



The Mathematics Department

10

Measuring

Shapes 3

Higher Tier

Version
3.0
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Name:

Contents

Chapter	Mathematics	Page Number
Trigonometry of Right-Angled Triangles	Labelling triangles. Calculating the length of a side. Calculating the size of an angle.	3
Transformations: Enlargement	Positive scale factor. Fractional scale factor. Centre of enlargement. Finding the centre of enlargement. Negative scale factor.	11
Perimeter and Area of Composite Shapes	Composite shapes. Length of an arc and the area of a sector.	17

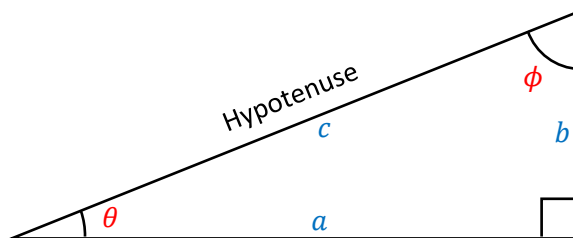


Trigonometry

Right-Angled Triangles

Any right-angled triangle has:

- One angle that is a right angle, or 90°;
- Two acute angles θ and ϕ ;
- A hypotenuse c , which is always opposite the right angle;
- Two sides a and b which are shorter than the hypotenuse.



In year 9, we introduced Pythagoras' Theorem, which connects the lengths a , b and c :

$$c^2 = a^2 + b^2$$

Given the length of any two sides in a triangle, we can use Pythagoras' Theorem to calculate the length of the third side.

Exercise 1

Use Pythagoras' Theorem to calculate the length of the third side in these right-angled triangles. Round off your answers to two decimal places.

(a)

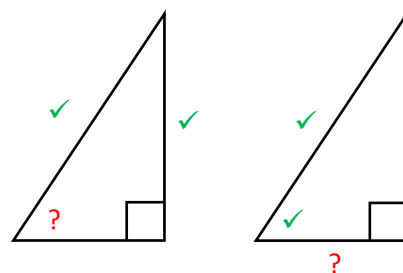
(b)

(c)

What is trigonometry?

Trigonometry is used for:

- Calculating the size of one of the **acute angles** in a right-angled triangle, given the length of **any two sides**;
- Calculating the length of **one of the sides** in a right-angled triangle given the length of **one other side** and the size of **one acute angle**.



How?

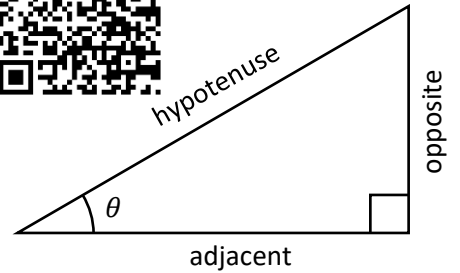
Trigonometry uses the relationship between the size of the angles and the lengths of the sides in any right-angled triangle.

Exercise 2

Draw any three right-angled triangles where one of the angles measures 30°. Measure the length of the hypotenuse and the length of the side opposite the 30° angle. What do you notice?

Labelling the sides of a right-angled triangle

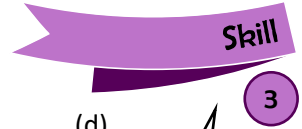
Let θ represent the size of one of the acute angles in a right-angled triangle. We follow these conventions when labelling the sides of the triangle.



- The **opposite** is the side opposite the angle θ .
- The **hypotenuse** is the side opposite the right angle.
- The **adjacent** is the side left over (it's close to the angle θ).

Exercise 3

Label the sides of these triangles using the words "opposite", "hypotenuse" and "adjacent".



(a)

(b)

(c)

(d)

(e)

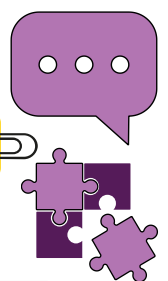
(f)

(g)

Sin, Cos, Tan


For a specific angle θ , we define the **sin**, **cos** and **tan** of the angle as follows.

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$



How to remember the formula...

In mathematics, we worship the King SOHCAHTOA.



S **O** **C** **A** **T** **O**
H **H** **A**

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

Finding lengths using trigonometry

Consider the right-angled triangle shown on the right.
Let us use trigonometry to calculate the length of the side x .

To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".

We see that we want to calculate the length of the **opposite** (x) side, and we know the length of the **hypotenuse** (5 cm). The trigonometric ratio that uses the words **opposite** and **hypotenuse** is sin, therefore we must use the formula

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

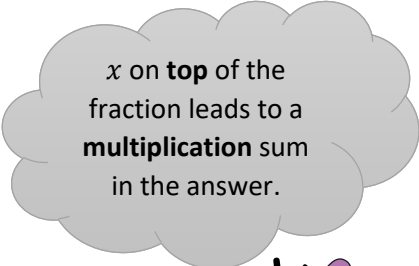
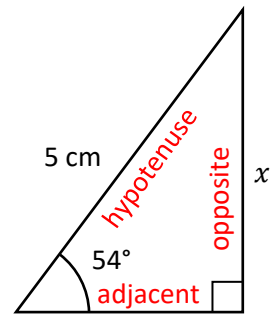
in this question. By substituting values into the formula, we obtain

$$\sin 54^\circ = \frac{x}{5}$$

By multiplying both sides of the equation by 5, we obtain

$$x = 5 \times \sin 54^\circ$$

By typing this sum on a calculator, we find that $x = 4.05$ cm, correct to two decimal places.



Exercise 4

For the following right-angled triangles, calculate the length of the side that is labelled with the variable x .

(a) (b) (c)

(d) (e) (f)

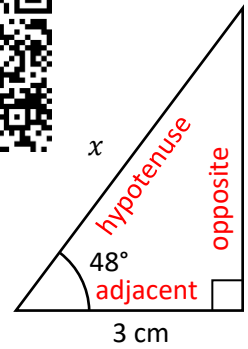
(g) (h) (i)



Example

Consider the right-angled triangle shown on the right.
Let us use trigonometry to calculate the length of the side x .

To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".



We see that we know the length of the **adjacent** (3 cm), and we want to calculate the length of the **hypotenuse** (x). The trigonometric ratio that uses the words **adjacent** and **hypotenuse** is cos, therefore we must use the formula

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

in this question. By substituting values into the formula, we obtain

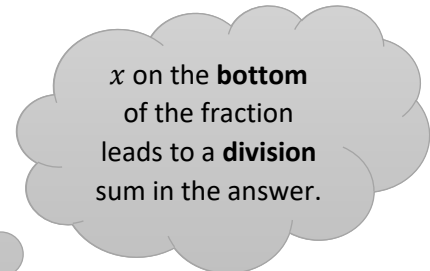
$$\cos 48^\circ = \frac{3}{x}$$

By multiplying both sides of the equation by x , we obtain

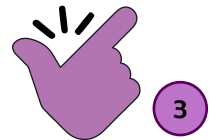
$$x \times \cos 48^\circ = 3.$$

By dividing both sides of the equation by $\cos 48^\circ$, we obtain

$$x = 3 \div \cos 48^\circ.$$



By typing this sum on a calculator, we find that $x = 4.48$ cm, correct to two decimal places.



Exercise 5

For the following right-angled triangles, calculate the length of the side that is labelled with the variable x .

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

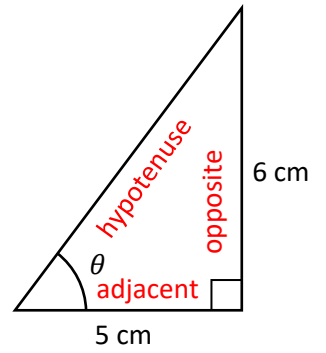


Finding angles using trigonometry

Consider the right-angled triangle shown on the right. Let us use trigonometry to calculate the size of the angle θ .

To start with, we label the sides of the triangle using the words "opposite", "adjacent" and "hypotenuse".

We see that we know the length of the **opposite** (6 cm) and the length of the **adjacent** (5 cm). The trigonometric ratio that uses the words **opposite** and **adjacent** is tan, therefore we must use the formula



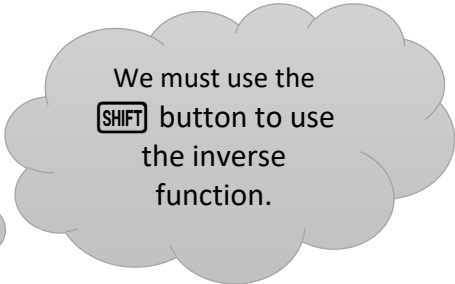
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

in this question. By substituting values into the equation, we obtain

$$\tan \theta = \frac{6}{5}$$

To find the size of the angle θ we must use the inverse tan function:

$$\theta = \tan^{-1}\left(\frac{6}{5}\right)$$



By typing this sum on a calculator, we find that $\theta = 50.19^\circ$, correct to two decimal places.

Exercise 6



For the following right-angled triangles, calculate the size of the angle θ .

