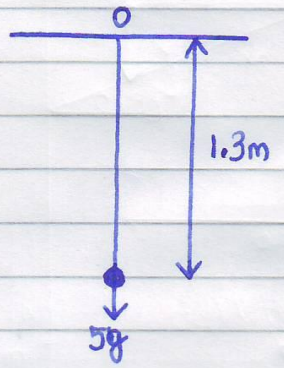
M2 Haf 2005

1



$l = 0.8m$
$\lambda = ?$
$x = 0.5m$

a) Using up forces = down forces at the particle, we have

$$T = 5g$$

But also $T = \dfrac{\lambda x}{l}$

So $5g = \dfrac{\lambda(0.5)}{0.8}$

$$\frac{5g(0.8)}{0.5} = \lambda$$

$$\underline{\underline{\lambda = 78.4N}}$$

b) Stored Elastic Energy $= \dfrac{\lambda x^2}{2l}$

$$= \frac{78.4 \times 0.5^2}{2 \times 0.8}$$

$$= 12.25J$$

2

$$\text{Displacement} \xleftarrow{\text{Integrate}} \text{Velocity} \xleftarrow{\text{Integrate}} \text{Acceleration}$$

$$\text{Displacement} \xrightarrow{\text{Differentiate}} \text{Velocity} \xrightarrow{\text{Differentiate}} \text{Acceleration}$$

(a) $a = 4 - 6t$

So $v = \int 4 - 6t \, dt$

$v = 4t - \dfrac{6t^2}{2} + K$

$v = 4t - 3t^2 + K$

When $t = 0$, we have $v = 4$, so

$4 = 4(0) - 3(0)^2 + K$ → $4 = K$.

Therefore $\underline{\underline{v = 4t - 3t^2 + 4}}$

(b) $r = \int v \, dt$

$r = \int 4t - 3t^2 + 4 \, dt$

$r = \frac{4t^2}{2} - \frac{3t^3}{3} + 4t + K$

$r = 2t^2 - t^3 + 4t + K$

When $t=0$, we have $r=0$, so

$0 = 2(0)^2 - (0)^3 + 4(0) + K$

$0 = K$.

Therefore $\underline{\underline{r = 2t^2 - t^3 + 4t}}$.

(c) If the particle at rest, then we must have $v=0$.

So $4t - 3t^2 + 4 = 0$

$0 = 3t^2 - 4t - 4$

$0 = (3t + 2)(t - 2)$

Either $3t + 2 = 0$ or $t = 2s$

$3t = -2$

$t = -\frac{2}{3}s$

Because it is stated that $t \geq 0$ we must have $\underline{\underline{t = 2s}}$.

When $t=2$, we have $r = 2(2)^2 - (2)^3 + 4(2)$

$\underline{\underline{r = 8m}}$

(d) When $t=3$, we have $v = 4(3) - 3(3^2) + 4$

$v = -11 \, ms^{-1}$.

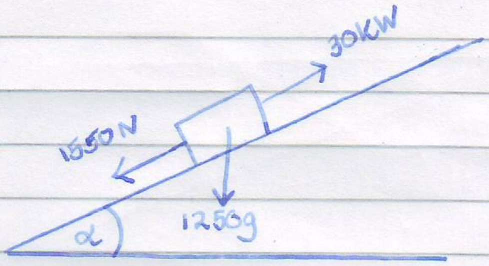
So the velocity is $-11ms^{-1}$ and the speed is $\underline{\underline{11ms^{-1}}}$!

To determine whether the speed is increasing or decreasing, we must look at the sign of the acceleration. When $t=3$, we have $a = 4 - 6(3)$

$a = -14ms^{-2}$.

The acceleration is negative so the velocity is decreasing, and so the speed is increasing.

③



Using Power = Force × Velocity

$30KW = F \times 7.5ms^{-1}$

$30000W = 7.5F$

$F = \frac{30000}{7.5}$

$F = 4000N$

Using $F = ma$ parallel to the slope

4000 − Frictional Force − Weight component down slope = ma

$4000 - 1550 - 1250g \sin\alpha = 1250a$

Because the speed is constant we have $a = 0$,

So $4000 - 1550 - 1250g \sin\alpha = 0$

$2450 = 1250g \sin\alpha$

$\frac{2450}{1250g} = \sin\alpha$

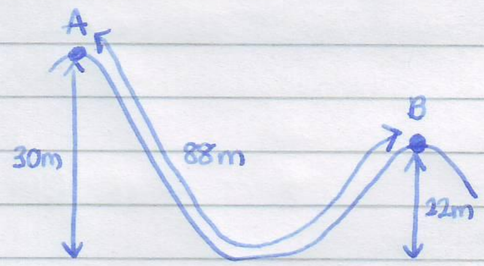
$0.2 = \sin\alpha$

$\alpha = \sin^{-1}(0.2)$

$\alpha = 11.536959030°$

$\alpha = 11.5°$ to one d.p.

