Hen Gwestiynau Arholiad — Rhifau Cymhlyg / Complex Numbers

Haf 2005

1. $|z+1| = 2|z-2i|$

Gadewch i $z = x+iy$. Yna

$|x+iy+1| = 2|x+iy-2i|$

$|(x+1)+iy| = 2|x+i(y-2)|$

$\sqrt{(x+1)^2+y^2} = 2\sqrt{x^2+(y-2)^2}$

$(x+1)^2+y^2 = 4(x^2+(y-2)^2)$

$x^2+2x+1+y^2 = 4(x^2+y^2-4y+4)$

$x^2+2x+1+y^2 = 4x^2+4y^2-16y+16$

$0 = 3x^2+3y^2-2x-16y+15$

$3x^2+3y^2-2x-16y+15 = 0$

Gaeaf 2006

1. ~~Gadewch i $z = \sqrt{3}+iy$~~

~~$|z| =$~~

Gadewch i $z = \sqrt{3}+i$

Yna $|z| = \sqrt{(\sqrt{3})^2+1^2}$

$|z| = \sqrt{3+1}$

$|z| = 2$

$\text{Arg}(z) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\text{Arg}(z) = \frac{\pi}{6}$ (neu 30^0)

Ffurf Cartesian $z = x+iy$

Ffurf Trigonometreg $|z|(\cos(\arg(z)) + i\sin(\arg(z)))$

$2(\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6}))$

Beth yw'r rhif positif lleiaf fel bod $(\sqrt{3}+i)^n$ yn real?

Yn defnyddio $\text{Arg}(z_1z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$

gwelwn mai $\text{Arg}((\sqrt{3}+i)^2)$ yw $\frac{2\pi}{6}$

$\text{Arg}((\sqrt{3}+i)^3) = \frac{3\pi}{6}$ ayb.

Er mwyn cael $(\sqrt{3}+i)^n$ yn real rhaid bod yr Arg yn luosrif cyfan o π fel bod cyfernod yr i yn y ffurf trigonometreg yn 0 (cofiwn bod $\sin(\pi) = \sin(2\pi) = \sin(3\pi) = \dots = 0$).

Felly mae $(\sqrt{3}+i)^n$ yn real os yw $n = 6, 12, 18, \dots$

Felly $n = 6$ yw'r rhif positif lleiaf fel bod $(\sqrt{3}+i)^n$ yn real.

4 $2z + \bar{z} = \frac{11+7i}{1+i}$

$2(x+iy) + x - iy = \frac{11+7i}{1+i}$

$2x + 2iy + x - iy = \frac{11+7i}{1+i}$

$3x + iy = \frac{(11+7i)(1-i)}{(1+i)(1-i)}$

$3x + iy = \frac{11 - 11i + 7i - 7i^2}{1 - i + i - i^2}$

$3x + iy = \frac{11 - 4i + 7}{1+1}$

$3x + iy = \frac{18 - 4i}{2}$

$3x + iy = 9 - 2i$

Felly $3x = 9$ $\quad y = -2$
$x = 3$

$\underline{\underline{z = 3 - 2i}}$

Haf 2006

3 $\frac{z}{z+1} = 2 + 3i$

$\frac{x+iy}{x+iy+1} = 2+3i$

$\frac{x+iy}{x+1+iy} = 2+3i$

$x + iy = (2+3i)(x+1+iy)$
$x + iy = 2x + 2 + 2yi + 3xi + 3i + 3yi^2$
$x + iy = 2x + 2 + 2yi + 3xi + 3i - 3y$
$x + iy = (2x + 2 - 3y) + (2y + 3x + 3)i$

Real

$x = 2x + 2 - 3y$
$0 = x + 2 - 3y$
$x = 3y - 2$ — 1

Dychmygol

$y = 2y + 3x + 3$
$0 = y + 3x + 3$ — 2

Amnewid 1 i 2: $0 = y + 3(3y-2) + 3$
$0 = y + 9y - 6 + 3$
$0 = 10y - 3$
$y = \frac{3}{10}$

Felly $x = 3\left(\frac{3}{10}\right) - 2$
$x = \frac{9}{10} - 2$
$x = \frac{-11}{10}$

$\boxed{z = \frac{-11}{10} + \frac{3}{10}i}$

Gaeaf 2007

(3) (a) $\frac{(3+4i)(1+2i)}{1+3i}$

$= \frac{3+6i+4i+8i^2}{1+3i}$

$= \frac{3+10i-8}{1+3i}$

$= \frac{-5+10i}{1+3i}$

$= \frac{(-5+10i)(1-3i)}{(1+3i)(1-3i)}$

$= \frac{-5+15i+10i-30i^2}{1-3i+3i-9i^2}$

$= \frac{-5+25i+30}{1+9}$

$= \frac{+25+25i}{10}$

$= +2.5+2.5i$

(b) (i) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

(c) $\arg(3+4i) = \tan^{-1}\left(\frac{4}{3}\right)$

$\arg(1+2i) = \tan^{-1}\left(\frac{2}{1}\right)$

$= \tan^{-1}(2)$

$\arg(1+3i) = \tan^{-1}\left(\frac{3}{1}\right)$

$= \tan^{-1}(3)$

Felly $\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}(2) - \tan^{-1}(3)$

