S3: Samplau a Dosraniadau

Haf 2006

(1) Dewis 3 rhif ar set $\{1, 1, 2, 3, 4\}$

(a) Dymar samplau anhrefnedig sy'n bosib:

(1, 1, 2) (1, 2, 3) (2, 3, 4)
(1, 1, 3) (1, 2, 4)
(1, 1, 4) (1, 3, 4)

(b)

Sampl	Tebygolrwydd	Cymedr	Canolrif
(1, 1, 2)	$({}^2C_2 \times {}^1C_1)/10 = 0.1$	$\frac{4}{3}$	1
(1, 1, 3)	$({}^2C_2 \times {}^1C_1)/10 = 0.1$	$\frac{5}{3}$	1
(1, 1, 4)	$({}^2C_2 \times {}^1C_1)/10 = 0.1$	2	1
(1, 2, 3)	$({}^2C_1 \times {}^1C_1 \times {}^1C_1)/10 = 0.2$	2	2
(1, 2, 4)	$({}^2C_1 \times {}^1C_1 \times {}^1C_1)/10 = 0.2$	$\frac{7}{3}$	2
(1, 3, 4)	$({}^2C_1 \times {}^1C_1 \times {}^1C_1)/10 = 0.2$	$\frac{8}{3}$	3
(2, 3, 4).	$({}^1C_1 \times {}^1C_1 \times {}^1C_1)/10 = 0.1$	3	3
	1 ✓		

$({}^5C_3 = 10)$

(i) Dosraniad samplu cymedr y sampl:

Cymedr	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
Tebygolrwydd	0.1	0.1	0.3	0.2	0.2	0.1

(ii) Dosraniad samplu canolrif y sampl:

Canolrif	1	2	3
Tebygolrwydd	0.3	0.4	0.3

Haf 2007

1 (a) Cyfansum 3: Samplau anhrefnedig (1, 2).

$$P(X=3) = \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2}$$

$$= \frac{2}{15} \checkmark$$

(b)

Sampl	Tebygolrwydd	Cyfansum
(1, 2)	$({}^1C_1 \times {}^2C_1)/15 = \frac{2}{15}$	3
(1, 3)	$({}^1C_1 \times {}^3C_1)/15 = \frac{3}{15}$	4
(2, 2)	$({}^2C_2)/15 = \frac{1}{15}$	4
(2, 3)	$({}^2C_1 \times {}^3C_1)/15 = \frac{6}{15}$	5
(3, 3)	$({}^3C_2)/15 = \frac{3}{15}$	6
	$\frac{15}{15}$ ✓	

Dosraniad Samplu X:

X	3	4	5	6
$P(X=x)$	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{6}{15}$	$\frac{3}{15}$

Haf 2008

(1) Dewis 3 darn arian ór set {£2, £2, £2, £2, £2, £2, £1, £1, £1, £1}.

(a) ${}^{10}C_3 = 120$

Sampl	Tebygolrwydd	Cyfanswm
(1, 1, 1)	$({}^4C_3)/120 = \frac{4}{120} = \frac{1}{30}$	£3
(1, 1, 2)	$({}^4C_2 \times {}^6C_1)/120 = \frac{36}{120} = \frac{3}{10}$	£4
(1, 2, 2)	$({}^4C_1 \times {}^6C_2)/120 = \frac{60}{120} = \frac{1}{2}$	£5
(2, 2, 2)	$({}^6C_3)/120 = \frac{20}{120} = \frac{1}{6}$	£6
	$\frac{\frac{120}{120}}{\frac{120}{120}}$ ✓	

Dosraniad Sampiu cyfanswm gwerth y tri darn arian

Cyfanswm	£3	£4	£5	£6
Tebygolrwydd	$\frac{1}{30}$	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{1}{6}$

(b) $E(\text{Cyfanswm}) = 3 \times \frac{1}{30} + 4 \times \frac{3}{10} + 5 \times \frac{1}{2} + 6 \times \frac{1}{6}$
$= £4.80.$

Cyfanswm y deg darn arian yn y bag = £16
Cymedr y deg darn arian yn y bag = £16 ÷ 10
= £1.60.

Gan fod £4.80 = 3 × £1.60 rydym wedi gwireddu fod gwerth disgwyliedig y cyfanswm deirgwaith gwerth cymedrig y deg darn arian yn y bag.

Haf 2009

① Dewis 5 pêl ór set $\{G, G, G, G, G, C, C, C\}$.

