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Matrix Algebra

(Haf 2005)

9. *(a)* The matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 3 & 5 & \lambda \end{bmatrix}.$$

Find the value of λ for which **A** is singular. [3]

(b) Consider the system of equations

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 3 & 5 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}.$$

(i) Given that $\lambda = 5$, find the general solution of this system of equations.

(ii) You are now given that $\lambda = 3$. By first finding the inverse of the matrix **A**, solve this system of equations. [12]

(Gaeaf 2006)

2. Show that the following matrix is non-singular for all values of the real constant λ.

$$\begin{bmatrix} 1 & -2 & \lambda \\ \lambda & 2 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

[6]

7. Consider the system of equations:

$$\begin{bmatrix} 2 & 5 & 3 \\ 1 & 2 & 2 \\ 1 & 1 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ \mu \end{bmatrix}$$

(a) Use reduction to echelon form to find the value of λ for which the equations do not have a unique solution. [5]

(b) For this value of λ, find the value of μ for which the equations are consistent. Find the general solution of the equations in this case. [5]

(Haf 2006)

5. The matrices **A** and **I** are given by

$$\mathbf{A} = \begin{bmatrix} -4 & -4 & 4 \\ -1 & 0 & 1 \\ -7 & -6 & 7 \end{bmatrix}; \quad \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Write down the matrix $\mathbf{A} + \lambda\mathbf{I}$. [2]

(b) Find the values of λ for which the matrix $\mathbf{A} + \lambda\mathbf{I}$ is singular. [7]

8. Use reduction to echelon form to solve the equations

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 7 \\ 4 \end{bmatrix}.$$

[7]

(Gaeaf 2007)

2. *(a)* Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

[6]

(b) **Hence** solve the equations

$$\begin{aligned} x + 2y + z &= 1 \\ 2x + 3y + z &= 4 \\ 3x + 4y + 2z &= 4. \end{aligned}$$

[2]

5. Consider the simultaneous equations

$$\begin{aligned} x + 2y - z &= 2 \\ 2x - y + z &= 3 \\ 4x - 7y + 5z &= 5. \end{aligned}$$

Given that these equations do not have a unique solution,

(a) show that the equations are consistent. [4]

(b) find the general solution to the equations. [3]

(Haf 2007)

7. *(a)* Show that the matrix **A** defined below is singular.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 4 & 1 \\ 1 & 8 & -5 \end{bmatrix}$$ [3]

(b) (i) Find the value of k for which the following equations are consistent.

$$\begin{aligned} 2x + y + 2z &= 3 \\ 3x + 4y + z &= 1 \\ x + 8y - 5z &= k \end{aligned}$$

(ii) For this value of k, find the general solution of these equations. [9]

(Gaeaf 2008)

1. Solve the following equations by reduction to echelon form.

$$\begin{aligned} x + 3y + 2z &= 14 \\ 2x + y + z &= 7 \\ 3x + 2y - z &= 7 \end{aligned}$$ [7]

3. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & \lambda & 2 \end{bmatrix}.$$

(a) Find the value of λ for which **A** is singular. [3]

(b) Given that $\lambda = 4$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse of **A**. [6]

(Haf 2008)

2. *(a)* Find the inverse of the matrix

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$ [6]

(b) **Hence** solve the equations

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \\ 5 \end{bmatrix}.$$ [2]

4. *(a)* Use reduction to echelon form to find the value of k for which the following equations are consistent.

$$\begin{aligned} 2x + \ \ y + 3z &= 5 \\ x - 2y + 2z &= 6 \\ 4x + 7y + 5z &= k \end{aligned}$$ [5]

(b) For this value of k, find the general solution to these equations. [3]

(Gaeaf 2009)

7. Given that **A** is a 2×2 matrix and k is a constant, show that

$$\det(k\mathbf{A}) = k^2 \det(\mathbf{A}).$$ [4]

9. The matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} \lambda+1 & 1 & \lambda \\ 1 & 2 & \lambda \\ 2 & \lambda & 1 \end{bmatrix}.$$

(a) (i) Find and simplify an expression for the determinant of **A**.

(ii) Show that **A** is singular when $\lambda = 1$ but there are no other real values of λ for which **A** is singular. [5]

(b) Now consider the system of equations

$$\mathbf{AX} = \mathbf{B}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \ \mathbf{B} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}.$$

(i) Given that $\lambda = 1$, show that these equations are consistent and find their general solution.

(ii) Given that $\lambda = -1$, find the inverse matrix $\mathbf{A}^{-1}$ and **hence** solve these equations. [7]

(Haf 2009)

3. *(a)* Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix}.$$ [6]

(b) Hence solve the system of equations

$$\begin{aligned} x + 2y + 3z &= 13 \\ 2x + 3y + \ \ z &= 13 \\ 3x + 5y + 2z &= 22. \end{aligned}$$ [2]

6. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 1 & 2 \\ 2 & -1 & \lambda \\ 3 & \lambda & 4 \end{bmatrix} .$$

(a) Show that $\lambda = 1$ is the only positive value of λ for which **A** is singular. [5]

(b) Consider the following equations.

$$\begin{aligned} x + y + 2z &= 2 \\ 2x - y + z &= -2 \\ 3x + y + 4z &= 2 \end{aligned}$$

(i) Show that these equations are consistent.