$= \arg(3+4i) + \arg(1+2i) - \arg(1+3i)$

$= \arg((3+4i)(1+2i)) - \arg(1+3i)$

$= \arg\left(\frac{(3+4i)(1+2i)}{(1+3i)}\right)$

$= \arg(2.5+2.5i)$ [darn (a)]

$= \tan^{-1}\left(\frac{2.5}{2.5}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$ (Felly $K = 4$)

Haf 2007

(2) $2z + \bar{z} = \dfrac{1+7i}{3+i}$

$2(x+iy) + x - iy = \dfrac{1+7i}{3+i}$

$2x + 2yi + x - iy = \dfrac{1+7i}{3+i}$

$3x + yi = \dfrac{1+7i}{3+i}$

$3x + yi = \dfrac{(1+7i)(3-i)}{(3+i)(3-i)}$

$3x + yi = \dfrac{3 - i + 21i - 7i^2}{9 - 3i + 3i - i^2}$

$3x + yi = \dfrac{3 + 20i + 7}{9 + 1}$

$3x + yi = \dfrac{10 + 20i}{10}$

$3x + yi = 1 + 2i$

Real

$3x = 1$

$x = \frac{1}{3}$

Dychmygol

$y = 2$

Felly $\underline{\underline{z = \frac{1}{3} + 2i}}$

Gaeaf 2008

(6) (a) $z = (3+2i)^2$

$z = (3+2i)(3+2i)$

$z = 9 + 6i + 6i + 4i^2$

$z = 9 + 12i - 4$

$z = 5 + 12i$

$|z| = \sqrt{5^2 + 12^2}$

$|z| = \sqrt{25 + 144}$

$|z| = \sqrt{169}$

$|z| = 13$

$\mathrm{Arg}(z) = \tan^{-1}\left(\frac{12}{5}\right)$

$\mathrm{Arg}(z) = 1.176$ i 3 ll.d.

(b) $\dfrac{1}{u} = \dfrac{1}{v} + \dfrac{1}{w}$

$\dfrac{1}{u} = \dfrac{1}{2+i} + \dfrac{1}{1-2i}$

$\dfrac{1}{u} = \dfrac{2-i}{(2+i)(2-i)} + \dfrac{1+2i}{(1-2i)(1+2i)}$

$\dfrac{1}{u} = \dfrac{2-i}{4-2i+2i-i^2} + \dfrac{1+2i}{1+2i-2i-4i^2}$

$\dfrac{1}{u} = \dfrac{2-i}{4+1} + \dfrac{1+2i}{1+4}$

$\dfrac{1}{u} = \dfrac{2-i}{5} + \dfrac{1+2i}{5}$

$\dfrac{1}{u} = \dfrac{2-i+1+2i}{5}$

$\dfrac{1}{u} = \dfrac{3+i}{5}$

$u = \dfrac{5}{3+i}$

$u = \dfrac{5(3-i)}{(3+i)(3-i)}$

$u = \dfrac{15-5i}{9-3i+3i-i^2}$

$u = \dfrac{15-5i}{9+1}$

$u = \dfrac{15-5i}{10}$

$\underline{\underline{u = 1.5 - 0.5i}}$

Haf 2008

3 (a) $z = (2-i)^2 + \frac{(7-4i)}{(2+i)} - 8$

$z = (2-i)(2-i) + \frac{(7-4i)(2-i)}{(2+i)(2-i)} - 8$

$z = 4 - 2i - 2i + i^2 + \frac{14 - 7i - 8i + 4i^2}{4 - 2i + 2i - i^2} - 8$

$z = 4 - 4i - 1 + \frac{14 - 15i - 4}{4+1} - 8$

$z = 3 - 4i + \frac{10 - 15i}{5} - 8$

$z = 3 - 4i + 2 - 3i - 8$

$z = -3 - 7i$

(b) $|z| = \sqrt{(-3)^2 + (-7)^2}$

$|z| = \sqrt{9 + 49}$

$|z| = \sqrt{58}$

$\text{Arg}(z) = \tan^{-1}\left(\frac{-7}{-3}\right)$

$= 1.166$ to 3 d.p.

Gaeaf 2009

4 (a) $2z - i\bar{z} = 1 + 4i$

$2(x+iy) - i(x-iy) = 1 + 4i$

$2x + 2yi - xi + yi^2 = 1 + 4i$

$2x + 2yi - xi - y = 1 + 4i$

$(2x - y) + (2y - x)i = 1 + 4i$

Real

$2x - y = 1$

$2x = 1 + y$

$2x - 1 = y$ — 1

Dychmygol

$2y - x = 4$ — 2

Amnewid 1 i 2

$2(2x - 1) - x = 4$

$4x - 2 - x = 4$

$3x - 2 = 4$

$3x = 6$

$x = 2$

Felly $2(2) - 1 = y$

$4 - 1 = y$

$y = 3$

$\boxed{z = 2 + 3i}$

(b) $\frac{1+3i}{2-i} = \frac{(1+3i)(2+i)}{(2-i)(2+i)}$

$= \frac{2 + i + 6i + 3i^2}{4 + 2i - 2i - i^2}$

$= \frac{2 + 7i - 3}{4 + 1}$

$z = \frac{-1 + 7i}{5}$

$|z| = \sqrt{\left(\frac{-1}{5}\right)^2 + \left(\frac{7}{5}\right)^2}$

$|z| = \sqrt{\frac{1 + 49}{25}}$

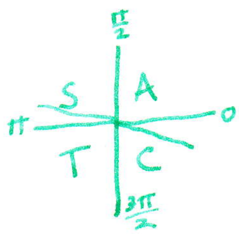
$|z| = \sqrt{2}$

$\text{Arg}(z) = \tan^{-1}\left(\frac{\frac{7}{5}}{\frac{-1}{5}}\right)$

$= \tan^{-1}(-7)$

$= -1.429$ i 3 ll.d.

