c) $\text{Var}(L)$. [3]

(Haf 2015)

7. The continuous random variable X is uniformly distributed on the interval $[a, b]$ where $0 < a < b$.

(a) Let $Y = \sqrt{X}$.

(i) Find an expression for $P(Y \leqslant y)$ for $\sqrt{a} \leqslant y \leqslant \sqrt{b}$.

(ii) Hence find the probability density function of Y. [6]

(b) Given that the mean and the variance of X are 5·5 and 3 respectively, find the values of a and b. [5]

(Haf 2016)

6. A piece of string of length 20 cm is cut at a random point. The length of the longer piece is denoted by X cm and the length of the shorter piece is denoted by Y cm. You may assume that X is uniformly distributed on the interval [10, 20].

(a) Determine $P(Y < 8)$. [2]

(b) (i) Express Y in terms of X.

(ii) Determine $P(XY > 64)$. [6]

(c) Calculate $E(XY)$. [5]

(Haf 2017)

6. The continuous random variable X is uniformly distributed on the interval $[a, b]$ where $0 < a < b$.

(a) (i) Write down the probability density function of X, stating its value for all real values of x.

(ii) Hence, using integration, show that

$$E(X^2) = \frac{a^2 + ab + b^2}{3} .$$

(iii) Hence, given that $E(X) = \frac{a+b}{2}$, show that

$$\text{Var}(X) = \frac{(b-a)^2}{12} . \qquad [8]$$

(b) Given that the random variable Y is defined by $Y = \frac{1}{X}$, $a \leqslant X \leqslant b$,

(i) determine $E(Y)$ in terms of a and b,

(ii) show that, for $\frac{1}{b} \leqslant y \leqslant \frac{1}{a}$,

$$P(Y \leqslant y) = \frac{b - \frac{1}{y}}{b-a} .$$

(iii) Hence find the probability density function of Y. [7]

(Haf 2018)

6. The continuous random variable X is uniformly distributed on the interval $[a, b]$.

(a) Given that $E(X) = 34$ and $\text{Var}(X) = 12$, determine the value of a and the value of b. [5]

(b) Determine the 95th percentile of X. [2]

(c) (i) State the Central Limit Theorem.

(ii) A random sample of 120 observations was taken from the distribution of X, and S denotes the sum of these 120 observations. Use the Central Limit Theorem to find an approximate value for $P(S > 4140)$. [7]

(Haf 2019)

1. The independent, continuous random variables X and Y are uniformly distributed on the interval [0, 4].

(a) Show that

$$E(X^n) = \frac{4^n}{n+1},$$

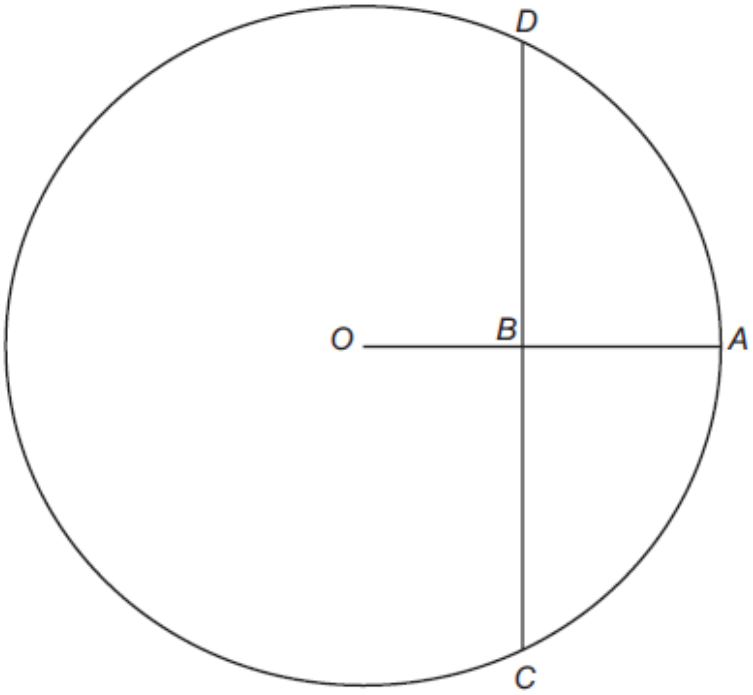
where n is a positive integer. [3]

(b) Determine the mean and the variance of

(i) X^2,

(ii) XY. [9]

6.



The diagram shows a fixed point A lying on a circle with centre O and fixed radius r. The random point B lies on OA such that the length OB is uniformly distributed on the interval $[0, r]$. The chord of the circle passing through B, perpendicular to OA, meets the circle at the points C and D. The length OB is denoted by X and the length CD is denoted by Y.

(a) Show that

$$Y^2 = 4(r^2 - X^2).$$ [2]

(b) Determine the value of $P(Y \geqslant r)$. [6]