④



Kinetic Energy at A

$= \frac{1}{2}mv^2$

$= \frac{1}{2} \times 240 \times 2^2$

$= 480J$

Potential Energy at A

$= mgh$

$= 240 \times 9.8 \times 30$

$= 70560J$

Potential Energy at B

$= mgh$

$= 240 \times 9.8 \times 22$

$= 51744J$

Potential Energy lost in going from A to B $= 70560 - 51744$

$= 18816J$

Work Done against resistance = Force × Distance

$= 132 \times 88$

$= 11616\text{ J}$

Using the principle of the conservation of energy,

PE lost = KE gained + work done against resistance

$18816 =$ KE gained $+ 11616$

$7200 =$ KE gained

$7200 =$ KE at B − KE at A

$7200 =$ KE at B − 480

$7680 =$ KE at B

$7680 = \frac{1}{2}mv^2$

$7680 = \frac{1}{2} \times 240 \times v^2$

$64 = v^2$

$8 = v$

So the speed at B is $\underline{\underline{8\text{ ms}^{-1}}}$.

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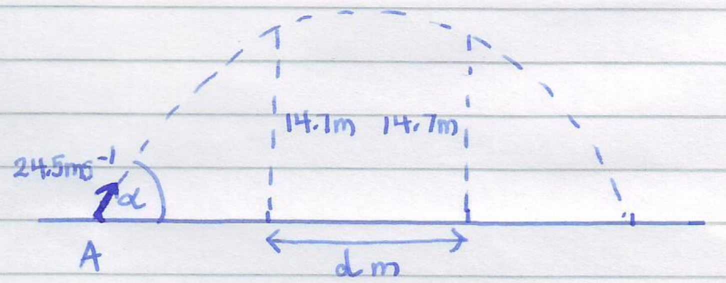


$\sin\alpha = 0.8$

(a) (i)

	Horizontal	Vertical
u	$24.5\cos\alpha$	$24.5\sin\alpha$
a	0	-9.8
v	$24.5\cos\alpha$	$24.5\sin\alpha - 9.8t$
S	$(24.5\cos\alpha)t$	$(24.5\sin\alpha)t - 4.9t^2$

We need to solve $S_y = 14.7$

$(24.5\sin\alpha)t - 4.9t^2 = 14.7$

$$(24.5\times 0.8)t - 4.9t^2 = 14.7$$
$$19.6t - 4.9t^2 = 14.7$$
$$0 = 4.9t^2 - 19.6t + 14.7$$
$$0 = t^2 - 4t + 3$$
$$0 = (t-1)(t-3)$$

Either $t-1=0$ or $t-3=0$

$t = 1s$ or $t = 3s$

The ball takes $\underline{1s}$ to reach the highest point of the first tree.

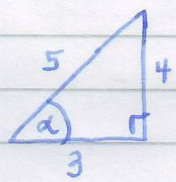
(ii) The ball travels for two seconds between the trees.

Horizontally, $S_x = (24.5\cos\alpha)t$

In two seconds, $S_x = (24.5\cos\alpha)\times 2$

$S_x = (24.5\times \frac{3}{5})\times 2$

$S_x = 29.4m$



Therefore $\underline{d = 29.4m}$

b) When $t = 0.75s$, $V_x = 24.5\cos\alpha$

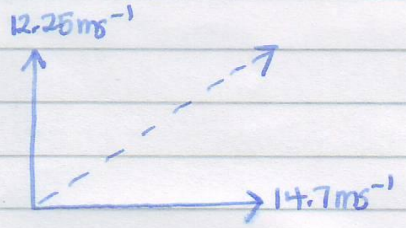
$= 24.5\times\frac{3}{5}$

$= 14.7ms^{-1}$

$V_y = 24.5\sin\alpha - 9.8t$

$= 24.5\times 0.8 - 9.8\times 0.75$

$= 12.25ms^{-1}$



Magnitude of resultant

$= \sqrt{14.7^2 + 12.25^2}$

$= 19.14\, ms^{-1}$ to 2 d.p.

Direction of resultant

$= \tan^{-1}\left(\frac{12.25}{14.7}\right)$

$= 39.81°$ to 2 d.p.

⑥ (a) $\underline{r} = (2t-5)\underline{i} + (t-3)\underline{j} + (7-2t)\underline{k}$

Distance = Magnitude of vector

Distance OP $= \sqrt{(2t-5)^2 + (t-3)^2 + (7-2t)^2}$

Distance OP $= \sqrt{(2t-5)(2t-5) + (t-3)(t-3) + (7-2t)(7-2t)}$

Distance OP $= \sqrt{4t^2 - 10t - 10t + 25 + t^2 - 3t - 3t + 9 + 49 - 14t - 14t + 4t^2}$

Distance OP $= \sqrt{9t^2 - 54t + 83}$

So $OP^2 = 9t^2 - 54t + 83$ ✓

P is closest to O when the distance is at a minimum, which happens when OP^2 is at a minimum.