(a) (i)

Sampl	Tebygolrwydd	X
(G, G, G, G, G)	$({}^5C_5)/56 = \frac{1}{56}$	5
(G, G, G, G, C)	$({}^5C_4 \times {}^3C_1)/56 = \frac{15}{56}$	4
(G, G, G, C, C)	$({}^5C_3 \times {}^3C_2)/56 = \frac{30}{56} = \frac{15}{28}$	3
(G, G, C, C, C)	$({}^5C_2 \times {}^3C_3)/56 = \frac{10}{56} = \frac{5}{28}$	2
	$\frac{56}{56}$ ✓	

$({}^8C_5 = 56.)$

Dosraniad samplu X:

X	2	3	4	5
$P(X=x)$	$\frac{5}{28}$	$\frac{15}{28}$	$\frac{15}{56}$	$\frac{1}{56}$

(ii) $E(X) = 2 \times \frac{5}{28} + 3 \times \frac{15}{28} + 4 \times \frac{15}{56} + 5 \times \frac{1}{56}$

$= \frac{25}{8}$

$= 3.125$

(b) (i) Mae X yn ddosraniad Binomial.

$X \sim B(5, \frac{5}{8})$

(ii) $E(X) = np$

$= 5 \times \frac{5}{8}$

$= \frac{25}{8}$

$= 3.125$ ✓

Haf 2010

(3) Dewis 3 darn arian ór set {20c, 10c, 10c, 10c, 5c, 5c}

$^6C_3 = 20$

Sampl	Tebygolrwydd	Cyfansum
(20, 10, 10)	$({}^1C_1 \times {}^3C_2)/20 = \frac{3}{20}$	40c
(20, 10, 5)	$({}^1C_1 \times {}^3C_1 \times {}^2C_1)/20 = \frac{6}{20} = \frac{3}{10}$	35c
(20, 5, 5)	$({}^1C_1 \times {}^2C_2)/20 = \frac{1}{20}$	30c
(10, 10, 10)	$({}^3C_3)/20 = \frac{1}{20}$	30c
(10, 10, 5)	$({}^3C_2 \times {}^2C_1)/20 = \frac{6}{20} = \frac{3}{10}$	25c
(10, 5, 5)	$({}^3C_1 \times {}^2C_2)/20 = \frac{3}{20}$	20c
	$\frac{20}{20}$ ✓	

Dosraniad samplu cyfansum y darnau arian:

Cyfansum	20c	25c	30c	35c	40c
Tebygolrwydd	$\frac{3}{20}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{20}$

Haf 2011

(3) Dewis 3 pêl ôr set $\{1, 1, 2, 3, 4\}$

Sampl	Tebygolrwydd	Cyfansum	Rhif Mwyaf
(1, 1, 2)	$({}^2C_2 \times {}^1C_1)/10 = \frac{1}{10}$	4	2
(1, 1, 3)	" $\frac{1}{10}$	5	3
(1, 1, 4)	" $\frac{1}{10}$	6	4
(1, 2, 3)	$({}^2C_1 \times {}^1C_1 \times {}^1C_1)/10 = \frac{2}{10} = \frac{1}{5}$	6	3
(1, 2, 4)	" $\frac{2}{10} = \frac{1}{5}$	7	4
(1, 3, 4)	" $\frac{2}{10} = \frac{1}{5}$	8	4
(2, 3, 4)	$({}^1C_1 \times {}^1C_1 \times {}^1C_1)/10 = \frac{1}{10}$	9	4
	$\frac{10}{10}$ ✓		

$({}^5C_3 = 10)$

(a) Dosraniad sampla Cyfansum y rhifau:

Cyfansum	4	5	6	7	8	9
Tebygolrwydd	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

(b) Dosraniad sampla y rhif mwyaf sy'n ymddangos

Rhif Mwyaf	2	3	4
Tebygolrwydd	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10}$

$$E(\text{Rhif Mwyaf}) = 2 \times \frac{1}{10} + 3 \times \frac{3}{10} + 4 \times \frac{6}{10}$$
$$= 3.5$$

Haf 2012

(1) Dewis 3 rhif ór set $\{1, 2, 3, 4, 5\}$

Sampl	Tebygolrwydd	Cymedr	Canolrif
(1, 2, 3)	$({}^1C_1 \times {}^1C_1 \times {}^1C_1)/10 = \frac{1}{10}$	$\frac{6}{3} = 2$	2
(1, 2, 4)	" $\frac{1}{10}$	$\frac{7}{3}$	2
(1, 2, 5)	" $\frac{1}{10}$	$\frac{8}{3}$	2
(1, 3, 4)	" $\frac{1}{10}$	$\frac{8}{3}$	3
(1, 3, 5)	" $\frac{1}{10}$	$\frac{9}{3} = 3$	3
(1, 4, 5)	" $\frac{1}{10}$	$\frac{10}{3}$	4
(2, 3, 4)	" $\frac{1}{10}$	$\frac{9}{3} = 3$	3
(2, 3, 5)	" $\frac{1}{10}$	$\frac{10}{3}$	3
(2, 4, 5)	" $\frac{1}{10}$	$\frac{11}{3}$	4
(3, 4, 5)	" $\frac{1}{10}$	$\frac{12}{3} = 4$	4
	$\frac{10}{10}$ ✓		