(a)

(b)

(c)

(d)

(e)

(f)

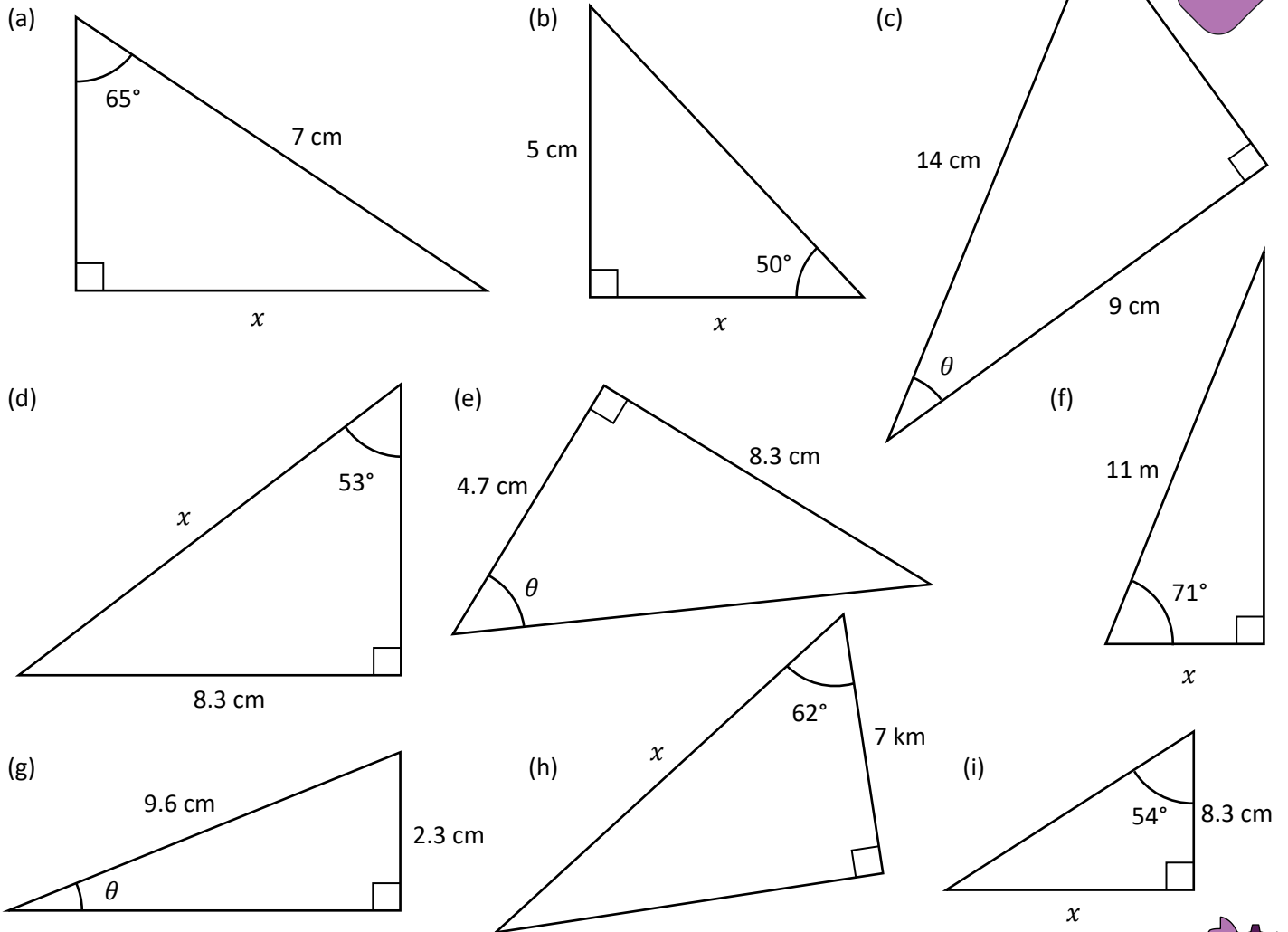
(g)

(h)

(i)

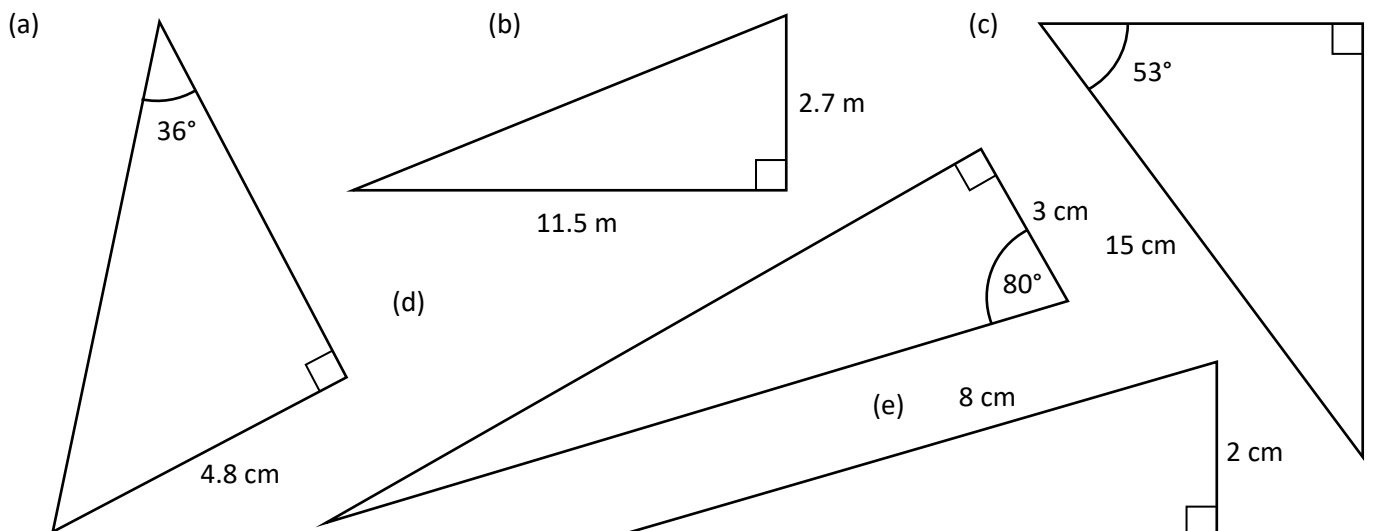
Exercise 7

For the following right-angled triangles, calculate the length of the side x , or the size of the angle θ . Round off your answers correct to two decimal places.



Exercise 8

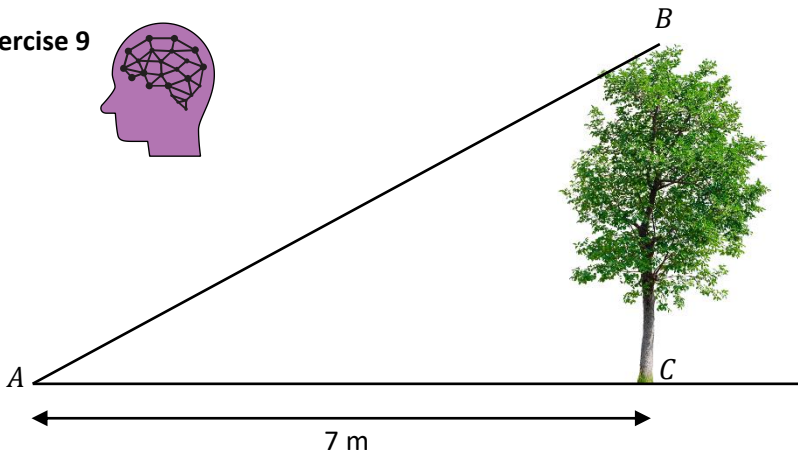
For the following right-angled triangles, find the size of **every** missing angle and the length of **every** missing side.



Exercise 9

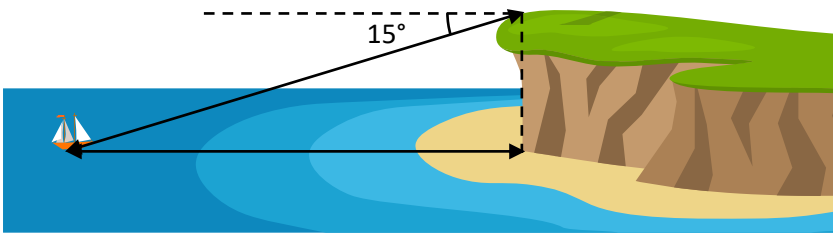


(a)



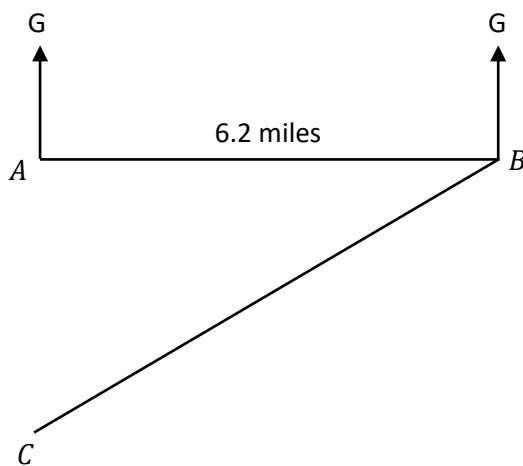
The vertical height of a tree is 3.2 metres. The horizontal distance from point A to the bottom of the tree is 7 metres. Calculate the angle of elevation of the top of the tree from the point A .

(b)



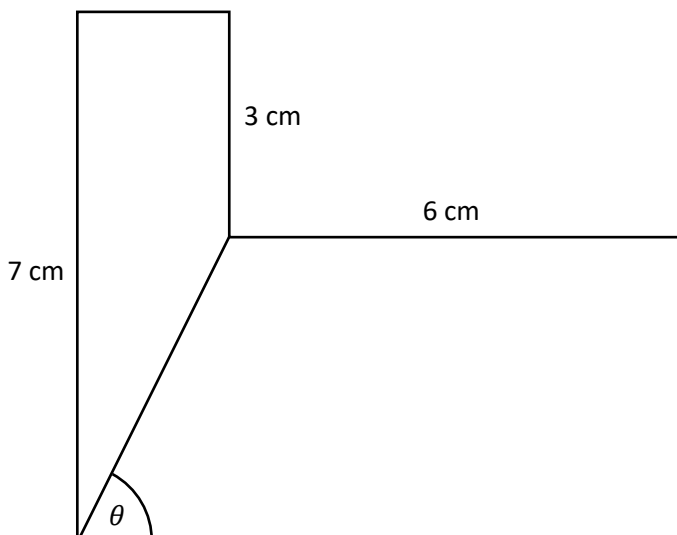
From the top of a vertical cliff, the angle of depression of a sailing boat is 15° . If the sailing boat is 200 m from the bottom of the cliff, calculate the height of the cliff above sea level.

