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Complex Transformations

(Haf 2005)

7. The complex numbers z and w are represented, respectively, by the points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{1}{z}.$$

(a) Show that

$$x = \frac{u}{u^2 + v^2}$$

and obtain an expression for y in terms of u and v. [5]

(b) The point P moves along the circle $x^2 + y^2 = 2$. Find the equation of the locus of Q in the (u, v) plane. [3]

(Gaeaf 2006)

10. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{z + 3}{z + 1}.$$

The point P moves around the circle with equation $|z| = 1$. Find the Cartesian equation of the locus of Q. Identify this locus. [8]

(Haf 2006)

10. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z^2.$$

The point P moves along the line $y = x - 1$. Find the Cartesian equation of the locus of Q. [8]

(Gaeaf 2007)

9. The complex number z is represented by the point $P(x, y)$ in an Argand diagram.

(a) Given that

$$|z - 3| = |z + i|,$$

find the Cartesian equation of the locus of P. [5]

(b) Find the coordinates of the two points lying on this locus for which $|z| = 4$. [7]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z^2.$$

    (a)   Obtain expressions for u and v in terms of x and y.   [3]

    (b)   The point P moves along the curve with equation $y^2 = 2x^2 - 1$. Find the Cartesian equation of the locus of Q.   [6]

8. The complex number z is represented by the point $P(x, y)$ in the Argand diagram. Given that

$$| z - 1 | = \sqrt{2} \, | z - i |,$$

show that the locus of P is a circle, and find its radius and the coordinates of its centre.   [7]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{1}{z + 1}.$$

    (a)   By first writing

$$z + 1 = \frac{1}{w},$$

       show that

$$x + 1 = \frac{u}{u^2 + v^2}$$

       and find an expression for y in terms of u and v.   [4]

    (b)   The point P moves along the circle $(x + 1)^2 + y^2 = 4$. Find the equation of the locus of Q.   [4]

8. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z(1 - z).$$

    (a)   Show that

$$v = y(1 - 2x)$$

       and find an expression for u in terms of x and y.   [4]

    (b)   The point P moves along the line $y = x$. Find the Cartesian equation of the locus of Q.   [4]

7. The complex number z is represented by the point $P(x, y)$ in the Argand diagram. Given that

$$|z - 1| = 2|z + 2|,$$

show that the locus of P is a circle, and find its radius and the coordinates of its centre. [7]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and $w = 1 + z^2$.

(a) Obtain expressions for u and v in terms of x and y. [4]

(b) The point P moves along the line $y = 2x$. Find the equation of the locus of Q. [4]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{1}{1 - z}.$$

(a) Obtain expressions for u and v in terms of x and y. [4]

(b) The point P moves along the line $y = 1 - x$. Find the equation of the locus of Q. [2]

(c) Find the coordinates of the points in the z-plane which are transformed to points with the same coordinates in the w-plane. [4]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and $w = z^2$.

(a) Obtain expressions for u and v in terms of x and y. [3]

(b) The point P moves along the curve with equation $y = x^2$. Find the equation of the locus of Q, giving your answer in the form $u = f(v)$. [3]

(c) The point $R(\alpha, 16)$ lies on the locus of Q.

(i) Find the value of α.

(ii) Find the coordinates of the point on the locus of P which corresponds to R. [4]

10. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = \frac{1}{z^2}.$$

(a) Show that

$$u = \frac{x^2 - y^2}{\left(x^2 + y^2\right)^2}$$

and obtain an expression for v in terms of x and y. [3]

(b) The point P moves along the line L with equation $y = mx$.

    (i) Show that the locus of Q is the line L' with equation of the form $v = m'u$ and find an expression for m' in terms of m.

    (ii) Determine the values of m for which L and L' have the same gradient. [7]

8. The complex number z is represented by the point $P(x, y)$ in the Argand diagram. Given that

$$|z - i| = 2|z + i|,$$

show that the locus of P is a circle and find its radius and the coordinates of its centre. [8]

9. The complex numbers z and w are represented by points $P(x, y)$ and $Q(u, v)$ respectively in Argand diagrams and

$$wz = 1.$$

(a) Show that

$$x = \frac{u}{u^2 + v^2}$$

and obtain an expression for y in terms of u and v. [3]

(b) The point P moves along the line $y = mx + 1$.

    (i) Show that the locus of Q is a circle.
    (ii) Determine the radius and the coordinates of the centre C of the circle.
    (iii) Write down the equation of the locus of C as m varies. [7]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z(z + 1).$$

(a) Show that

$$v = (2x + 1)y$$

and obtain an expression for u in terms of x and y. [3]

(b) The point P moves along the line $y = x + 1$. Find the equation of the locus of Q, giving your answer in the form $v = au^2 + bu$ where a, b are positive integers. [7]

8. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z^2.$$

(a) Obtain expressions for u and v in terms of x and y. [4]

(b) The point P moves along the curve with equation $y^2 - 2x^2 = 1$. Find the equation of the locus of Q. [5]

9. The complex number z is represented by the point $P(x, y)$ in the Argand diagram. Given that

$$|z - 2| = 2|z + i|,$$

(a) show that it can be deduced immediately that the locus of P passes through the origin, [2]

(b) show that the locus of P is a circle, and find its radius and the coordinates of its centre. [7]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z(z - 1).$$

(a) Obtain expressions for u and v in terms of x and y. [4]

(b) The point P moves along the line $x + y = 0$. Find the equation of the locus of Q. [5]

10. The complex number z is represented by the point $P(x, y)$ in the Argand diagram and

$$|z + 3| = k|z - i|,$$

where k is a real positive constant.

    (a) When $k \neq 1$, the locus of P is a circle. Find, in terms of k,

        (i)   the equation of the circle,

        (ii)   the coordinates of the centre of the circle.   **[7]**

    (b) (i)   Write down the equation of the locus of P when $k = 1$.

        (ii)   Give a geometric interpretation of this locus.   **[2]**

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = (z + 2i)^2.$$

    (a) Obtain expressions for u and v in terms of x and y.   **[4]**

    (b) The point P moves along the line $y = x - 1$. Find the equation of the locus of Q.   **[4]**

8. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$wz = 1.$$

    (a) Obtain expressions for x and y in terms of u and v.   **[4]**

    (b) Given that the point P moves along the line $x + y = 1$,

        (i)   show that the locus of Q is a circle,

        (ii)   determine the radius and the coordinates of the centre C of the circle.   **[6]**

    (c) Given that P and Q have the same coordinates, find the two possible positions of P and Q.   **[3]**

8. The complex number z is represented by the point $P(x, y)$ in the Argand diagram and

$$|z + 2i| = 2|z - 3|.$$

    (a)   Show that the locus of P is a circle.   [4]

    (b)   Find its radius and the coordinates of its centre.   [3]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z^2 - z - i.$$

    (a)   Show that

$$u = x^2 - y^2 - x$$

       and find an expression for v in terms of x and y.   [4]

    (b)   The point P moves along the line $y = x - 1$. Determine the equation of the locus of Q. [4]