$({}^5C_3 = 10)$

(a) Dosraniad Samplu Cymedr y tri rhif

Cymedr	2	$\frac{7}{3}$	$\frac{8}{3}$	3	$\frac{10}{3}$	$\frac{11}{3}$	4
Tebygolrwydd	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

(b) Dosraniad Samplu Canolrif y tri rhif

Canolrif	2	3	4
Tebygolrwydd	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{3}{10}$

Haf 2013

(2) Dewis 4 pêl ór set $\{C, C, C, G, G, G, G, G, G, G\}$.

Sampl	Tebygolrwydd	X	Y	$\lvert X-Y \rvert$
(C, C, C, G)	$({}^3C_3 \times {}^7C_1)/210 = \frac{7}{210} = \frac{1}{30}$	3	1	2
(C, C, G, G)	$({}^3C_2 \times {}^7C_2)/210 = \frac{63}{210} = \frac{3}{10}$	2	2	0
(C, G, G, G)	$({}^3C_1 \times {}^7C_3)/210 = \frac{105}{210} = \frac{1}{2}$	1	3	2
(G, G, G, G)	$({}^7C_4)/210 = \frac{35}{210} = \frac{1}{6}$	0	4	4

$({}^{10}C_4 = 210)$

Dosraniad Sampla $\lvert X-Y \rvert$

$\lvert X-Y \rvert$	0	2	4
Tebygolrwydd	$\frac{3}{10}$	$\frac{8}{15}$	$\frac{1}{6}$

S3 Haf 2015

① Dewis 3 pêl ór set $\{1, 2, 2, 4, 6, 6\}$. ${}^6C_3 = 20$

Sampl	Tebygolrwydd	Amrediad	Canolrif
(1, 2, 2)	$({}^1C_1 \times {}^2C_2)/20 = \frac{1}{20}$	$2-1=1$	2
(1, 2, 4)	$({}^1C_1 \times {}^2C_1 \times {}^1C_1)/20 = \frac{2}{20} = \frac{1}{10}$	$4-1=3$	2
(1, 2, 6)	$({}^1C_1 \times {}^2C_1 \times {}^2C_1)/20 = \frac{4}{20} = \frac{1}{5}$	$6-1=5$	2
(1, 4, 6)	$({}^1C_1 \times {}^1C_1 \times {}^2C_1)/20 = \frac{2}{20} = \frac{1}{10}$	$6-1=5$	4
(1, 6, 6)	$({}^1C_1 \times {}^2C_2)/20 = \frac{1}{20}$	$6-1=5$	6
(2, 2, 4)	$({}^2C_2 \times {}^1C_1)/20 = \frac{1}{20}$	$4-2=2$	2
(2, 2, 6)	$({}^2C_2 \times {}^2C_1)/20 = \frac{2}{20} = \frac{1}{10}$	$6-2=4$	2
(2, 4, 6)	$({}^2C_1 \times {}^1C_1 \times {}^2C_1) = \frac{4}{20} = \frac{1}{5}$	$6-2=4$	4
(2, 6, 6)	$({}^2C_1 \times {}^2C_2) = \frac{2}{20} = \frac{1}{10}$	$6-2=4$	6
(4, 6, 6)	$({}^1C_1 \times {}^2C_2) = \frac{1}{20}$	$6-4=2$	6
	$\frac{20}{20} = 1$ ✓		

Dosraniad Sampla R, amrediad y sampl

Amrediad	1	2	3	4	5
Tebygolrwydd	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{7}{20}$

Dosraniad Sampla M, canolrif y sampl

Canolrif	2	4	6
Tebygolrwydd	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{5}$

S3 Haf 2016

① Dewis 3 darn arian ar set {50c, 10c, 10c, 2c, 2c, 2c}.