(c)

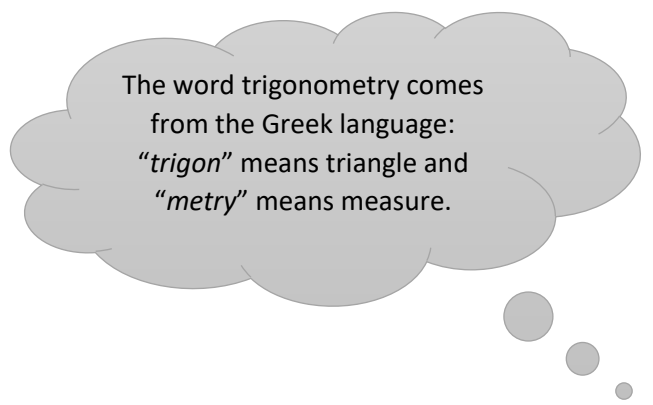


A ship leaves a port and sails 6.2 miles at a bearing of 090° to reach B . Then it turns and sails at a bearing of 224° until it reaches a point C , which is south of the port A . Calculate the distance between the point C and the port A .

(d)

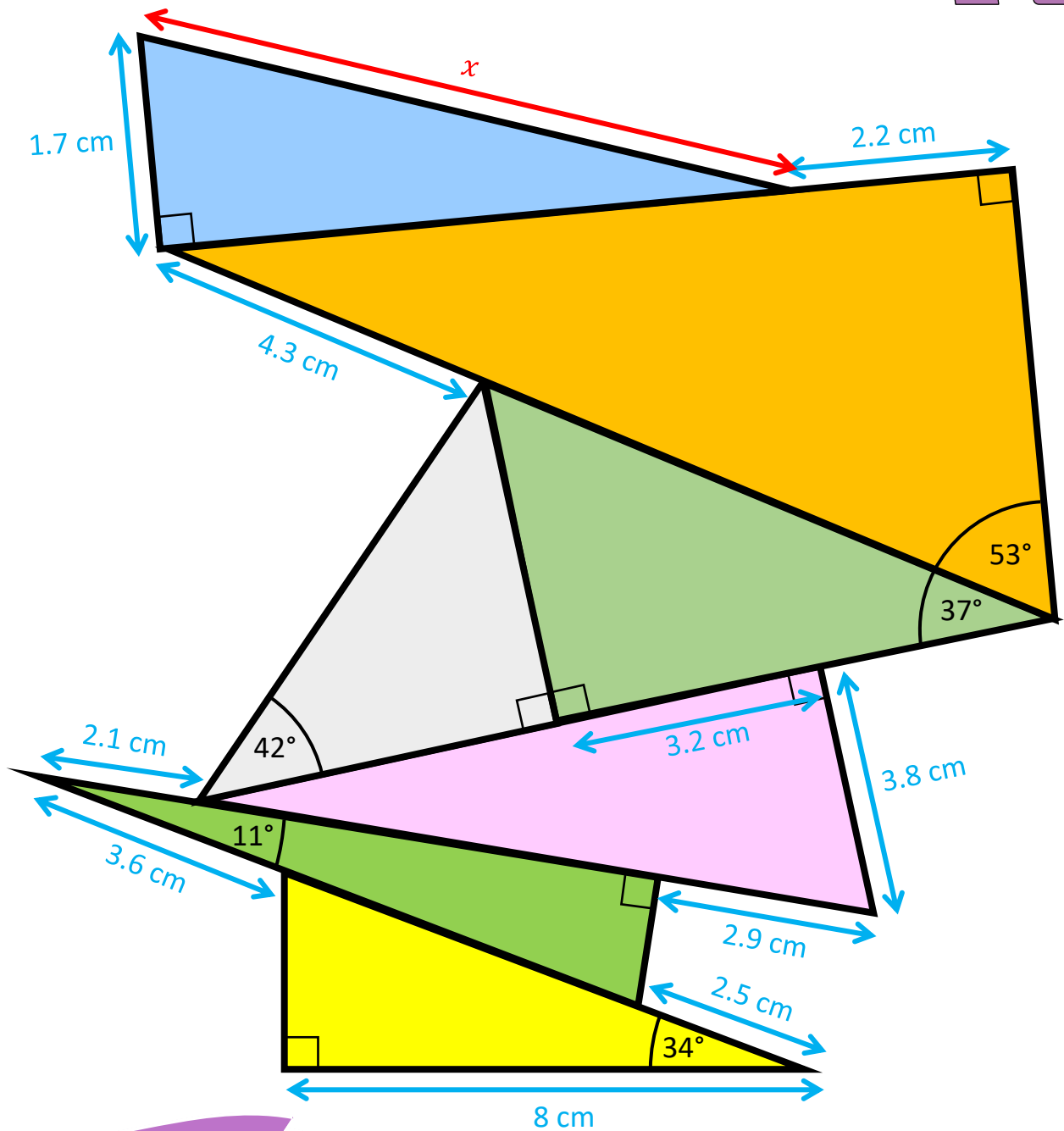
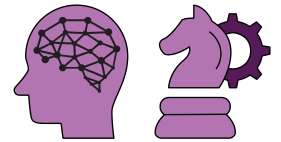


The diagram shows an irregular hexagon split into two trapezia. The edges of the hexagon are either vertical or horizontal. Calculate the size of the angle θ .



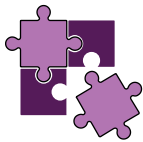
Exercise 10: A Pile of Triangles

Use Trigonometry and Pythagoras' Theorem to find the length of x .



Evaluation

Key words	Corrections	I am happy with...	I need to revise...



Enlargement



Enlargement is one of the four transformations.

Year 7	Year 8	Year 9	Year 10
Translation	Rotation	Reflection	Enlargement

When a shape is enlarged, the **size** of the shape changes. The **scale factor** decides how the shape changes. For example, if the scale factor is 2, then the size of the shape doubles. If the scale factor is $\frac{1}{2}$, then the size of the shape halves.

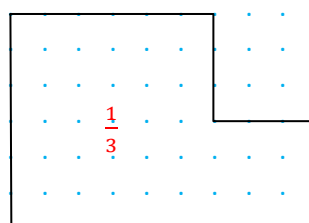
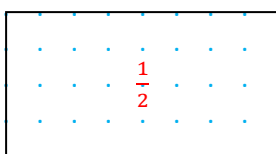
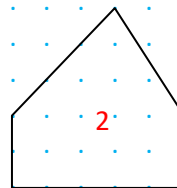
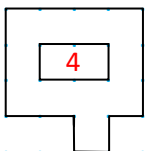
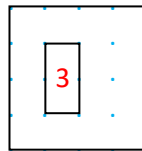
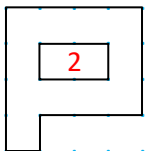


Skill

Exercise 11

Enlarge the following shapes using the scale factor that is given in the centre of each shape.

2



Exercise 12



Enlarge the following shapes using the scale factor that is given in the centre of each shape.

Centre of Enlargement

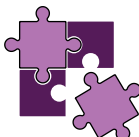
If a question states a point as the **centre of enlargement**, then the enlargement must appear in a certain location (it cannot appear *anywhere* like in Exercises 11 and 12).