$= 1.713$ i 3 ll.d.



Haf 2009

4 $z = \frac{9+7i}{3-i}$

(a) $z = \frac{(9+7i)(3+i)}{(3-i)(3+i)}$

$z = \frac{27+9i+21i+7i^2}{9+3i-3i-i^2}$

$z = \frac{27+30i-7}{9+1}$

$z = \frac{20+30i}{10}$

$\underline{\underline{z = 2+3i}}$

(b) $|z| = \sqrt{2^2+3^2}$

$|z| = \sqrt{4+9}$

$|z| = \sqrt{13}$

$\text{Arg}(z) = \tan^{-1}\left(\frac{3}{2}\right)$

$\text{Arg}(z) = 0.983$ i 3 ll.d.

Gaeaf 2010

1 (a) $(1+2i)^3 + (1+2i) + 10$

$= (1+2i)(1+2i)(1+2i) + (1+2i) + 10$

$= (1+2i+2i+4i^2)(1+2i) + (1+2i) + 10$

$= (1+4i-4)(1+2i) + (1+2i) + 10$

$= (-3+4i)(1+2i) + 1+2i+10$

$= (-3-6i+4i+8i^2) + 11 + 2i$

$\cancel{= (-3+2i-8) + 11 + 2i}$

$\cancel{= -8+2i+}$

$= (-3-2i-8) + 11 + 2i$

$= -8-2i-3+11+2i$

$= 0$ ✓

(b) Gwreiddyn 1af $1+2i$
2il wreiddyn $1-2i$

Mae gwreiddiau cymhlyg yn dod mewn parau felly mae'r 3ydd gwreiddyn yn real. Trwy edrych, mae $(-2)^3+(-2)+10$
$= -8-2+10$
$= 0$

Felly -2 yw'r 3ydd gwreiddyn.

③ $z = \frac{1+8i}{1-2i}$

$z = \frac{(1+8i)(1+2i)}{(1-2i)(1+2i)}$

$z = \frac{1+2i+8i+16i^2}{1+2i-2i-4i^2}$

$z = \frac{1+10i-16}{1+4}$

$z = \frac{-15+10i}{5}$

$\underline{\underline{z = -3+2i}}$

(b) $|z| = \sqrt{(-3)^2+2^2}$

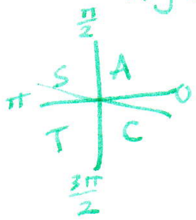
$|z| = \sqrt{9+4}$

$|z| = \sqrt{13}$

$Arg(z) = \tan^{-1}\left(\frac{2}{-3}\right)$

$Arg(z) = -0.588$ i 3 ll.d.

$Arg(z) = 2.554$ i 3 ll.d.



Haf 2010

② $z - \frac{5\bar{z}}{z}$

$= 2-i - \frac{5(2+i)}{2-i}$

$= 2-i - \frac{5(2+i)(2+i)}{(2-i)(2+i)}$

$= 2-i - \frac{5(4+2i+2i+i^2)}{4+2i-2i-i^2}$

$= 2-i - \frac{5(4+4i-1)}{4+1}$

$= 2-i - \frac{\cancel{5}(3+4i)}{\cancel{5}}$

$= 2-i-(3+4i)$

$= -1-5i.$

Gadewch i $w = -1-5i$

$|w| = \sqrt{(-1)^2+(-5)^2}$

$|w| = \sqrt{1+25}$

$|w| = \sqrt{26}$

$Arg(w) = \tan^{-1}\left(\frac{-5}{-1}\right)$

$Arg(w) = 1.373$ i 3 ll.d.

Gaeaf 2011

3 (a) $\frac{1}{z} - 4(1-i) = (2+i)(-1+i)$

$\frac{1}{z} - 4 + 4i = -2 + 2i - i + i^2$

$\frac{1}{z} - 4 + 4i = -2 + i - 1$

$\frac{1}{z} = -3 + i + 4 - 4i$

$\frac{1}{z} = 1 - 3i$

$z = \frac{1}{1-3i}$

$z = \frac{1+3i}{(1-3i)(1+3i)}$

$z = \frac{1+3i}{1+3i-3i-9i^2}$

$z = \frac{1+3i}{1+9}$

$z = \frac{1}{10} + \frac{3}{10}i$

(b) $|z| = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{3}{10}\right)^2}$

$|z| = \sqrt{\frac{1+9}{100}}$

$|z| = \frac{\sqrt{10}}{10}$

$Arg(z) = \tan^{-1}\left(\frac{\frac{3}{10}}{\frac{1}{10}}\right)$

$Arg(z) = \tan^{-1}(3)$

$Arg(z) = 1.249$ i 3ll d.

