

FPL: Differm o Egwyddorion Sylfaenol

Haf 2005

$$f(x) = \frac{1}{x^2 + x}$$

$$\begin{aligned}f(x+\delta x) &= \frac{1}{(x+\delta x)^2 + (x+\delta x)} \\&= \frac{1}{x^2 + 2x\delta x + (\delta x)^2 + x + \delta x} \\&= \frac{1}{x^2 + x + (\delta x)^2 + 2x\delta x + \delta x}\end{aligned}$$

$$\begin{aligned}f(x+\delta x) - f(x) &= \frac{1}{x^2 + x + (\delta x)^2 + 2x\delta x + \delta x} - \frac{1}{x^2 + x} \\&= \frac{(x^2 + x) - (x^2 + x + (\delta x)^2 + 2x\delta x + \delta x)}{(x^2 + x + (\delta x)^2 + 2x\delta x + \delta x)(x^2 + x)} \\&= \frac{\cancel{x^2 + x} - \cancel{x} - \cancel{x} - (\delta x)^2 - 2x\delta x - \delta x}{x^4 + x^3 + x^2(\delta x)^2 + 2x^3\delta x + x^2\delta x + x^3 + x^2 + x(\delta x)^2 + 2x^2\delta x + x\delta x} \\&= \frac{-(\delta x)^2 - 2x\delta x - \delta x}{x^4 + 2x^3 + (x^2 + x)(\delta x)^2 + (2x^3 + 3x^2 + x)\delta x + x^2}\end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{-\delta x - 2x - 1}{x^4 + 2x^3 + (x^2 + x)(\delta x)^2 + (2x^3 + 3x^2 + x)\delta x + x^2}$$

$$\frac{dy}{dx} = \frac{-2x - 1}{x^4 + 2x^3 + x^2}$$

Gaeaf 2006

$$f(x) = \frac{1}{1-x^2}$$

$$\begin{aligned}f(x+\delta x) &= \frac{1}{1-(x+\delta x)^2} \\&= \frac{1}{1-(x^2+2x\delta x+(\delta x)^2)} \\&= \frac{1}{1-x^2-2x\delta x-(\delta x)^2}\end{aligned}$$

$$\begin{aligned}f(x+\delta x) - f(x) &= \frac{1}{1-x^2-2x\delta x-(\delta x)^2} - \frac{1}{1-x^2} \\&= \frac{(1-x^2) - (1-x^2-2x\delta x-(\delta x)^2)}{(1-x^2-2x\delta x-(\delta x)^2)(1-x^2)} \\&= \frac{\cancel{1-x^2}-\cancel{1+x^2}+2x\delta x+(\delta x)^2}{1-x^2-2x\delta x-(\delta x)^2 - x^2+x^4+2x^3\delta x+x^2(\delta x)^2} \\&= \frac{2x\delta x+(\delta x)^2}{1-2x^2+x^4+(-2x+2x^3)\delta x+(-1+x^2)(\delta x)^2}\end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x)-f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{2x + \delta x}{1-2x^2+x^4+(-2x+2x^3)\delta x+(-1+x^2)(\delta x)^2}$$

$$\frac{dy}{dx} = \frac{2x}{1-2x^2+x^4}$$

Haf 2006

$$f(x) = \frac{1}{2x-3}$$

$$\begin{aligned}f(x+\delta x) &= \frac{1}{2(x+\delta x)-3} \\&= \frac{1}{2x+2\delta x-3}\end{aligned}$$

$$\begin{aligned}f(x+\delta x) - f(x) &= \frac{1}{2x+2\delta x-3} - \frac{1}{2x-3} \\&= \frac{(2x-3) - (2x+2\delta x-3)}{(2x+2\delta x-3)(2x-3)} \\&= \frac{\cancel{2x-3} - \cancel{2x} - \cancel{2\delta x} + \cancel{3}}{4x^2 + 4x\delta x - 6x - 6x - 6\delta x + 9} \\&= \frac{-2\delta x}{4x^2 - 12x + 9 + (4x-6)\delta x}\end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{-2}{4x^2 - 12x + 9 + (4x-6)\delta x}$$

$$\frac{dy}{dx} = \frac{-2}{4x^2 - 12x + 9}$$

Haf 2007

$$f(x) = x^4$$

$$f(x+\delta x) = (x+\delta x)^4$$

$$= (x+\delta x)^2 (x+\delta x)^2$$

$$= (x^2 + 2x\delta x + (\delta x)^2)(x^2 + 2x\delta x + (\delta x)^2)$$

$$= x^4 + 2x^3\delta x + x^2(\delta x)^2 + 2x^3\delta x + 4x^2(\delta x)^2 + 2x(\delta x)^3 + x^2(\delta x)^2 + 2x(\delta x)^3 + (\delta x)^4$$