Example

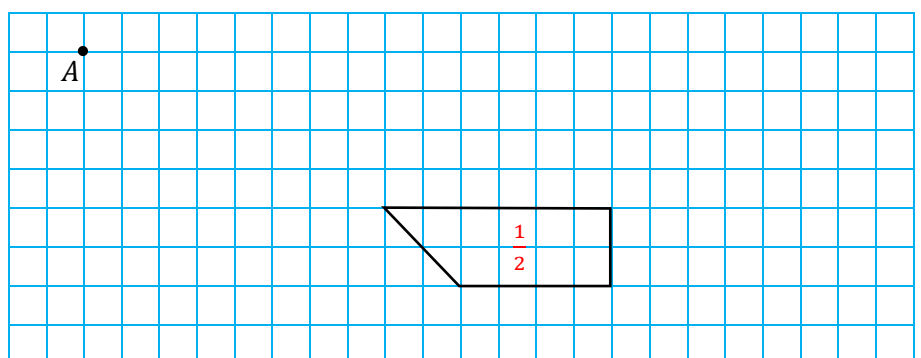
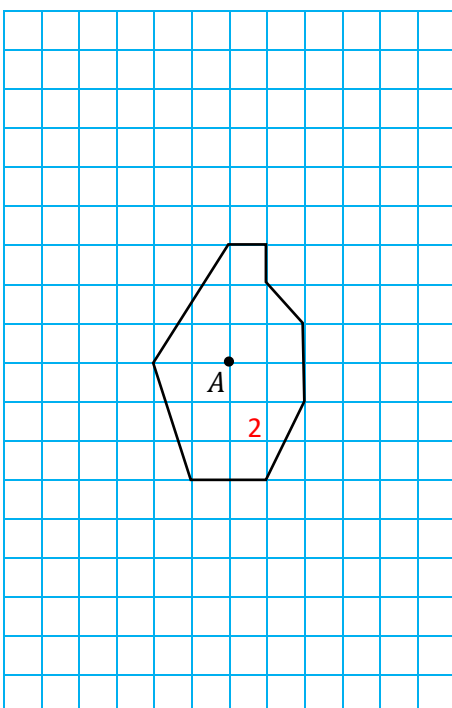
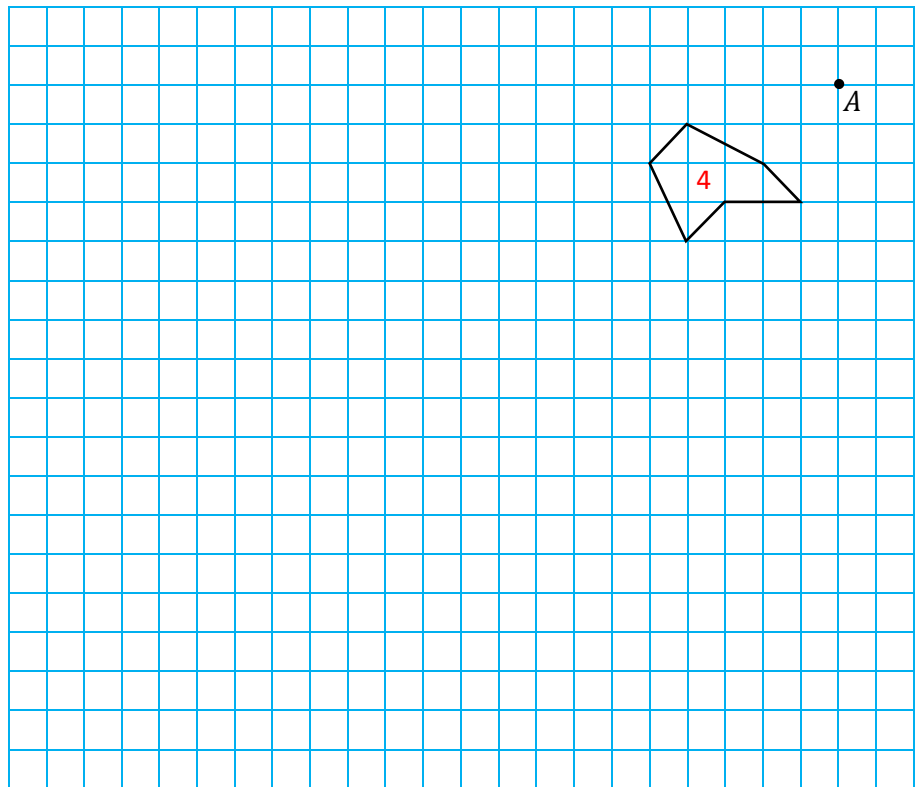
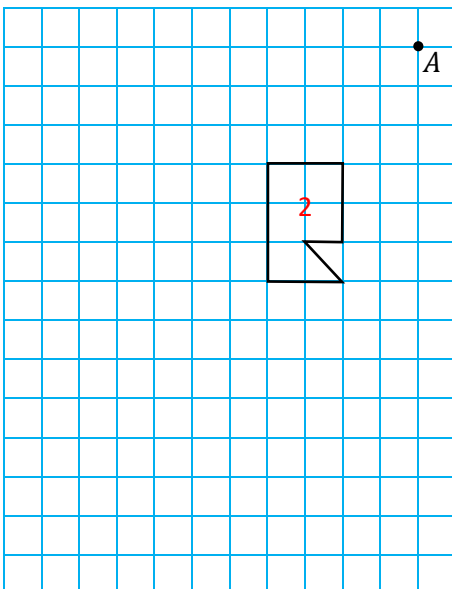
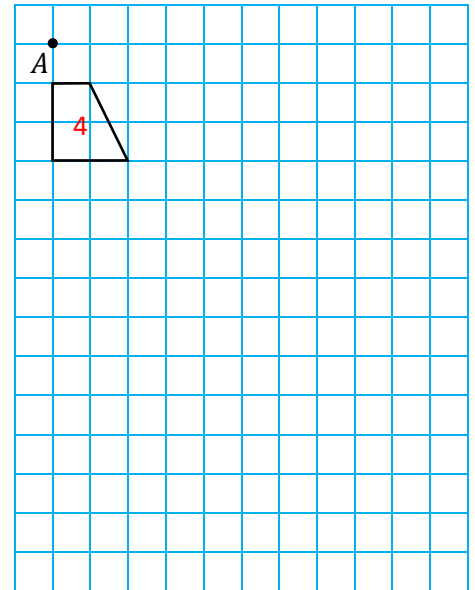
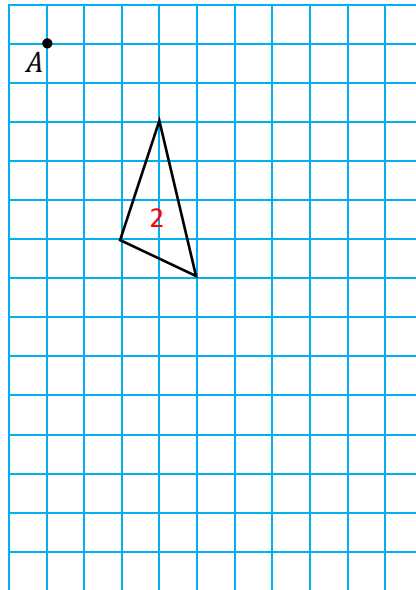
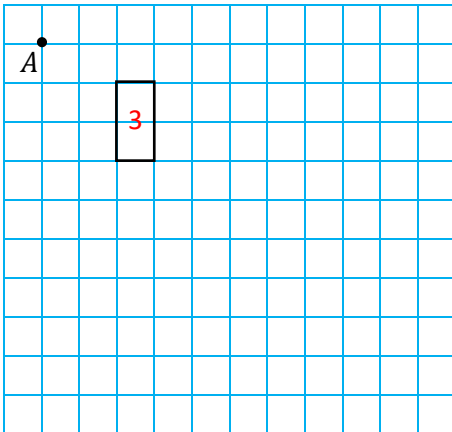
Enlarge the following triangle using a scale factor of 3 and the point *A* as the centre of enlargement.

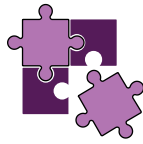
To go from the point *A* to the vertex *P* in the original triangle, we must go **2 units to the right and 1 unit down**. Since the scale factor is 3, to go from the point *A* to the vertex *P'* in the new triangle, we must go **$2 \times 3 = 6$ units to the right, and $1 \times 3 = 3$ units down**. We can repeat this with the other vertices (*Q* and *R*), or you can start at the vertex *P'* and draw a triangle that is three times larger.



Exercise 13

Enlarge the following shapes using the point *A* as the centre of enlargement and the number in the centre of each shape as the scale factor.

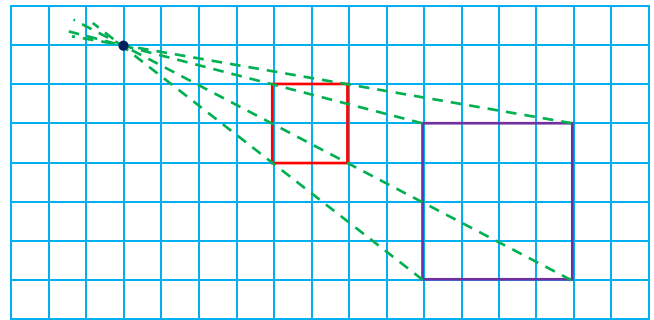




Finding the centre of enlargement

Given a **shape** and its **enlargement**, **connect the corresponding vertices and extend the lines** to find the location of the centre of enlargement.

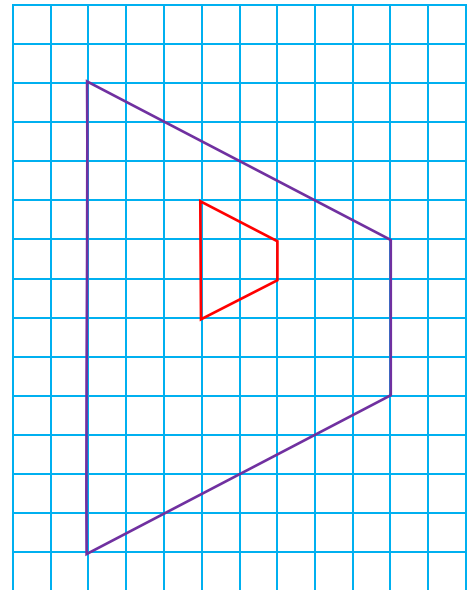
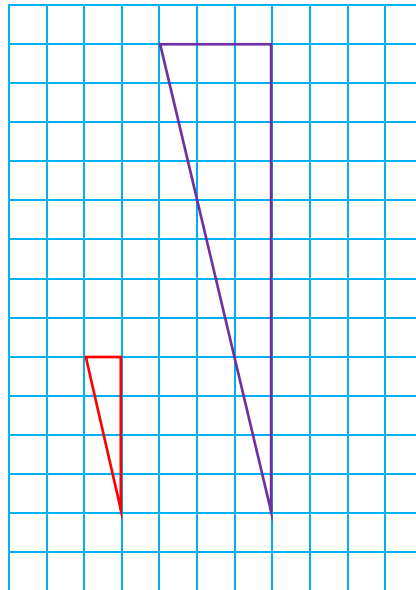
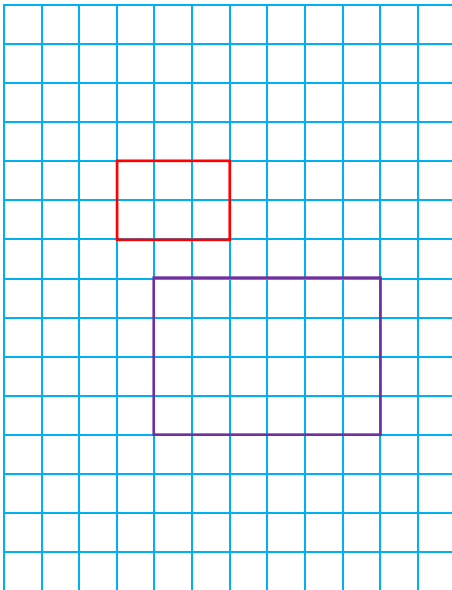
You can find the scale factor by comparing the sizes of the shapes.



