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Matrix Transformations

(Haf 2005)

4. The transformation T of the plane is equivalent to a reflection in the line $y = x$ followed by the translation in which the point (x, y) is translated to the point $(x + 1, y + 2)$.

(a) Find the 3×3 matrix representing T. [4]

(b) Show that T has no fixed points. [3]

(Gaeaf 2006)

5. The transformations T_1 and T_2 in the plane are defined as follows.

T_1 : A translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$.

T_2 : An anti-clockwise rotation through $\frac{\pi}{2}$ about the origin.

The single transformation T is equivalent to T_1 followed by T_2.

(a) Find the 3 x 3 matrix representing T. [5]

(b) Find the coordinates of the fixed point of T. [4]

(Haf 2006)

6. The transformations T_1 and T_2 in the plane are defined as follows.

T_1 : A translation in which the point (x, y) is transformed to the point $(x - 1, y + 1)$.

T_2 : A reflection in the line $y = x$.

The single transformation T is equivalent to T_1 followed by T_2.

(a) Find the 3×3 matrix representing T. [4]

(b) Find the equation of the locus of the fixed points of T. [4]

(Gaeaf 2007)

8. The transformation T in the plane consists of an anticlockwise rotation about the origin through an angle θ followed by a translation in which the point (x, y) is transformed to the point $(x + h, y + k)$.

(a) Find the 3×3 matrix corresponding to T. [4]

(b) Given that T maps the point (0, 1) to (1, 2) and the point (3, 0) to (4, 3), find the values of h, k and θ. [7]

(Haf 2007)

8. *(a)* The transformation T_1 in the plane transforms the point (x, y) to the point (x', y') and is defined by

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}.$$

Write down the images under T_1 of the points $(1, 0)$ and $(0, 1)$. Given that T_1 is a reflection in the line $x + y = 0$, use your results to find the values of a, b, c and d. [4]

(b) The transformation T_2 is a translation in which the point (x, y) is transformed to the point $(x + 2, y + 2)$. The transformation T is defined as T_1 followed by T_2.

(i) Find the 3×3 matrix representing T.

(ii) Show that the fixed points of T lie on a straight line and state the equation of this line.

(iii) Describe in words the transformation T. [8]

(Gaeaf 2008)

5. The transformation T in the plane consists of a translation in which the point (x, y) is transformed to the point $(x + a, y + b)$, followed by a reflection in the line $y = x$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & b \\ 1 & 0 & a \\ 0 & 0 & 1 \end{bmatrix}. \qquad [3]$$

(b) Given that $a + b = 0$,

(i) determine the set of fixed points of T,

(ii) describe, in words, the single transformation that is equivalent to T followed by T. [7]

(Haf 2008)

7. The transformation T in the plane consists of an anticlockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}. \qquad [3]$$

(b) Find the coordinates of the fixed point of T. [4]

(c) Find the equation of the image under T of the line $y = 2x - 1$. [5]

(Gaeaf 2009)

5. The rotation T in the plane has matrix

$$\begin{bmatrix} 0{\cdot}6 & 0{\cdot}8 & 2 \\ -0{\cdot}8 & 0{\cdot}6 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Find the coordinates of the fixed point of T. [4]

(b) Determine the centre and the angle of this rotation. [4]

(Haf 2009)

8. The transformation T in the plane consists of a reflection in the line $x + y = 0$ followed by a translation in which the point (x, y) is transformed to the point $(x + h, y + k)$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & -1 & h \\ -1 & 0 & k \\ 0 & 0 & 1 \end{bmatrix}. \quad [3]$$

(b) Given that the image of the point $(1, 2)$ under T is the point $(2, 1)$,

(i) find the values of h and k,

(ii) find the equation of the image under T of the line $y = 3x + 2$. [8]

(Gaeaf 2010)

8. The transformation T in the plane consists of a reflection in the line $y = x$ followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y - 1)$ followed by a clockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad [5]$$

(b) Show that T has no fixed points. [3]

(c) Find the equation of the image under T of the line $y = 2x + 1$. [5]

(Haf 2010)