Haf 2011

3 $2\bar{z} + iz = (1+2i)(2-3i)$

$2(x-iy) + i(x+iy) = 2 - 3i + 4i - 6i^2$

$2(x-iy) + i(x+iy) = 2 + i + 6$

$2x - 2yi + xi + i^2y = 8 + i$

$2x - 2yi + xi - y = 8 + i$

$(2x-y) + (x-2y)i = 8 + i$

Real

$2x - y = 8$ — 1

Dychmygol

$x - 2y = 1$

$x = 1 + 2y$ — 2

Amnewid o 2 i 1

$2(1+2y) - y = 8$

$2 + 4y - y = 8$

$2 + 3y = 8$

$3y = 6$

$\underline{\underline{y = 2}}$

Felly $x = 1 + 2(2)$

$\underline{\underline{x = 5}}$

$\boxed{z = 5 + 2i}$

Gaeaf 2012

2 $\dfrac{1+3i}{1+2i} = \dfrac{(1+3i)(1-2i)}{(1+2i)(1-2i)}$

$= \dfrac{1 - 2i + 3i - 6i^2}{1 - 2i + 2i - 4i^2}$

$= \dfrac{1 + i + 6}{1 + 4}$

$= \dfrac{7+i}{5}$

$= \frac{7}{5} + \frac{1}{5}i.$

Gadewch i $z = \frac{7}{5} + \frac{1}{5}i.$

$|z| = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{1}{5}\right)^2}$

$|z| = \sqrt{\frac{49+1}{25}}$

$|z| = \frac{\sqrt{50}}{5}$

$|z| = \frac{\sqrt{25} \times \sqrt{2}}{5}$

$|z| = \sqrt{2}$

$\mathrm{Arg}(z) = \tan^{-1}\left(\frac{\frac{1}{5}}{\frac{7}{5}}\right)$

$\mathrm{Arg}(z) = \tan^{-1}\left(\frac{1}{7}\right)$

$\mathrm{Arg}(z) = 0.142$ i 3 ll.d.

(4) (a) $(2+3i)^3 = (2+3i)(2+3i)(2+3i)$

$= (4+6i+6i+9i^2)(2+3i)$

$= (4+12i-9)(2+3i)$

$= (-5+12i)(2+3i)$

$= -10-15i+24i+36i^2$

$= -10+9i-36$

$= -46+9i$

(b)(i) $x^3-3x+52=0$

$(2+3i)^3-3(2+3i)+52$

$= (-46+9i)-6-9i+52$

$= (-46-6+52)+9i-9i$

$= 0$ ✓

(ii) $2+3i$ yw'r gwreiddyn cyntaf

$2-3i$ yw'r ail wreiddyn

Mae gwreiddiau cymhlyg o hyd yn dod mewn parau felly mae'r 3ydd gwreiddyn yn real.

Trwy edrych, mae ~~$4^3-3x4+52$~~

~~$=64-12$~~

$(-4)^3-3\times-4+52$

$= -64+12+52$

$= 0$

Felly -4 yw'r 3ydd gwreiddyn.

Haf 2012

(1) (a) $z(2+i)=(1+2i)^2$

$z=\dfrac{(1+2i)^2}{(2+i)}$

$z=\dfrac{(1+2i)(1+2i)(2-i)}{(2+i)(2-i)}$

$z=\dfrac{(1+2i+2i+4i^2)(2-i)}{4-2i+2i-i^2}$

$z=\dfrac{(1+4i-4)(2-i)}{4+1}$

$z=\dfrac{(-3+4i)(2-i)}{5}$

$z=\dfrac{-6+3i+8i-4i^2}{5}$

$z=\dfrac{-6+11i+4}{5}$

$z=-\frac{2}{5}+\frac{11}{5}i$

(b) $|z|=\sqrt{(-\frac{2}{5})^2+(\frac{11}{5})^2}$

$|z|=\sqrt{\dfrac{4+121}{25}}$

$|z|=\dfrac{\sqrt{125}}{\sqrt{25}}$

$|z|=\dfrac{\cancel{\sqrt{25}}\times\sqrt{5}}{\cancel{\sqrt{25}}}$

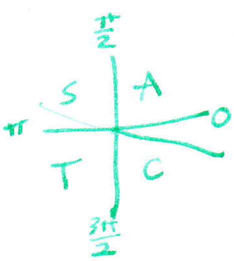
$|z|=\sqrt{5}$

$\text{Arg}(z)=\tan^{-1}\left(\dfrac{\frac{11}{5}}{-\frac{2}{5}}\right)$

$\text{Arg}(z)=\tan^{-1}(-\frac{11}{2})$

$\text{Arg}(z)=-1.391$ i 3 ll.d.

$\text{Arg}(z)=1.751$ i 3 ll.d.