Exercise 14

Find the scale factor and the centre of enlargement for the following enlargements.

2

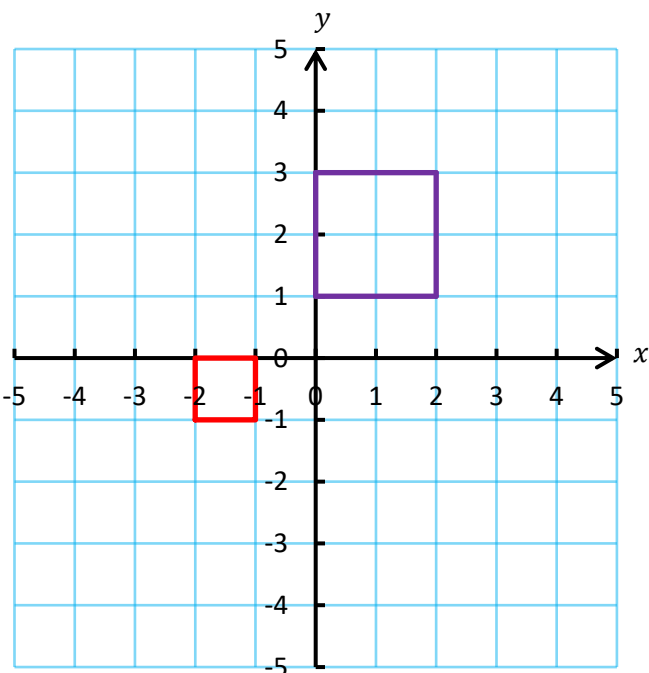


Exercise 15

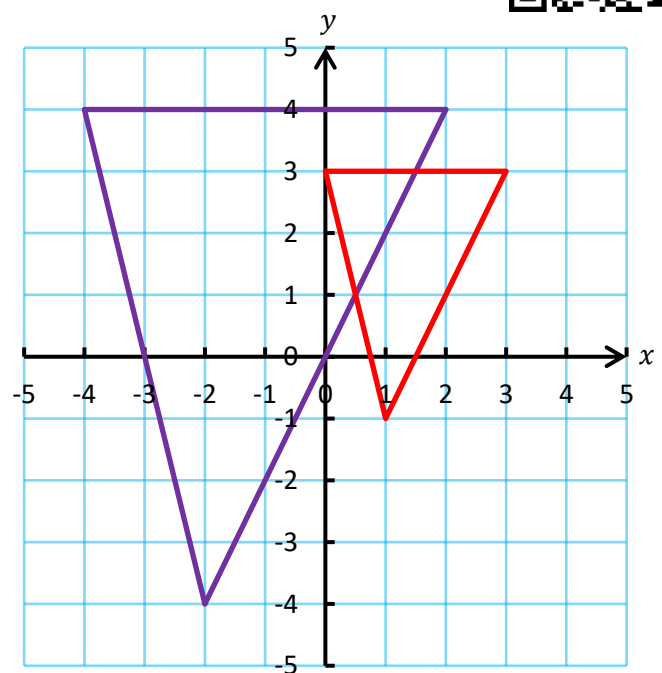
Find the scale factor and the centre of enlargement for the following enlargements.



(a)



(b)



Negative Scale Factor

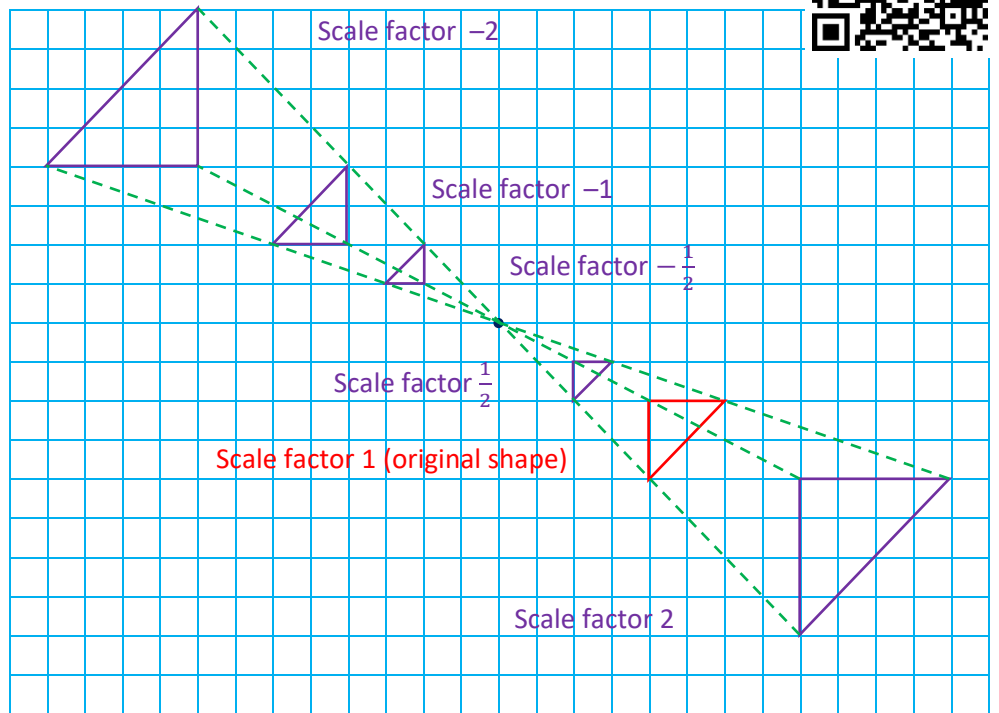
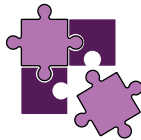


A **negative** scale factor means working from the centre of enlargement to the opposite direction.

For example, consider the diagram on the right. To go from the centre of enlargement to the top left vertex of the **original triangle**, we must go 4 units right and 2 units down.

With a scale factor of 2, we must go $4 \times 2 = 8$ units right and $2 \times 2 = 4$ units down.

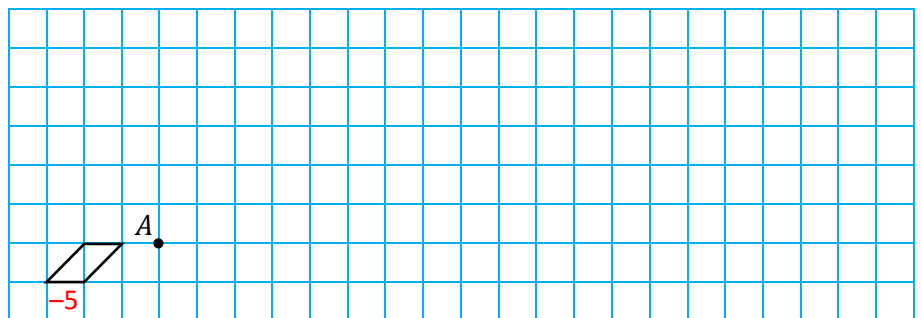
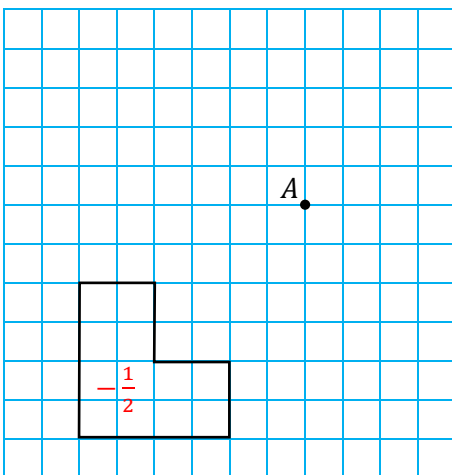
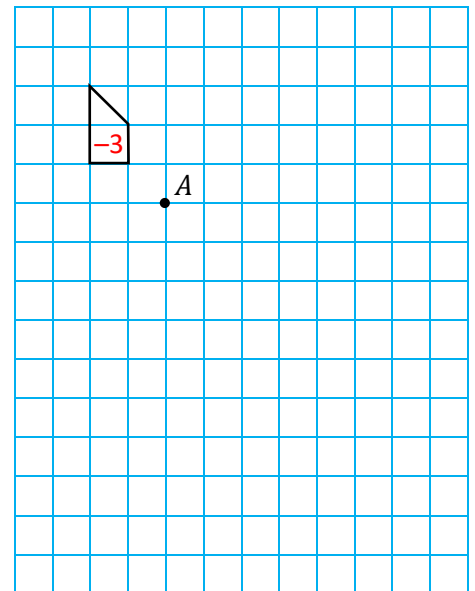
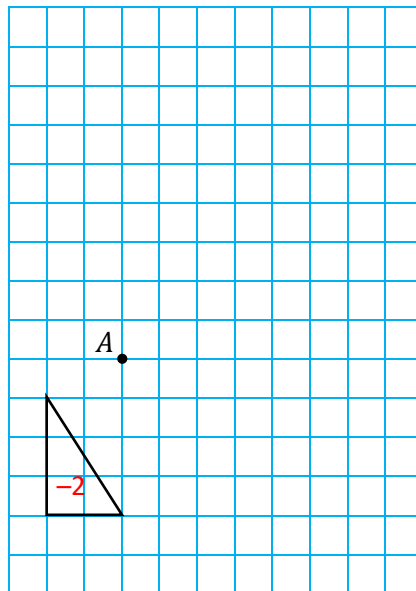
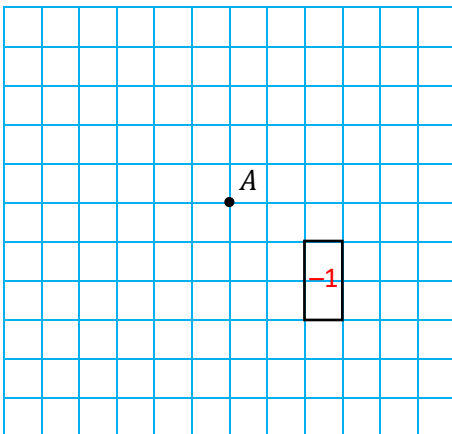
With a scale factor of -2 , we must go $4 \times 2 = 8$ units **left** and $2 \times 2 = 4$ units **up**.