So we need to solve $\frac{d}{dt}(OP^2) = 0$

$$\frac{d}{dt}(9t^2 - 54t + 83) = 0$$

$$18t - 54 = 0$$

$$18t = 54$$

$$\underline{\underline{t = 3s}}$$

(b) $\underline{v} = \frac{d}{dt}(\underline{r})$

$\underline{v} = \frac{d}{dt}((2t-5)\underline{i} + (t-3)\underline{j} + (7-2t)\underline{k})$

$\underline{v} = 2\underline{i} + \underline{j} - 2\underline{k}$

The magnitude of the velocity is given by

$\sqrt{2^2 + 1^2 + (-2)^2}$

$= \sqrt{9}$

$= 3$

(c) When P is closest to O, we have

$\underline{r} = (2(3)-5)\underline{i} + (3-3)\underline{j} + (7-2(3))\underline{k}$

$\underline{r} = \underline{i} + \underline{k}$

We must now show that $\underline{r} \cdot \underline{v} = 0$ in order to show that the direction of the velocity is perpendicular to OP.

But $\underline{r} \cdot \underline{v} = (\underline{i} + \underline{k}) \cdot (2\underline{i} + \underline{j} - 2\underline{k})$

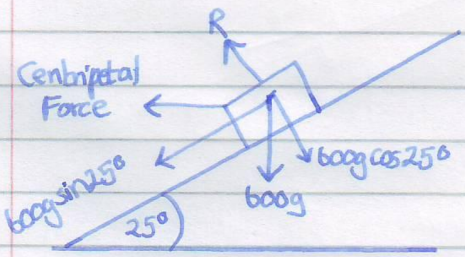
$= (1 \times 2) + (0 \times 1) + (1 \times -2)$

$= 2 + 0 - 2$

$= 0.$

So we have shown that the direction of the velocity of P is perpendicular to OP.

7



Constant speed $42ms^{-1}$
No side slip

(a) Resolving Vertically,

$R \cos 25° = 600g$

$$R = \frac{600 \times 9.8}{\cos 25°}$$

$R = 6487.86N$ to 2 d.p.

(b) Resolving Horizontally,

$$R \sin 25° = \frac{mv^2}{r}$$

$$R \sin 25° = \frac{600 \times 42^2}{r}$$

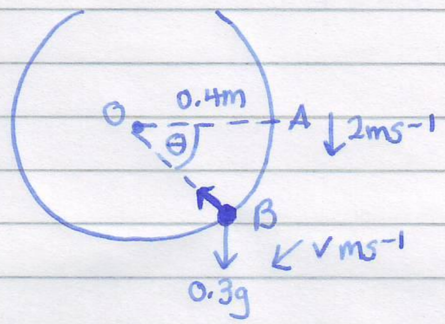
$r(R \sin 25°) = 600 \times 42^2$

$r(6487.86 \times \sin 25°) = 1058400$

$$r = \frac{1058400}{6487.86 \times \sin 25°}$$

$r = 386.01m$ to 2 d.p.

8



(a) By the principle of conservation of energy, the total energy at A is equal to the total energy at B.
Therefore KE at A + PE at A = KE at B + PE at B

$\frac{1}{2}\cancel{m}(2)^2 + \cancel{m}g(r - r\cos\theta) = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(r - r\cos\theta)$

$\frac{1}{2}(2)^2 + 9.8(0.4 - 0.4\cos 90°) = \frac{1}{2}v^2 + 9.8(0.4 - 0.4\cos(90-\theta))$

angle between vertical and A

angle between vertical and shown θ

$2 + 9.8(0.4 - 0.4(0)) = \frac{1}{2}v^2 + 3.92 - 3.92\cos(90° - \theta)$

$2 + \cancel{3.92} = \frac{1}{2}v^2 + \cancel{3.92} - 3.92\cos(90° - \theta)$

$2 + 3.92\cos(90° - \theta) = \frac{1}{2}v^2$

$4 + 7.84\cos(90° - \theta) = v^2$

$4 + 7.84\sin(\theta) = v^2$

↳ (Note that $\cos(90° - \theta) = \sin\theta$.)

(b)

Using $\frac{mv^2}{r} = T - mg\cos\theta$ θ here = angle with vertical

$\frac{0.3v^2}{0.4} = T - 0.3 \times 9.8 \times \cos(90° - \theta)$ θ here = as given in question

$0.75v^2 = T - 2.94\cos(90° - \theta)$

$0.75(4 + 7.84\sin\theta) = T - 2.94\sin\theta$

$3 + 5.88\sin\theta = T - 2.94\sin\theta$

$3 + 8.82\sin\theta = T$.

(c) The maximum value for θ happens when $R = 0$.

So $3 + 8.82\sin\theta = 0$

$$8.82\sin\theta = -3$$

$$\sin\theta = \frac{-3}{8.82}$$

$$\theta = -19.89^\circ \text{ to 2 d.p.}$$

$$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$$

or $\theta = 199.89^\circ$ to 2 d.p.

The marble leaves the bowl after moving through an angle of 199.89°. The marble then moves under the action of gravity, behaving like a projectile.