FP1 Ionawr 2013

(3) $iz + 2\bar{z} = \dfrac{4+6i}{1+i}$

(a) $i(x+iy) + 2(x-iy) = \dfrac{4+6i}{1+i}$

$ix + i^2y + 2x - 2iy = \dfrac{(4+6i)(1-i)}{(1+i)(1-i)}$

$ix - y + 2x - 2iy = \dfrac{4 - 4i + 6i - 6i^2}{1 - i + i - i^2}$

$(2x-y) + i(x-2y) = \dfrac{4 + 2i + 6}{1 - -1}$

$(2x-y) + i(x-2y) = \dfrac{10+2i}{2}$

$(2x-y) + i(x-2y) = 5 + i$

Real

$2x - y = 5$

$2x = 5 + y$

$2x - 5 = y$ — (1)

Dychmygol

$x - 2y = 1$ — (2)

Amnewid am y o (1) i (2):

$x - 2(2x-5) = 1$

$x - 4x + 10 = 1$

$-3x = -9$

$\underline{x = 3}$

Felly

$2x - 5 = y$

$2(3) - 5 = y$

$6 - 5 = y$

$\underline{y = 1}$

Casgliad:

$z = x + iy$

$\underline{z = 3 + i}$

(b) $z = 3 + i$

$$|z| = \sqrt{3^2 + 1^2}$$
$$|z| = \sqrt{9 + 1}$$
$$|z| = \sqrt{10}$$

$$\text{Arg}(z) = \text{Tan}^{-1}\left(\frac{y}{x}\right)$$
$$\text{Arg}(z) = \text{Tan}^{-1}\left(\frac{1}{3}\right)$$

$\text{Arg}(z) = 18.4°$ i un lle degol

neu $\text{Arg}(z) = 0.32$ rad i ddau le degol.

FP1 Haf 2013

(2) $\frac{1}{w} = \frac{1}{u} + \frac{1}{v}$

a) $u = 1-i \qquad v = 1+2i$

Felly $\frac{1}{w} = \frac{1}{1-i} + \frac{1}{1+2i}$

$\frac{1}{w} = \frac{(1+i)}{(1-i)(1+i)} + \frac{(1-2i)}{(1+2i)(1-2i)}$

$\frac{1}{w} = \frac{1+i}{1+i-i-i^2} + \frac{1-2i}{1-2i+2i-4i^2}$

$\frac{1}{w} = \frac{1+i}{1+1} + \frac{1-2i}{1+4}$

$\frac{1}{w} = \frac{1+i}{2} + \frac{1-2i}{5}$

$\frac{1}{w} = \frac{1}{2} + \frac{i}{2} + \frac{1}{5} - \frac{2}{5}i$

$\frac{1}{w} = \frac{7}{10} + \frac{1}{10}i$

$\frac{1}{w} = \frac{7+i}{10}$

$w = \frac{10}{7+i}$

$w = \frac{10(7-i)}{(7+i)(7-i)}$

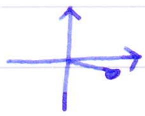
$w = \frac{10(7-i)}{49-7i+7i-i^2}$

$w = \frac{10(7-i)}{49+1}$

$w = \frac{10(7-i)}{50}$

$w = \frac{1}{5}(7-i)$

$w = \frac{7}{5} - \frac{1}{5}i$



b) $|w| = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{-1}{5}\right)^2}$

$|w| = \sqrt{\frac{49}{25} + \frac{1}{25}}$

$|w| = \sqrt{\frac{50}{25}}$

$|w| = \sqrt{2}$

$\text{Arg}(w) = \tan^{-1}\left(\frac{-\frac{1}{5}}{\frac{7}{5}}\right)$

$\text{Arg}(w) = \tan^{-1}\left(-\frac{1}{7}\right)$

$\text{Arg}(w) = -8.1^\circ$ felly $\underline{351.9^\circ}$

FP1 Gaeaf 2014

3 a) $(1+2i)^4 = (1+2i)^2(1+2i)^2$
$= (1+4i+4i^2)(1+4i+4i^2)$
$= (1+4i-4)(1+4i-4)$
$= (-3+4i)(-3+4i)$
$= 9-12i-12i+16i^2$
$= 9-24i-16$
$= -7-24i$

b) i) $x^4+12x-5=0.$
Amnewid $x=1+2i$ i'r ochr chwith:
$(1+2i)^4+12(1+2i)-5$
$= -7-24i+12+24i-5$
$= -7-24i+7+24i$
$= 0.$
Felly mae $1+2i$ yn wreiddyn i'r hafaliad cwartig

ii) Un gwreiddyn $1+2i$
Ail wreiddyn $1-2i$ (mae gwreiddiau cymhlyg o hyd yn ymddangos mewn parau cyfiau).

Felly mae $(x-(1+2i))(x-(1-2i))$ yn ffactor.
$= x^2-x(1-2i)-x(1+2i)+(1+2i)(1-2i)$
$= x^2-x+\cancel{2i}-x-\cancel{2i}+(1-2i+2i-4i^2)$
$= x^2-2x+1+4$
$= x^2-2x+5$

Trosodd

Rhannu'r ffactor x^2-2x+5 allan:

$$
\begin{array}{r l}
 & \quad\quad\quad x^2+2x-1 \\
x^2-2x+5 \,) & x^4 \quad\quad\quad\quad +12x-5 \\
 & \underline{x^4-2x^3+5x^2} \\
 & \quad 2x^3-5x^2+12x-5 \\
 & \quad \underline{2x^3-4x^2+10x} \\
 & \quad\quad -x^2+2x-5 \\
 & \quad\quad \underline{\underline{-x^2+2x-5}}
\end{array}
$$

Felly $x^4+12x-5 = (x^2-2x+5)(x^2+2x-1)$

gwreiddiau $1+2i,\ 1-2i$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2-4\times1\times-1}}{2\times1}$$

$$x = \frac{-2 \pm \sqrt{8}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$

Unai $x=-1+\sqrt{2}$ neu $x=-1-\sqrt{2}$.