Exercise 16

2

Enlarge the following shapes using the point *A* as the centre of enlargement and the number in the centre of each shape as the scale factor.

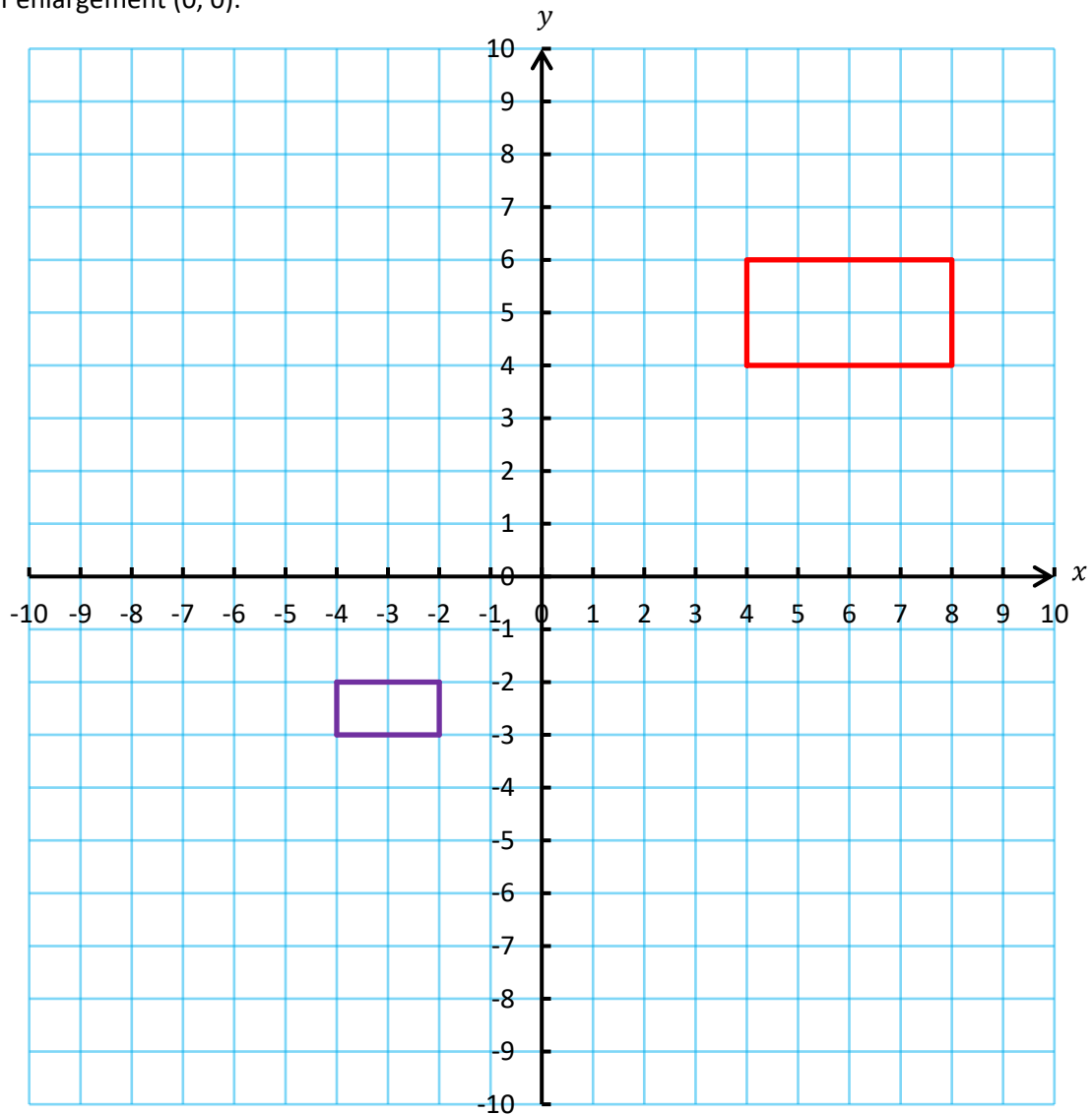


Exercise 17

2

The **larger** rectangle is transformed to the **smaller** rectangle. The co-ordinates of the centre of enlargement are $(0, 0)$. Complete the following sentence to fully describe this transformation.

The transformation from the larger rectangle to the smaller rectangle is an enlargement using scale factor _____ and centre of enlargement $(0, 0)$.



Evaluation

Key words	Corrections	I am happy with...	I need to revise...

Composite Shapes



Revision

13

Exercise 18

Complete the following table.

Shape	Name of the shape	Formula to find the area of the shape	Calculate the area of the shape

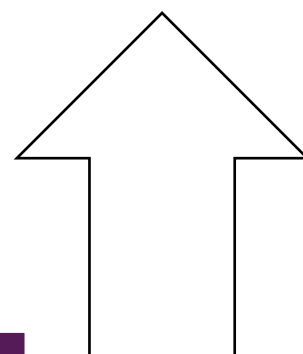
Exercise 19

Calculate the perimeter of each shape in the above table.



