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Induction

(Haf 2005)

5. Use mathematical induction to prove that

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^n = \begin{bmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{bmatrix}$$

for all positive integers n. [7]

(Gaeaf 2006)

6. Consider the proposition P given by

'$\sum_{r=1}^{n}(2r+1) = (n+1)^2$ where n is a positive integer.'

(a) Show that if P is true for $n = k$, then it is true for $n = k + 1$. [5]

(b) Explain why it cannot be deduced, using mathematical induction, that P is true for all positive integers n. Show that P is in fact false. [2]

(Haf 2006)

7. Use mathematical induction to show that $9^n - 5^n$ is divisible by 4 for all positive integers n. [7]

(Gaeaf 2007)

4. Use mathematical induction to show that $6^n + 4$ is divisible by 5 for all positive integers n. [7]

(Haf 2007)

5. Use mathematical induction to show that

$$\sum_{r=1}^{n}\left[r \times \left(\frac{1}{2}\right)^r\right] = 2 - (n+2)\left(\frac{1}{2}\right)^n$$

for all positive integers n. [8]

(Gaeaf 2008)

7. Use mathematical induction to show that

$$\sum_{r=1}^{n} r \times 2^r = 2^{n+1}(n-1) + 2$$

for all positive integers n. [8]

(Haf 2008)

5. Use mathematical induction to show that $7^n + 5$ is divisible by 6 for all positive integers n. [7]

(Gaeaf 2009)

6. Use mathematical induction to show that

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n & 2n^2 \\ 0 & 1 & 2n \\ 0 & 0 & 1 \end{bmatrix}$$

for all positive integers n. [8]

(Haf 2009)

5. Use mathematical induction to prove that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1} \quad .$$

for all positive integers n. [8]

(Gaeaf 2010)

6. *(a)* Use mathematical induction to prove that

$$1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + n \times n! = (n+1)! - 1$$

for all positive integers n. [6]

(b) Given that

$$S_n = \sum_{r=1}^{n} r(3r+1) ,$$

obtain an expression for S_n in terms of n, simplifying your answer. [5]

(Haf 2010)

5. Use mathematical induction to prove that $4^{2n} - 1$ is divisible by 15 for all positive integers n. [6]

(Gaeaf 2011)

5. Use mathematical induction to prove that

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^n = \begin{bmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{bmatrix}$$

for all positive integers n. [7]

(Haf 2011)

6. Use mathematical induction to prove that $6^n + 4$ is divisible by 10 for all positive integers n. [7]

(Gaeaf 2012)

6. Use mathematical induction to prove that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}.$$ [6]

(Haf 2012)

6. Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3 for all positive integers n. [7]

(Gaeaf 2013)

6. Use mathematical induction to prove that

$$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

for all positive integers n. [7]

(Haf 2013)

5. Using mathematical induction, prove that $7^n - 1$ is divisible by 6 for all positive integers n. [6]

(Gaeaf 2014)

6. *(a)* Use mathematical induction to prove that

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^n = \begin{bmatrix} 1 & 3^n - 1 \\ 0 & 3^n \end{bmatrix}$$

for all positive integers n. [7]

(b) Determine whether or not this result is true for $n = -1$. [3]

(Haf 2014)

8. Using mathematical induction, prove that

$$\sum_{r=1}^{n} \left(r \times 2^{r-1}\right) = 1 + 2^n(n-1),$$

for all positive integers n. [7]

(Haf 2015)

8. The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$

(a) Show that

$$\mathbf{A}^2 = 2\mathbf{A} - \mathbf{I},$$

where $\mathbf{I}$ denotes the 2×2 identity matrix. [2]

(b) Using mathematical induction, prove that

$$\mathbf{A}^n = n\mathbf{A} - (n-1)\mathbf{I}$$

for all positive integers n. [6]

(Haf 2016)

7. The sequence $x_1, x_2, x_3, \ldots$ is generated by the relationship

$$x_{n+1} = 2x_n - n + 1 \quad \text{where} \quad x_1 = 3.$$

Use mathematical induction to prove that

$$x_n = 2^n + n$$

for all positive integers n. [6]

(Haf 2017)

6. Use mathematical induction to prove that $9^n - 1$ is divisible by 8 for all positive integers n. [7]

(Haf 2018)

7. Use mathematical induction to prove that

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

for all positive integers n. [7]

(Haf 2019)

8. Use mathematical induction to prove that $5^n - (-1)^n$ is divisible by 6 for all positive integers n. [6]