Felly pedwar gwreiddyn yr hafaliad $x^4+12x-5=0$ yw $1+2i,\ 1-2i,\ -1+\sqrt{2},\ -1-\sqrt{2}$

FP1 Haf 2014

(4) $z = \dfrac{1+2i}{1-i}$

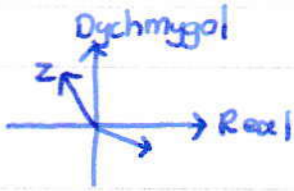
$z = \dfrac{(1+2i)(1+i)}{(1-i)(1+i)}$

$z = \dfrac{1+i+2i+2i^2}{1+i-i-i^2}$

$z = \dfrac{1+3i+-2}{1--1}$

$z = \dfrac{3i-1}{2}$

$z = \frac{3}{2}i - \frac{1}{2}$



$|z| = \sqrt{(\frac{3}{2})^2+(-\frac{1}{2})^2}$

$= \sqrt{\frac{9}{4}+\frac{1}{4}}$

$= \dfrac{\sqrt{10}}{2}$

$Arg(z) = \tan^{-1}\left(\dfrac{\frac{3}{2}}{-\frac{1}{2}}\right)$

$= \tan^{-1}(-3)$

$= -1.25$ rad i 2 ledegol

Felly $Arg(z) = \pi - 1.25$

$= 1.89$ rad i 2 ledegol

(neu $108.43°$ i 2 le degol)

FP1 Haf 2015

3) a) $2z - i\bar{z} = \dfrac{2+i}{1-i}$

$$2(x+iy) - i(x-iy) = \frac{(2+i)(1+i)}{(1-i)(1+i)}$$

$$2x + 2yi - xi + i^2y = \frac{2+2i+i+i^2}{1+i-i-i^2}$$

$$2x + 2yi - xi - y = \frac{2+3i-1}{1--1}$$

$$2x - y + i(2y-x) = \frac{1+3i}{2}$$

$$(2x-y) + i(2y-x) = \tfrac{1}{2} + \tfrac{3}{2}i$$

Comparing real and imaginary parts:

$2x - y = \frac{1}{2}$ $\qquad\qquad$ $2y - x = \frac{3}{2}$ — 1

$2x - \frac{1}{2} = y$ — 2

substituting for y from 2 into 1

$$2(2x - \tfrac{1}{2}) - x = \tfrac{3}{2}$$

$$4x - 1 - x = \tfrac{3}{2}$$

$$3x = \tfrac{3}{2} + 1$$

$$3x = \tfrac{5}{2}$$

$$\underline{x = \frac{5}{6}}$$

so

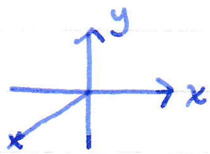
$$y = 2(\tfrac{5}{6}) - \tfrac{1}{2}$$

$$y = \tfrac{10}{6} - \tfrac{3}{6}$$

$$\underline{y = \frac{7}{6}}$$

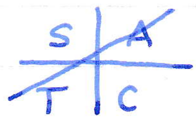
Therefore $\underline{\underline{z = \frac{5}{6} + \frac{7}{6}i}}$

b) $z = -20 - 21i$



$$|z| = \sqrt{(-20)^2 + (-21)^2}$$
$$= \sqrt{400 + 441}$$
$$= \sqrt{841}$$
$$= \underline{\underline{29}}$$

$$\text{Arg}(z) = \tan^{-1}\left(\frac{-21}{-20}\right)$$



$= 46.4^\circ$ or $\underline{\underline{226.4^\circ}}$ (to one decimal place)

(3.95 radians to 2 d.p.)

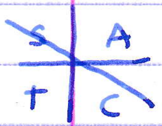
FP1 Haf 2016

(4) $z_1 = -\sqrt{3} + i \qquad z_2 = 1 + i$

a) $|z_1| = \sqrt{(-\sqrt{3})^2 + 1^2}$
$= \sqrt{(-\sqrt{3}) \times (-\sqrt{3}) + 1}$
$= \sqrt{3+1}$
$= \sqrt{4}$
$= 2$

$|z_2| = \sqrt{1^2 + 1^2}$
$= \sqrt{1+1}$
$= \sqrt{2}$

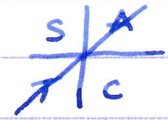
$\text{Arg}(z_1) = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)$