8. The transformation T in the plane consists of an anti-clockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x - 3, y + 1)$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & -1 & -3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

[3]

(b) Find the coordinates of the fixed point of T. [4]

(c) The image of the line L under T has equation $x + 2y = 3$. Find the equation of L. [4]

(Gaeaf 2011)

8. The transformation T in the plane consists of a reflection in the line $y - x = 0$, followed by a translation in which the point (x, y) is transformed to the point $(x + 2, y - 1)$, followed by a reflection in the line $y + x = 0$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$

[5]

(b) Find the coordinates of the fixed point of T. [3]

(Haf 2011)

7. The transformation T in the plane consists of an anticlockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x - 2, y + 1)$ followed by a reflection in the line $x + y = 0$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

[5]

(b) Find the equation of the image under T of the line $y = 2x - 1$. [5]

(Gaeaf 2012)

7. The transformation T in the plane consists of a translation in which the point (x, y) is transformed to the point $(x + h, y + k)$ followed by a clockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & k \\ -1 & 0 & -h \\ 0 & 0 & 1 \end{bmatrix}. \qquad [3]$$

(b) Given that the fixed point of T is (1, 3),

(i) find the values of h and k,

(ii) find the equation of the image of the line $y = 3x + 1$ under T. [8]

(Haf 2012)

7. The transformation T in the plane consists of a reflection in the line $y = x$ followed by a translation in which the point (x, y) is transformed to the point $(x - 2, y + 2)$ followed by a reflection in the x-axis.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}. \qquad [5]$$

(b) Find the coordinates of the fixed point of T. [4]

(Gaeaf 2013)

8. The transformation T in the plane consists of an anticlockwise rotation through 45° about the origin followed by a reflection in the line $x + y = 0$.

(a) Show that the 2 × 2 matrix representing T is

$$\frac{1}{\sqrt{2}}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}. \qquad [3]$$

(b) (i) Find the equation of the image under T of the line $y = mx$.

(ii) Given that the line $y = mx$ is transformed into itself under T, determine the possible values of m. [6]

(Haf 2013)

4. The transformation T in the plane consists of an anticlockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x + 2, y + 1)$ followed by a reflection in the line $y = x$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}. \quad [5]$$

(b) Show that T has no fixed points. [3]

(Gaeaf 2014)

5. The transformation T in the plane consists of a reflection in the line $x + y = 0$, followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$, followed by a clockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}. \quad [5]$$

(b) Find the equation of the image under T of the line $y = 2x - 1$. [5]

(Haf 2014)

7. The transformation T in the plane consists of a clockwise rotation through 90° about the origin, followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$, followed by a reflection in the y-axis.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}. \quad [5]$$

(b) Find the equation of the image under T of the line $y = 2x + 1$. [4]

(Haf 2015)

2. The transformation T in the plane consists of a reflection in the line $y = x$ followed by a reflection in the line $y = -x$.

(a) Determine the 2×2 matrix which represents T. [4]

(b) Identify the single transformation that is equivalent to T. [1]

(Haf 2016)

2. The transformation T in the plane consists of an anticlockwise rotation through 90° about the origin followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$.

(a) Determine the 3×3 matrix which represents T. [4]

(b) Find the fixed point of T. [4]

(Haf 2017)

4. The transformation T in the plane consists of a reflection in the x-axis, followed by a translation in which the point (x, y) is transformed to the point $(x - 2, y + 1)$, followed by an anticlockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}. \quad [5]$$

(b) Show that T has no fixed points. [3]

(Haf 2018)

4. The transformation T in the plane consists of a clockwise rotation through 90° about the origin, followed by a translation in which the point (x, y) is transformed to the point $(x - 1, y + 2)$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}. \quad [3]$$

(b) Determine the coordinates of the point which is transformed to the point (1, –1) under T. [3]

(Haf 2019)

4. The transformation T in the plane consists of an anticlockwise rotation through 45° about the origin, followed by a reflection in the line $y = x$, followed by a clockwise rotation through 45° about the origin.

(a) Find the 2×2 matrix representing T. [6]

(b) Identify T as a single transformation. [1]