$$= x^4 + (4x^3)\delta x + (6x^2)(\delta x)^2 + (4x)(\delta x)^3 + (\delta x)^4$$

$$f(x+\delta x) - f(x) = x^4 + 4x^3\delta x + 6x^2(\delta x)^2 + 4x(\delta x)^3 + (\delta x)^4 - x^4$$

$$= 4x^3\delta x + 6x^2(\delta x)^2 + 4x(\delta x)^3 + (\delta x)^4$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} 4x^3 + 6x^2\delta x + 4x(\delta x)^2 + (\delta x)^3$$

$$\frac{dy}{dx} = 4x^3$$

$$\textcircled{1} \quad (\text{a}) \quad y = 2^x$$

$$\ln(y) = x \ln(2)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(x \ln(2))$$

$$\frac{dy}{dx} \frac{d}{dy}(\ln(y)) = \frac{d}{dx}(x \ln(2))$$

$$\frac{dy}{dx} \left(\frac{1}{y} \right) = \ln(2)$$

$$\frac{dy}{dx} = y \ln(2)$$

$$\frac{dy}{dx} = 2^x \ln(2)$$

$\frac{dy}{dx}$

$$(\text{b}) \quad f(x) = \frac{x}{x+1}$$

$$f(x+\delta x) = \frac{x+\delta x}{x+\delta x+1}$$

$$f(x+\delta x) - f(x) = \frac{x+\delta x}{x+\delta x+1} - \frac{x}{x+1}$$

$$= \frac{(x+\delta x)(x+1) - x(x+\delta x+1)}{(x+\delta x+1)(x+1)}$$

$$= \frac{x^2 + x + x\delta x + \delta x - (x^2 + x\delta x + x)}{x^2 + x\delta x + x + x + \delta x + 1}$$

$$= \frac{\cancel{x^2 + x + x\delta x + \delta x} - \cancel{x^2 + x\delta x - x}}{x^2 + 2x + 1 + (x+1)\delta x}$$

$$= \frac{\delta x}{x^2 + 2x + 1 + (x+1)\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{1}{x^2 + 2x + 1 + (x+1)\delta x}$$

$$\frac{dy}{dx} = \frac{1}{x^2 + 2x + 1}$$

Haf 2010

$$f(x) = \frac{1}{1+x^2}$$

$$\begin{aligned} f(x+\delta x) &= \frac{1}{1+(x+\delta x)^2} \\ &= \frac{1}{1+x^2+2x\delta x+(\delta x)^2} \end{aligned}$$

$$\begin{aligned} f(x+\delta x) - f(x) &= \frac{1}{1+x^2+2x\delta x+(\delta x)^2} - \frac{1}{1+x^2} \\ &= \frac{(1+x^2) - (1+x^2+2x\delta x+(\delta x)^2)}{(1+x^2+2x\delta x+(\delta x)^2)(1+x^2)} \\ &= \frac{\cancel{1+x^2} - \cancel{1+x^2} - 2x\delta x - (\delta x)^2}{1+x^2+2x\delta x+(\delta x)^2 + x^2+x^4+2x^3\delta x+x^2(\delta x)^2} \\ &= \frac{-2x\delta x - (\delta x)^2}{1+2x^2+x^4+(2x+2x^3)\delta x+(1+x^2)(\delta x)^2} \end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{-2x - \delta x}{1+2x^2+x^4+(2x+2x^3)\delta x+(1+x^2)(\delta x)^2}$$

$$\frac{dy}{dx} = \frac{-2x}{1+2x^2+x^4}$$

$$f(x) = \frac{1}{x^3}$$

$$\begin{aligned} f(x+\delta x) &= \frac{1}{(x+\delta x)^3} \\ &= \frac{1}{(x+\delta x)^2(x+\delta x)} \\ &= \frac{1}{(x^2+2x\delta x+(\delta x)^2)(x+\delta x)} \\ &= \frac{1}{x^3+2x^2\delta x+x(\delta x)^2+x^2\delta x+2x(\delta x)^2+(\delta x)^3} \\ &= \frac{1}{x^3+3x^2\delta x+3x(\delta x)^2+(\delta x)^3} \end{aligned}$$

$$\begin{aligned} f(x+\delta x) - f(x) &= \frac{1}{x^3+3x^2\delta x+3x(\delta x)^2+(\delta x)^3} - \frac{1}{x^3} \\ &= \frac{x^3 - (x^3+3x^2\delta x+3x(\delta x)^2+(\delta x)^3)}{(x^3+3x^2\delta x+3x(\delta x)^2+(\delta x)^3)x^3} \\ &= \frac{x^3 - x^3 - 3x^2\delta x - 3x(\delta x)^2 - (\delta x)^3}{x^6 + 3x^5\delta x + 3x^4(\delta x)^2 + x^3(\delta x)^3} \\ &= \frac{-3x^2\delta x - 3x(\delta x)^2 - (\delta x)^3}{x^6 + 3x^5\delta x + 3x^4(\delta x)^2 + x^3(\delta x)^3} \end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{-3x^2 - 3x\delta x - (\delta x)^2}{x^6 + 3x^5\delta x + 3x^4(\delta x)^2 + x^3(\delta x)^3}$$