Sampl	Tebygolrwydd	Darn Mwyaf
50, 10, 10	$({}^1C_1 \times {}^2C_2)/20 = \frac{1}{20}$	50c
50, 10, 2	$({}^1C_1 \times {}^2C_1 \times {}^3C_1)/20 = \frac{6}{20}$	50c
50, 2, 2	$({}^1C_1 \times {}^3C_2)/20 = \frac{3}{20}$	50c
10, 10, 2	$({}^2C_2 \times {}^3C_1)/20 = \frac{3}{20}$	10c
10, 2, 2	$({}^2C_1 \times {}^3C_2)/20 = \frac{6}{20}$	10c
2, 2, 2	${}^3C_3/20 = \frac{1}{20}$	2c
	$\frac{20}{20}$ ✓	

$({}^6C_3 = 20)$

Gwerth disgwyliedig y darn arian uchaf ei werth

$= 50 \times \frac{1}{20} + 50 \times \frac{6}{20} + 50 \times \frac{3}{20} + 10 \times \frac{3}{20} + 10 \times \frac{6}{20} + 2 \times \frac{1}{20}$

$= 50 \times \frac{10}{20} + 10 \times \frac{9}{20} + 2 \times \frac{1}{20}$

$= \frac{500}{20} + \frac{90}{20} + \frac{2}{20}$

$= \frac{592}{20}$

$= \underline{\underline{29.6c}}$

S3 Haf 2017

2) a) $P(X \leq x) = P($Rhif ar y dis cyntaf yn $\leq x$,
Rhif ar yr ail ddis yn $\leq x$,
Rhif ar y trydydd dis yn $\leq x)$

$= P($Rhif ar y dis cyntaf yn $\leq x) \times$
$P($Rhif ar yr ail ddis yn $\leq x) \times$
$P($Rhif ar y trydydd dis yn $\leq x)$

$$= \left(\frac{x}{6}\right) \times \left(\frac{x}{6}\right) \times \left(\frac{x}{6}\right)$$

$P(X \leq x) = \left(\frac{x}{6}\right)^3$ ar gyfer $x = 1, 2, 3, 4, 5, 6.$ ✓

b) $P(X = x) = P(X \leq x) - P(X \leq x-1)$

$$= \left(\frac{x}{6}\right)^3 - \left(\frac{x-1}{6}\right)^3$$

$$= \frac{x^3}{216} - \left(\frac{(x-1)(x-1)(x-1)}{216}\right)$$

$$= \frac{x^3}{216} - \left(\frac{(x^2-2x+1)(x-1)}{216}\right)$$

$$= \frac{x^3}{216} - \left(\frac{x^3-2x^2+x-x^2+2x-1}{216}\right)$$

$$= \frac{x^3}{216} - \left(\frac{x^3-3x^2+3x-1}{216}\right)$$

$P(X = x) = \dfrac{3x^2 - 3x + 1}{216}$ ar gyfer $x = 1, 2, 3, 4, 5, 6$

c) Dosraniad samplu X:

Rhif uchaf	1	2	3	4	5	6
Tebygolrwydd	$\frac{1}{216}$	$\frac{7}{216}$	$\frac{19}{216}$	$\frac{37}{216}$	$\frac{61}{216}$	$\frac{91}{216}$

Gwerth mwyaf tebygol X yw $\underline{\underline{6}}$

S3 Haf 2018

1) Dewis 3 pêl ôr set $\{1, 1, 1, 2, 2, 3\}$

${}^6C_3 = 20$

Sampl	Tebygolrwydd	Rhif Mwyaf
(1, 1, 1)	$({}^3C_3)/20 = \frac{1}{20}$	1
(1, 1, 2)	$({}^3C_2 \times {}^2C_1)/20 = \frac{6}{20}$	2
(1, 2, 2)	$({}^3C_1 \times {}^2C_2)/20 = \frac{3}{20}$	2
(1, 1, 3)	$({}^3C_2 \times {}^1C_1)/20 = \frac{3}{20}$	3
(1, 2, 3)	$({}^3C_1 \times {}^2C_1 \times {}^1C_1)/20 = \frac{6}{20}$	3
(2, 2, 3)	$({}^2C_2 \times {}^1C_1)/20 = \frac{1}{20}$	3
	$\frac{20}{20}$ ✓	

Dosraniad Sampla y rhif mwyaf

Rhif Mwyaf	1	2	3
Tebygolrwydd	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{10}{20}$

$$E(\text{Rhif mwyaf}) = 1 \times \frac{1}{20} + 2 \times \frac{9}{20} + 3 \times \frac{10}{20}$$

$$= \frac{1}{20} + \frac{18}{20} + \frac{30}{20}$$

$$= \frac{49}{20}$$

$$= \underline{\underline{2.45}}$$