$= -30°, \underline{150°}, 330°$

$\therefore \text{Arg}(z_1) = \frac{5\pi}{6}$

$\text{Arg}(z_2) = \tan^{-1}\left(\frac{1}{1}\right)$



$= \underline{45°}, 225°$

$\therefore \text{Arg}(z_2) = \frac{\pi}{4}$

b) $w = \frac{z_1^2}{z_2}$

$w = \frac{z_1 \times z_1}{z_2}$

$|w| = \frac{|z_1| \times |z_1|}{|z_2|}$

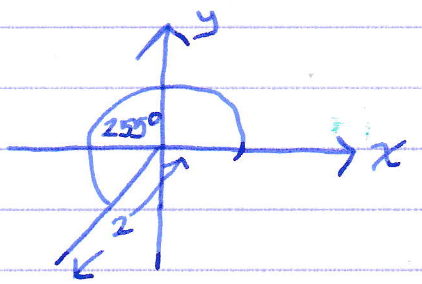
$|w| = \frac{2 \times 2}{\sqrt{2}}$

$|w| = \frac{4}{\sqrt{2}}$

$|w| = \frac{4\sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$|w| = 2\sqrt{2}$

$\text{Arg}(w) = \text{Arg}(z_1) + \text{Arg}(z_1) - \text{Arg}(z_2)$
$= \frac{5\pi}{6} + \frac{5\pi}{6} - \frac{\pi}{4}$
$= \frac{17\pi}{12}$



Real component $|w| \times \cos(\mathrm{Arg}(w))$

$$= 2\sqrt{2} \times \cos\left(\frac{17\pi}{12}\right)$$

$$= 2\sqrt{2}\left(\frac{-\sqrt{6}+\sqrt{2}}{4}\right)$$

$$= 1-\sqrt{3}$$

Imaginary component $|w| \times \sin(\mathrm{Arg}(w))$

$$= 2\sqrt{2} \times \sin\left(\frac{17\pi}{12}\right)$$

$$= 2\sqrt{2}\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right)$$

$$= -1-\sqrt{3}$$

$\therefore$ $w = 1-\sqrt{3} - (1+\sqrt{3})i$

$w = -0.73 - 2.73i$ to 2 d.p.

FP1 Haf 2017

3) $z = \dfrac{(1+2i)(-3+i)}{(1+3i)}$

$z = \dfrac{(-3+i-6i+2i^2)}{(1+3i)}$

$z = \dfrac{(-3-5i-2)}{(1+3i)}$

$z = \dfrac{(-5-5i)}{(1+3i)}$

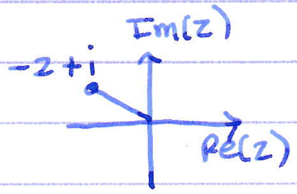
$z = \dfrac{(-5-5i)(1-3i)}{(1+3i)(1-3i)}$

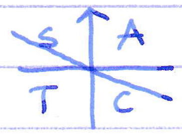
$z = \dfrac{-5+15i-5i+15i^2}{1-3i+3i-9i^2}$

$z = \dfrac{-5+10i-15}{1+9}$

$z = \dfrac{-20+10i}{10}$

$z = -2+i$





$|z| = \sqrt{(-2)^2+1^2}$

$|z| = \sqrt{4+1}$

$|z| = \underline{\underline{\sqrt{5}}}$

$\mathrm{Arg}(z) = \tan^{-1}\left(\dfrac{1}{-2}\right)$

$= -26.57^\circ$ neu $\underline{\underline{153.43^\circ}}$

(neu 2.68 rad)

FP1 Haf 2018

2) a) $(2+i)^4$

$= (2+i)(2+i)(2+i)(2+i)$

$= (4+2i+2i+i^2)(4+2i+2i+i^2)$

$= (4+4i-1)(4+4i-1)$

$= (3+4i)(3+4i)$

$= 9+12i+12i+16i^2$

$= 9+24i-16$

$= \underline{\underline{-7+24i}}$

(Felly $a=-7$, $b=24$)

b) Amnewid $x=2+i$ i mewn i ochr chwith yr hafaliad:

$x^4+2x^2-32x+65$

$= (2+i)^4+2(2+i)^2-32(2+i)+65$

$= -7+24i+2(3+4i)-64-32i+65$

$= -7+24i+6+8i-64-32i+65$

$= 0+0i$

$= 0$

Felly mae $2+i$ yn wreiddyn i'r hafaliad.

c) Mae gwreiddiau cymhlyg yn ymddangos mewn parau cyfiau (conjugate pairs) felly mae $2-i$ yn wreiddyn i'r hafaliad.

Trwy theorem y ffactor, mae $(x-(2+i))$ a $(x-(2-i))$ yn ffactorau o'r hafaliad.
Felly mae $(x-(2+i))(x-(2-i))$ yn ffactor.