(ii) Find the general solution. [6]

(Gaeaf 2010)

2. The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix} ; \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(a) Find the inverse of **A**. [3]

(b) Find the 2×2 matrix **X** that satisfies the equation

$$\mathbf{AX} = \mathbf{B}.$$ [3]

4. *(a)* Show that the following matrix is singular.

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 4 & 5 & 7 \end{bmatrix}$$ [2]

(b) Consider the following equations

$$\begin{aligned} x + 2y + 2z &= 1 \\ 2x + y + 3z &= 3 \\ 4x + 5y + 7z &= \lambda \end{aligned}$$

(i) Find the value of λ for which these equations are consistent.

(ii) Find the general solution corresponding to this value of λ. [7]

(Haf 2010)

3. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 2 & \lambda & 3 \\ 1 & 2 & \lambda \\ 4 & 5 & 5 \end{bmatrix}$$

(a) Find the values of λ for which **A** is singular. [4]

(b) Given that $\lambda = 3$,

(i) find the inverse of **A**,

(ii) **hence** solve the equations

$$\begin{aligned} 2x + 3y + 3z &= 2 \\ x + 2y + 3z &= -1 \\ 4x + 5y + 5z &= 4. \end{aligned}$$

[6]

(Gaeaf 2011)

2. Consider the following equations.

$$\begin{aligned} x + 2y + z &= 1 \\ 2x + 3y + z &= 3 \\ 3x + 4y + z &= \lambda \end{aligned}$$

Given that these equations are consistent,

(a) find the value of λ, [4]

(b) find the general solution. [3]

6. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ \lambda & 1 & -2 \\ 2 & 1 & \lambda \end{bmatrix}.$$

(a) (i) Find and simplify an expression for the determinant of **A**.

(ii) Show that **A** is non-singular for all real values of λ. [4]

(b) Given that $\lambda = 1$,

(i) find $\mathbf{A}^{-1}$, the inverse of **A**,

(ii) hence solve the equation $\mathbf{AX} = \mathbf{B}$,

where $\mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix}$. [7]

(Haf 2011)

4. *(a)* Show that the following matrix is singular.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 4 & -1 & 7 \end{bmatrix}$$ [2]

(b) Given that the following system of equations is consistent,

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 4 & -1 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ \lambda \end{bmatrix}$$

(i) find the value of λ,

(ii) find the general solution. [7]

8. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

(a) Evaluate $\mathbf{A}^2$ and show that

$$\mathbf{A}^2 = 5\mathbf{A} + 2\mathbf{I},$$

where **I** denotes the identity matrix. [4]

(b) Using the result in *(a)*, show that

$$\mathbf{A}^3 = \lambda\mathbf{A} + \mu\mathbf{I}$$

where λ, μ are constants to be determined. [3]

(Gaeaf 2012)

5. The matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} k & 1 & 6 \\ 1 & k & 4 \\ 0 & 1 & 1 \end{bmatrix}.$$

(a) Show that **A** is non-singular for all real values of k. [4]

(b) Given that $k = 3$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse matrix of **A**,

(iii) **hence** solve the equations

$$\begin{aligned} 3x + y + 6z &= 1, \\ x + 3y + 4z &= -1, \\ y + z &= -1. \end{aligned}$$ [7]

(Haf 2012)

4. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 1 & 4 \\ 4 & 5 & 7 \end{bmatrix}.$$

(a) (i) Find the adjugate matrix of **A**.
(ii) Find the inverse of **A**. [6]

(b) **Hence** solve the equations

$$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 1 & 4 \\ 4 & 5 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix}.$$ [2]

5. *(a)* Determine the value of k for which the following system of equations is consistent.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ k \end{bmatrix}$$ [5]

(b) Find the general solution for this value of k. [3]

(Gaeaf 2013)

2. Consider the equations

$$\begin{aligned} x + 2y + 3z &= 4, \\ 2x - y + z &= 2, \\ x + 7y + 8z &= k. \end{aligned}$$

Given that these equations are consistent,

(a) find the value of the constant k, [4]

(b) find the general solution of the equations. [3]

4. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 1 & 1 \\ 1 & 3 & \lambda \\ 4 & 7 & 5 \end{bmatrix}.$$

(a) Find the values of λ for which **A** is singular. [5]

(b) Given that $\lambda = 1$,

(i) determine the adjugate matrix of **A**,
(ii) determine the inverse matrix $\mathbf{A}^{-1}$. [5]

(Haf 2013)

6. Consider the system of equations **AX** = **B**, where

$$\mathbf{A} = \begin{bmatrix} 1 & \lambda & 3 \\ 2 & 1 & \lambda \\ 5 & 4 & 7 \end{bmatrix}; \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}.$$

(a) (i) Find the determinant of **A** in terms of the constant λ.