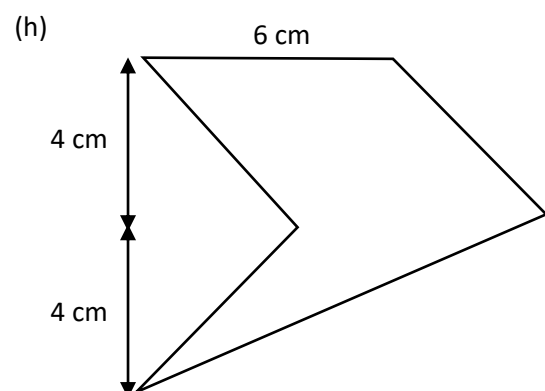
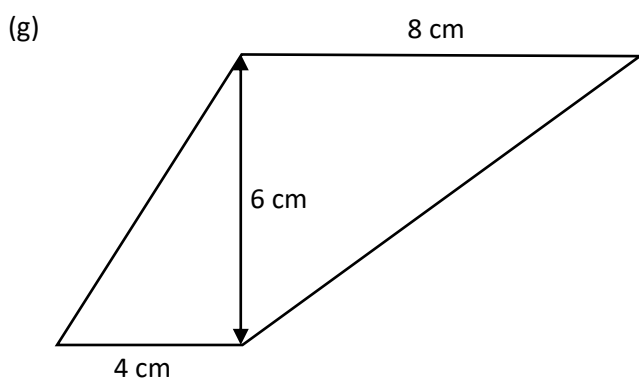
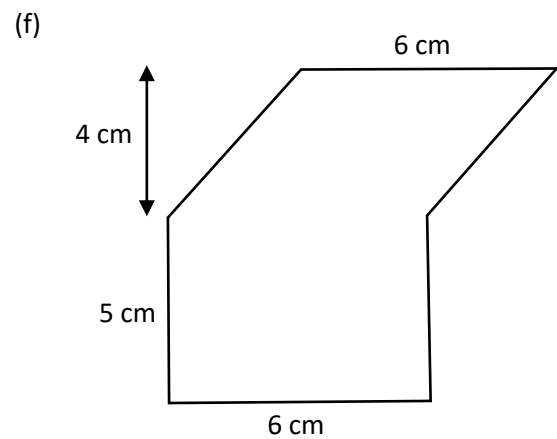
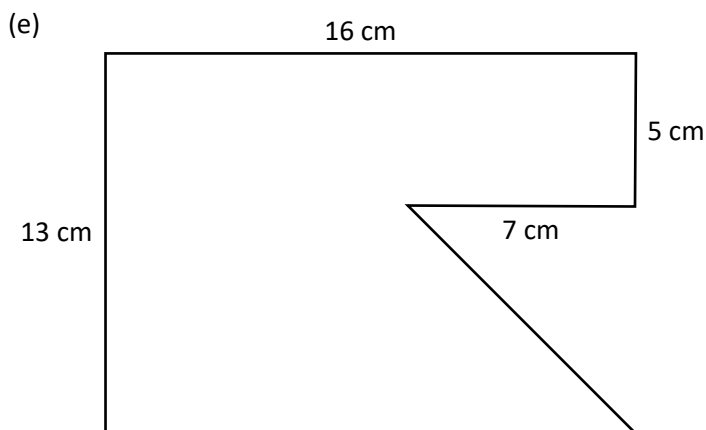
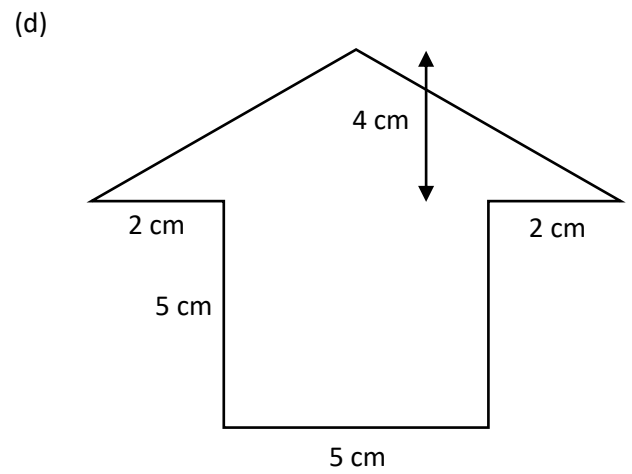
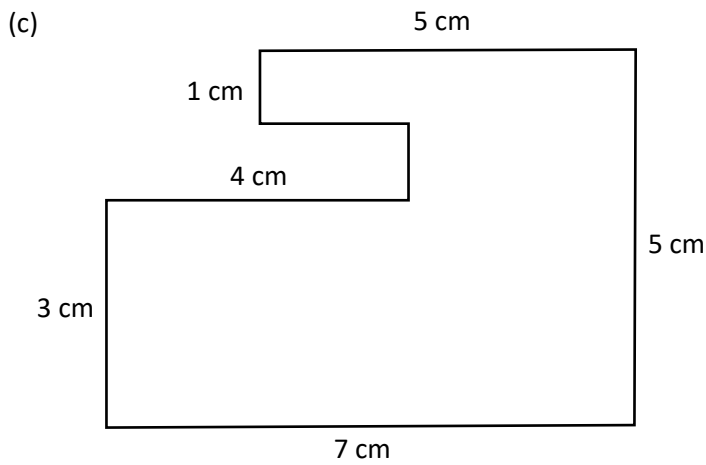
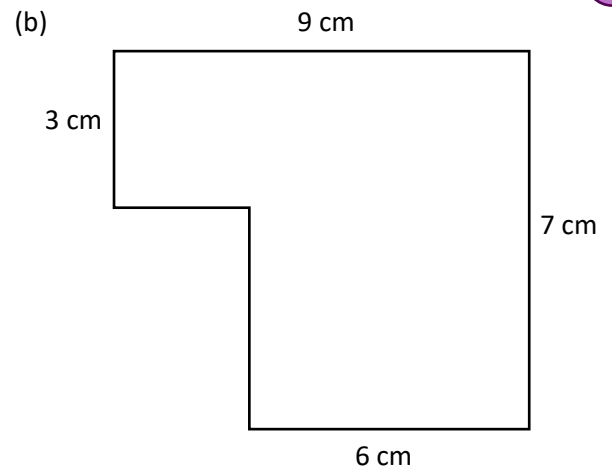
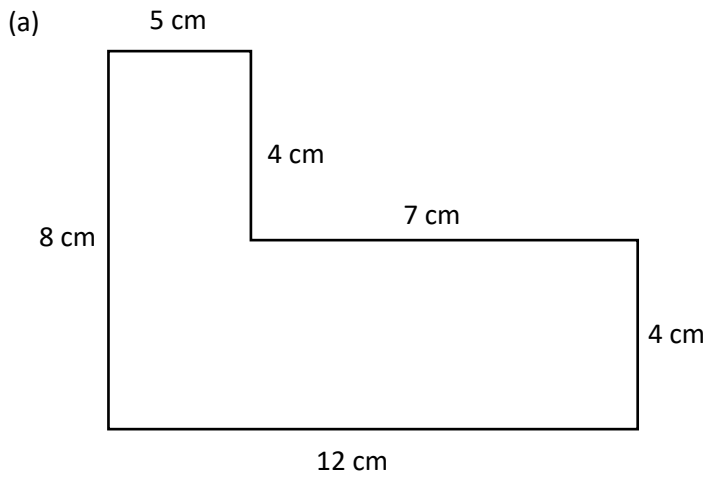
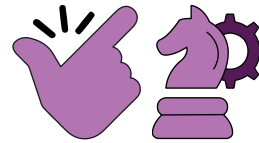
Composite Shapes

Composite shapes are shapes that you can split into simpler shapes, like the shapes in the above table. For example, the shape on the right is a composite shape – it is possible to split the shape into a rectangle (at the bottom) and a triangle (at the top). We can calculate the area of the composite shape by adding the area of the rectangle to the area of the triangle.



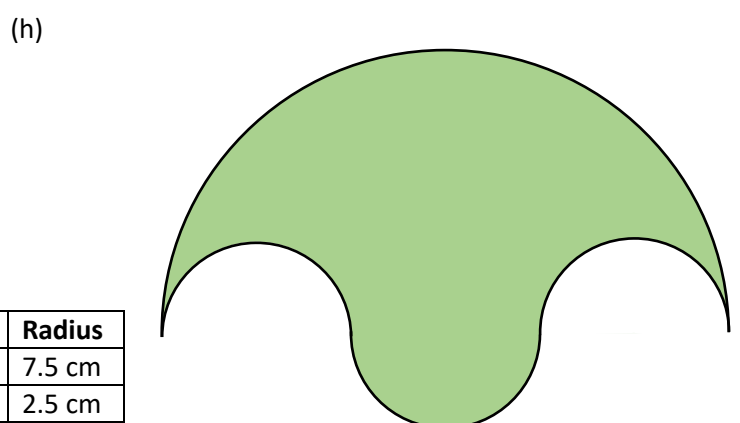
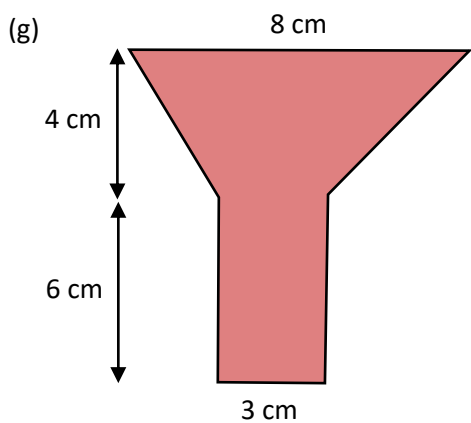
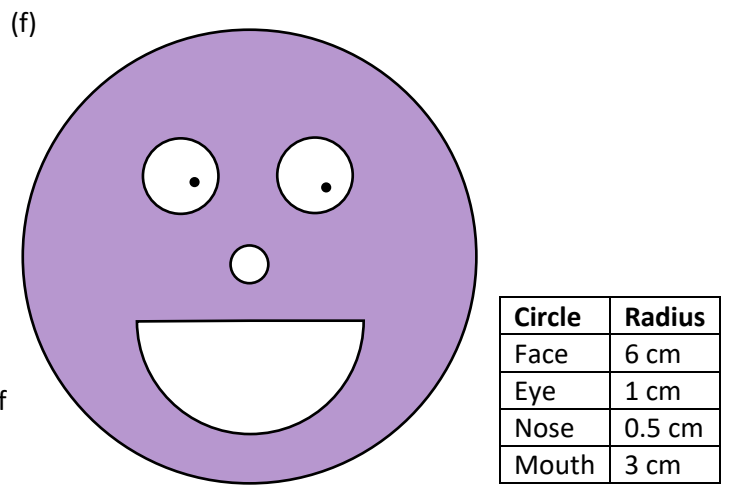
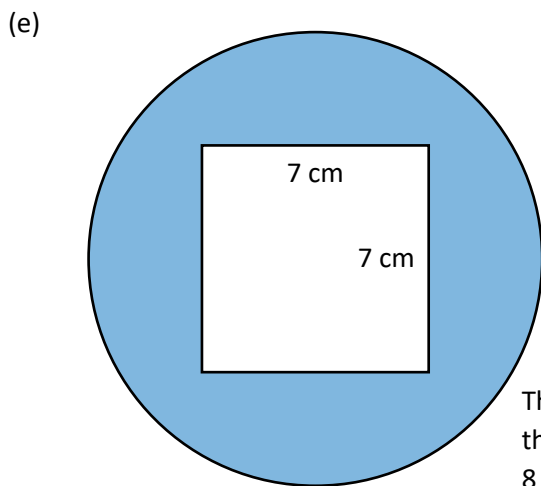
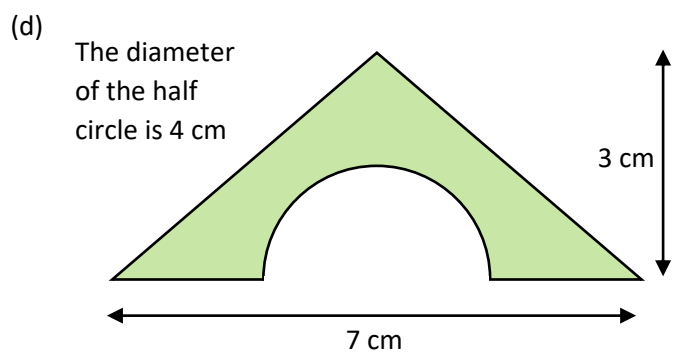
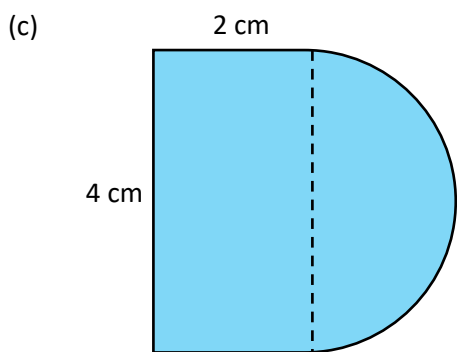
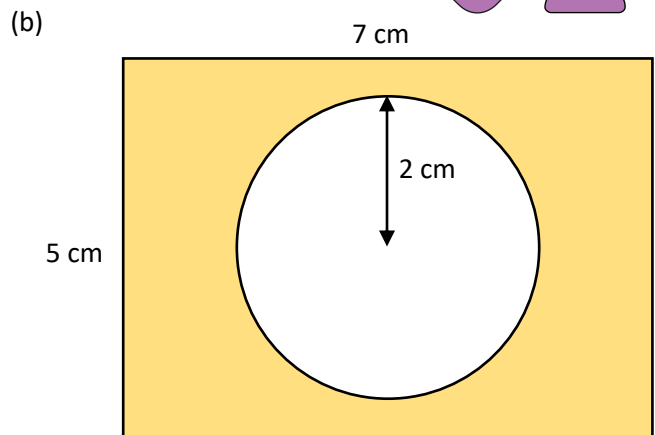
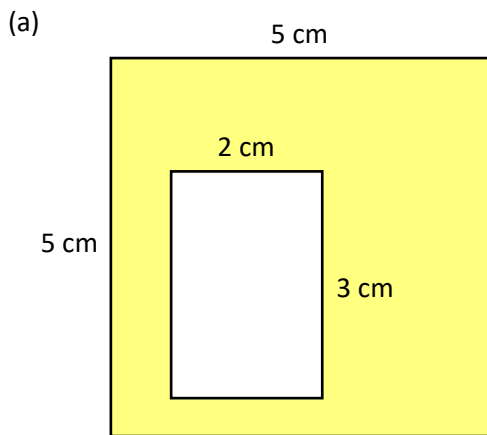
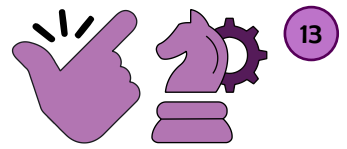
Exercise 20