$$\frac{dy}{dx} = \frac{-3x^2}{x^6}$$

$$\frac{dy}{dx} = \frac{-3}{x^4}$$

Gaeaf 2012

$$f(x) = \frac{1}{1-x}$$

$$\begin{aligned}f(x+\delta x) &= \frac{1}{1-(x+\delta x)} \\&= \frac{1}{1-x-\delta x}\end{aligned}$$

$$\begin{aligned}f(x+\delta x) - f(x) &= \frac{1}{1-x-\delta x} - \frac{1}{1-x} \\&= \frac{(1-x) - (1-x-\delta x)}{(1-x-\delta x)(1-x)} \\&= \frac{1-x-1+x+\delta x}{1-x-\delta x -x+x^2+x\delta x} \\&= \frac{\delta x}{1-2x+x^2+(-1+x)\delta x}\end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{1}{1-2x+x^2+(-1+x)\delta x}$$

$$\frac{dy}{dx} = \frac{1}{1-2x+x^2}$$

FPI Ionawr 2013

$$\textcircled{1} \quad f(x) = \frac{1}{2+x^2}$$

$$f(x+\delta x) = \frac{1}{2+(x+\delta x)^2}$$

$$= \frac{1}{2+x^2+2x\delta x+(\delta x)^2}$$

$$f(x+\delta x) - f(x) = \frac{1}{2+x^2+2x\delta x+(\delta x)^2} - \frac{1}{2+x^2}$$

$$= \frac{(2+x^2) - (2+x^2+2x\delta x+(\delta x)^2)}{(2+x^2+2x\delta x+(\delta x)^2)(2+x^2)}$$

$$= \frac{-2x\delta x - (\delta x)^2}{(2+x^2+2x\delta x+(\delta x)^2)(2+x^2)}$$

$$= \frac{-2x\delta x - (\delta x)^2}{4+2x^2+4x\delta x+2(\delta x)^2+2x^2+x^4+2x^3\delta x+x^2(\delta x)^2}$$

$$= \frac{-2x\delta x - (\delta x)^2}{x^4+2x^3\delta x+4x^2+x^2(\delta x)^2+4x\delta x+2(\delta x)^2+4}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{-2x - \delta x}{x^4+2x^3\delta x+4x^2+x^2(\delta x)^2+4x\delta x+2(\delta x)^2+4}$$

$$\frac{dy}{dx} = \frac{-2x}{x^4+4x^2+4}$$

FPI Graef 2014

①

$$f(x) = \frac{x}{1+x}$$

$$f(x+\delta x) = \frac{x+\delta x}{1+x+\delta x}$$

$$\begin{aligned} f(x+\delta x) - f(x) &= \frac{x+\delta x}{1+x+\delta x} - \frac{x}{1+x} \\ &= \frac{(x+\delta x)(1+x) - x(1+x+\delta x)}{(1+x+\delta x)(1+x)} \\ &= \frac{\cancel{x+x^2+\delta x+x\delta x} - \cancel{x+x^2+\delta x}}{1+x+\delta x+x+x^2+x\delta x} \\ &= \frac{\delta x}{1+2x+x^2+\delta x(1+x)} \end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{1}{1+2x+x^2+\delta x(1+x)}$$

$$\frac{dy}{dx} = \frac{1}{1+2x+x^2}$$

$$\frac{dy}{dx} = \frac{1}{(1+x)^2}$$

FPI Haf 2014

①

$$(a) f(x) = \frac{1}{x^2}$$

$$f(x+\delta x) = \frac{1}{(x+\delta x)^2}$$

$$= \frac{1}{x^2 + 2x\delta x + (\delta x)^2}$$

$$\begin{aligned} f(x+\delta x) - f(x) &= \frac{1}{x^2 + 2x\delta x + (\delta x)^2} - \frac{1}{x^2} \\ &= \frac{x^2 - (x^2 + 2x\delta x + (\delta x)^2)}{x^2(x^2 + 2x\delta x + (\delta x)^2)} \\ &= \frac{-2x\delta x - (\delta x)^2}{x^4 + 2x^3\delta x + x^2(\delta x)^2} \end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{-2x - \delta x}{x^4 + 2x^3\delta x + x^2(\delta x)^2}$$

$$\frac{dy}{dx} = \frac{-2x}{x^4}$$

$$\frac{dy}{dx} = \frac{-2}{x^3}$$

$$\left(\frac{dy}{dx} = -2x^{-3} \right)$$

FPI Haf 2015

1)