(ii) Show that **A** is singular when $\lambda = 2$ and determine the other value of λ for which **A** is singular. [4]

(b) Given that $\lambda = 2$,

(i) show that the equations are consistent,

(ii) determine the general solution of the equations. [7]

(c) Given that $\lambda = 1$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse of **A**,

(iii) hence solve the equations. [7]

(Gaeaf 2014)

7.

(a) Given that $\mathbf{A} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse of **A**. [5]

(b) **Hence** solve the equations

$$\begin{aligned} 2x + 3y + z &= 13, \\ x + 2y + 3z &= 13, \\ 2x + 3y + 4z &= 19. \end{aligned}$$

[2]

(Haf 2014)

3. Consider the following equations.

$$\begin{aligned} x + 2y + 4z &= 3, \\ x - y + 2z &= 4, \\ 4x - y + 10z &= k. \end{aligned}$$

Given that the equations are consistent,

(a) find the value of k, [5]

(b) determine the general solution of the equations. [3]

6. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 2 & 3 \\ -1 & 1 & 1 \\ 2 & \lambda & 2 \end{bmatrix}.$$

(a) Find the values of λ for which **A** is singular. [4]

(b) Given that $\lambda = -1$,

(i) find the adjugate matrix of **A**,
(ii) find the inverse of **A**. [5]

(Haf 2015)

4. *(a)* The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

Show that $\mathbf{M}$ is singular. [3]

(b) (i) Find the value of μ for which the following system of equations is consistent.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ \mu \end{bmatrix}$$

(ii) For this value of μ, find the general solution to this system of equations. [7]

6. The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 4 \\ 3 & 3 & 6 \\ 2 & 2 & 3 \end{bmatrix}; \ \mathbf{B} = \begin{bmatrix} 3 & -2 & 0 \\ -3 & -1 & 6 \\ 0 & 2 & -3 \end{bmatrix}.$$

(a) Evaluate the matrix $\mathbf{AB}$. [2]

(b) Hence, or otherwise, find the inverse matrix $\mathbf{A}^{-1}$. [2]

(c) Hence solve the simultaneous equations

$$\begin{aligned} 3x + 2y + 4z &= 14 \\ 3x + 3y + 6z &= 18 \\ 2x + 2y + 3z &= 11 \end{aligned}$$

[2]

(Haf 2016)

5. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 2 & 5 & \lambda \\ 0 & \lambda & -1 \\ \lambda & 2 & 1 \end{bmatrix}.$$

(a) (i) Show that

$$\det \mathbf{M} = 4 - 3\lambda - \lambda^3.$$

(ii) Hence show that $\mathbf{M}$ is singular when $\lambda = 1$ and is not singular for any other real values of λ.

(iii) Show that the following system of equations is consistent and find the general solution. [12]

$$\begin{bmatrix} 2 & 5 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

(b) Suppose now that $\lambda = -1$. By first finding the adjugate matrix of $\mathbf{M}$, determine the inverse matrix $\mathbf{M}^{-1}$. [5]

(Haf 2017)

1. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}.$$

(a) Evaluate the determinant of $\mathbf{M}$. [2]

(b) (i) Find the adjugate matrix of $\mathbf{M}$.

(ii) Deduce the inverse matrix $\mathbf{M}^{-1}$. [3]

(c) Hence solve the system of equations

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \\ 17 \end{bmatrix}.$$

[2]

5. Consider the following equations.

$$x + 3y - z = 1,$$
$$2x - y + 2z = 3,$$
$$3x - 5y + 5z = \lambda.$$

(a) Find the value of λ for which the equations are consistent. [4]

(b) For this value of λ, find the general solution of the equations. [3]

(Haf 2018)

6. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} \lambda & 1 & 2 \\ 4 & \lambda & 1 \\ 5 & 2 & 3 \end{bmatrix}, \text{ where } \lambda \text{ is a constant.}$$

(a) (i) Find an expression for the determinant of $\mathbf{M}$ in terms of λ.

(ii) Show that $\mathbf{M}$ is singular when $\lambda = 3$ and state the other value of λ for which $\mathbf{M}$ is singular. [4]

(b) Given that $\lambda = 3$, determine the value of μ for which the following system of equations is consistent.

$$\begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & 1 \\ 5 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ \mu \\ 2 \end{bmatrix}$$ [4]

(c) Suppose now that $\lambda = 2$ so that

$$\mathbf{M} = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 1 \\ 5 & 2 & 3 \end{bmatrix}.$$

(i) Determine the adjugate matrix of $\mathbf{M}$.

(ii) Hence determine the inverse matrix $\mathbf{M}^{-1}$. [5]

(Haf 2019)

6. The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix}.$$

(a) Show that $\mathbf{A}$ is non-singular. [3]

(b) Show that

$$\mathbf{A}^3 = 2\mathbf{A}^2 + 5\mathbf{A} + 2\mathbf{I}.$$ [5]

(c) Hence obtain a quadratic expression in $\mathbf{A}$ for $\mathbf{A}^4$. [4]