$= (x-2-i)(x-2+i)$

$= x^2-2x\cancel{+ix}-2x+4\cancel{-2i}\cancel{-ix}\cancel{+2i}-i^2$

$= x^2-4x+4--1$

$= x^2-4x+5$

Gallwn rannu'r polynomial $x^4+2x^2-32x+65$ efo'r ffactor x^2-4x+5.

$$
\begin{array}{r}
x^2+4x+13 \\
x^2-4x+5\ \overline{)\,x^4+2x^2-32x+65} \\
\underline{x^4-4x^3+5x^2} \\
4x^3-3x^2-32x+65 \\
\underline{4x^3-16x^2+20x} \\
13x^2-52x+65 \\
\underline{\underline{13x^2-52x+65}}
\end{array}
$$

Felly $x^4+2x^2-32x+65$

$$= \underbrace{(x^2+4x+13)}\underbrace{(x^2-4x+5)}_{\text{Gwreiddiau } 2+i,\ 2-i}$$

Gwreiddiau $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{-4 \pm \sqrt{4^2-4(1)(13)}}{2 \times 1}$$

$$x = \frac{-4 \pm \sqrt{-36}}{2}$$

$$x = \frac{-4 \pm \sqrt{(-1)(36)}}{2}$$

$$x = \frac{-4 \pm 6i}{2}$$

$$x = -2 \pm 3i$$

Casgliad: Gwreiddiau'r hafaliadau yw
$2+i$, $2-i$, $-2+3i$, $-2-3i$.

FP1 Haf 2018

3) a) $\dfrac{1+17i}{1+2i} = \dfrac{(1+17i)(1-2i)}{(1+2i)(1-2i)}$

$= \dfrac{1-2i+17i-34i^2}{1-2i+2i-4i^2}$

$= \dfrac{1+15i-34(-1)}{1-4(-1)}$

$= \dfrac{35+15i}{5}$

$= \underline{\underline{7+3i}}$ (Felly $a=7$, $b=3$.)

b) $2iz + 3\bar{z} = \dfrac{1+17i}{1+2i}$

$2iz + 3\bar{z} = 7+3i$

Gadewch i $z = x+iy$ fel bod $\bar{z} = x-iy$

$2i(x+iy) + 3(x-iy) = 7+3i$

$2xi + 2yi^2 + 3x - 3yi = 7+3i$

$2xi + 2y(-1) + 3x - 3yi = 7+3i$

$(3x-2y) + (2x-3y)i = 7+3i$

Yn cymharu darnau real a dychmygol:

$3x - 2y = 7$ $\qquad$ $2x - 3y = 3$

$6x - 4y = 14$ $\qquad$ $6x - 9y = 9$

$6x = 4y + 14$ $\qquad$ $6x = 9y + 9$

$4y + 14 = 9y + 9$

$14 - 9 = 9y - 4y$

$5 = 5y$

$\underline{y = 1}$

Felly $6x = 4y + 14$

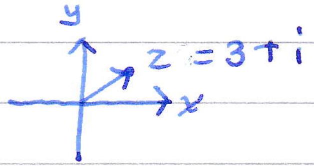
$6x = 4(1) + 14$

$6x = 18$

$\underline{x = 3}$

Felly $z = x + iy$

$z = 3 + i$



$|z| = \sqrt{3^2 + 1^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

$\text{Arg}(z) = \tan^{-1}\left(\frac{1}{3}\right)$

$= 0.3217505544\ldots$

Ffurf trigonometrig : $r = \sqrt{10}$ $\quad \theta = 0.322$

$r = 3.16$ $\quad$ i 3 ff. yst

i 3 ff. yst

$z = r(\cos\theta + i\sin\theta)$

$$\underline{\underline{z = 3.16(\cos(0.322) + i\sin(0.322))}}$$

FP1 May 2019

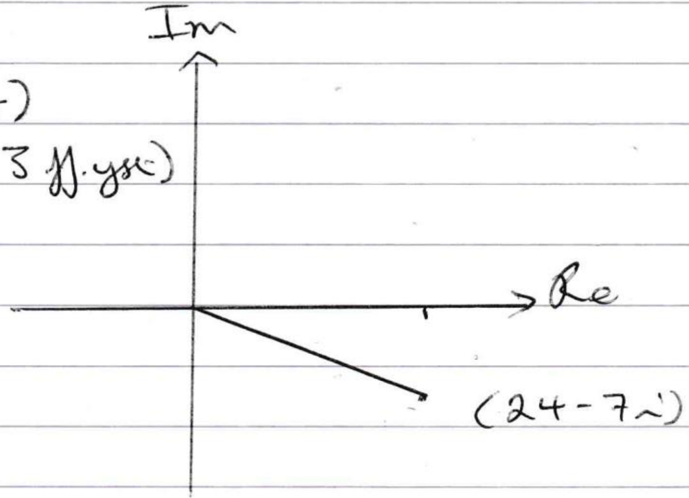
2) a) $z = (4-i)^2 + \frac{7+i}{3-i} + 7$

$(4-i)^2 = 16 - 8i + i^2$
$= 16 - 8i - 1$
$= 15 - 8i$

$\frac{7+i}{3-i} = \frac{(7+i)(3+i)}{(3-i)(3+i)} = \frac{21 + 7i + 3i + i^2}{9 + 3i - 3i - i^2}$
$= \frac{20 + 10i}{10}$
$= 2 + i$

Felly $z = 15 - 8i + (2+i) + 7$
$z = 24 - 7i$

b) $\text{Arg}(z) = \tan^{-1}(-7/24)$
$= -0.284$ (3 ff.ygs)



$|z| = \sqrt{x^2+y^2}$
$= \sqrt{(-7)^2 + 24^2}$
$= \sqrt{49 + 576}$
$= \sqrt{625}$
$|z| = 25$