$$f(x) = \frac{1}{x^2 - x}$$

$$f(x + \delta x) = \frac{1}{(x + \delta x)^2 - (x + \delta x)}$$

$$= \frac{1}{x^2 + 2x\delta x + (\delta x)^2 - x - \delta x}$$

$$\begin{aligned} f(x + \delta x) - f(x) &= \frac{1}{x^2 + 2x\delta x + (\delta x)^2 - x - \delta x} - \frac{1}{x^2 - x} \\ &= \frac{(x^2 - x) - (x^2 + 2x\delta x + (\delta x)^2 - x - \delta x)}{(x^2 + 2x\delta x + (\delta x)^2 - x - \delta x)(x^2 - x)} \\ &= \frac{x^2 - x - x^2 - 2x\delta x - (\delta x)^2 + x + \delta x}{(x^2 + 2x\delta x + (\delta x)^2 - x - \delta x)(x^2 - x)} \\ &= \frac{-2x\delta x - (\delta x)^2 + \delta x}{x^4 + 2x^3\delta x + x^2(\delta x)^2 - x^3 - x^2\delta x - x^3 - 2x^2\delta x - x(\delta x)^2 + x^2 + x\delta x} \\ &= \frac{-2x\delta x - (\delta x)^2 + \delta x}{x^4 - 2x^3 + x^2 + (2x^3 - 3x^2 + x)\delta x + (x^2 - x)(\delta x)^2} \end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{-2x - (\delta x)^2 + 1}{x^4 - 2x^3 + x^2 + (2x^3 - 3x^2 + x)\delta x + (x^2 - x)(\delta x)^2}$$

$$\frac{dy}{dx} = \frac{-2x + 1}{x^4 - 2x^3 + x^2}$$

$$\frac{dy}{dx} = \frac{1 - 2x}{x^2(x^2 - 2x + 1)}$$

$$\frac{dy}{dx} = \frac{1 - 2x}{x^2(x - 1)^2}$$

FPI Haf 2016

$$\textcircled{1} \quad f(x) = \frac{x^2}{x+1}$$

$$f(x+\delta x) = \frac{(x+\delta x)^2}{(x+\delta x)+1}$$
$$= \frac{x^2 + 2x\delta x + (\delta x)^2}{x + \delta x + 1}$$

$$f(x+\delta x) - f(x) = \frac{x^2 + 2x\delta x + (\delta x)^2}{x + \delta x + 1} - \frac{x^2}{x+1}$$
$$= \frac{(x^2 + 2x\delta x + (\delta x)^2)(x+1) - x^2(x+\delta x+1)}{(x+\delta x+1)(x+1)}$$
$$= \cancel{x^3 + 2x^2\delta x + x(\delta x)^2 + x^2 + 2x\delta x + (\delta x)^2} - \cancel{x^3} - \cancel{x^2\delta x} - \cancel{x^2}$$
$$(x+\delta x+1)(x+1)$$
$$= \frac{x^2\delta x + x(\delta x)^2 + 2x\delta x + (\delta x)^2}{(x+\delta x+1)(x+1)}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$= \lim_{\delta x \rightarrow 0} \frac{x^2 + x\delta x + 2x + \delta x}{(x+\delta x+1)(x+1)}$$
$$= \frac{x^2 + 2x}{(x+1)(x+1)}$$

$$\frac{dy}{dx} = \frac{x(x+2)}{(x+1)^2}$$

F81 May 2019

$$1) f(x) = \frac{1}{x^3}$$

$$f(x+\delta x) = \frac{1}{(x+\delta x)^3}$$

$$\begin{aligned} f(x+\delta x) - f(x) &= \frac{1}{(x+\delta x)^3} - \frac{1}{x^3} \\ &= \frac{x^3 - (x+\delta x)^3}{(x+\delta x)^3 (x^3)} \\ &= \frac{x^3 - (x^3 + 3x^2\delta x + 3x(\delta x)^2 + (\delta x)^3)}{x^3 (x+\delta x)^3} \\ &= \frac{-3x^2\delta x - 3x(\delta x)^2 - (\delta x)^3}{x^3 (x+\delta x)^3} \end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$= \lim_{\delta x \rightarrow 0} \frac{-3x^2\delta x - 3x(\delta x)^2 - (\delta x)^3}{(\delta x)x^3(x+\delta x)^3}$$

$$= \lim_{\delta x \rightarrow 0} \frac{-3x^2 - 3x\delta x - (\delta x)^2}{x^3(x+\delta x)^3}$$

$$= \frac{-3x^2}{x^3(x)^3}$$

$$= \frac{-3x^2}{x^6}$$

$$= \frac{-3}{x^4}$$