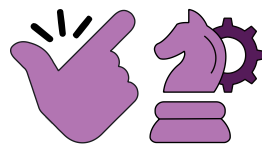
Calculate the area of each of the following composite shapes.



Exercise 21

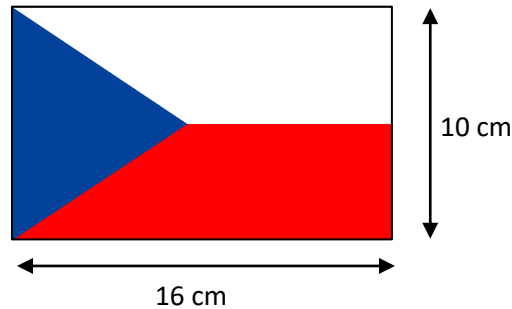
Calculate the area of the coloured region.



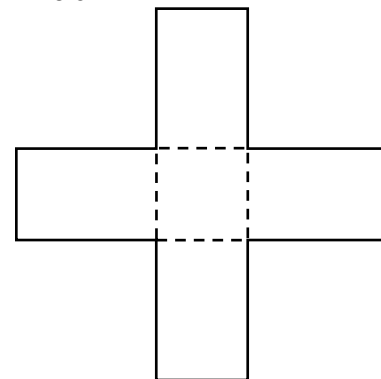


Exercise 22

(a) The flag of the Czech Republic is shown on the right. Given that the vertex of the blue triangle appears in the middle of the flag, calculate the area of the red region.



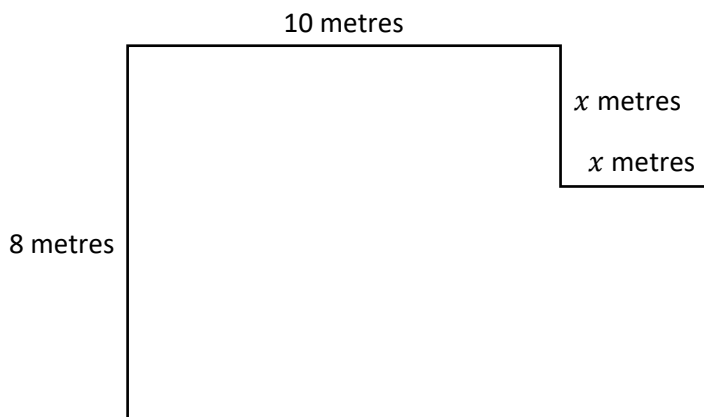
(b) A pattern in the form of a cross is formed by arranging four identical rectangles around the sides of a square, as shown in the diagram on the right. (The diagram is not drawn to scale.)



The area of the square is 36 cm^2 . The area of each rectangle is one and a half times the area of the square.

- (i) Find the area of the pattern in the form of a cross.
- (ii) Find the perimeter of the pattern in the form of a cross.

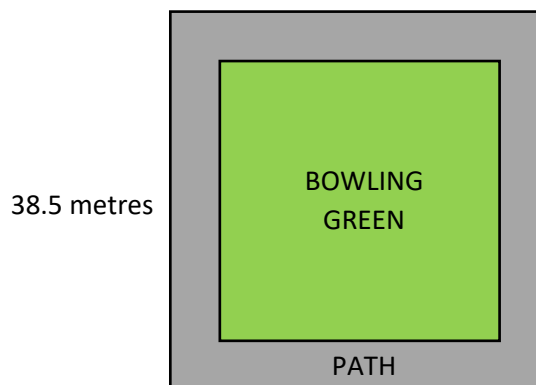
(c) Gerry has drawn a sketch of a room's floor plan. Each one of the corners of the room is either 90° or 270° . Gerry has forgotten some of the measurements but knows that two of the measurements are the same, and so labels them with x metres.



Show that the area of the floor, in square metres, is $80 + 8x - x^2$.

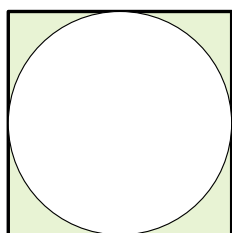
(d) The local council decides to place a concrete path of width $1\frac{1}{2}$ metres around a square bowling green, as shown in the diagram.

Calculate the area of the concrete path, in square metres.



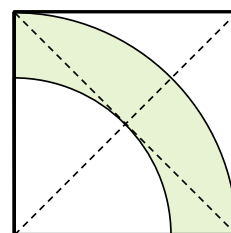
Challenge!

Which fraction of the square is shaded?



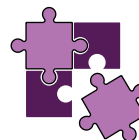
Challenge 2!

Which fraction of the square is shaded?



Length of an Arc and the Area of a Sector

The length of an arc is a fraction of the circumference of a circle, whilst the area of a sector is a fraction of the area of the circle.

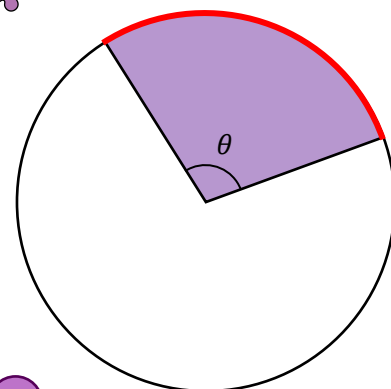


Arc Length = $\frac{\theta}{360^\circ} \times \text{circle circumference}$

Area of a Sector = $\frac{\theta}{360^\circ} \times \text{circle area}$

Arc Length = $\frac{\theta}{360^\circ} \times \pi \times \text{diameter}$

Area of a Sector = $\frac{\theta}{360^\circ} \times \pi \times \text{radius}^2$



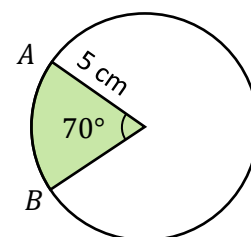
Exercise 23

For the circle shown on the right,

- Calculate the length of the minor (*smaller*) arc AB .
- Calculate the area of the minor sector AB .
- Calculate the length of the major (*larger*) arc AB .
- Calculate the area of the major sector AB .
- What fraction of the circle is shaded in green? Give your answer in its simplest form.



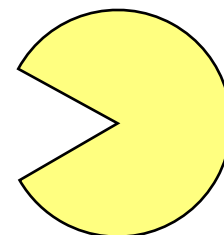
13



Exercise 24

Pacman, the computer game character, is in the shape of a sector. The angle in the centre of the shape is 300° . If the radius of Pacman is 6 cm,

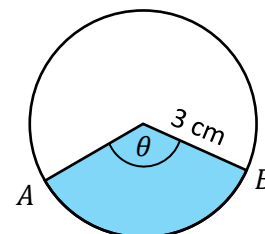
- What is the area of Pacman?
- What is the perimeter of Pacman?



Exercise 25

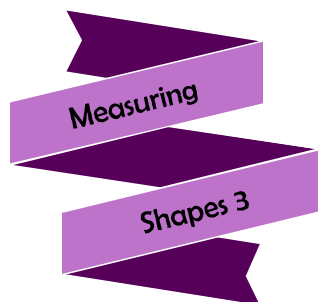
The length of the minor arc AB in the diagram on the right is 7 cm.

- What is the size of the angle θ ?
- Calculate the area of the blue sector.



Evaluation



Key words	Corrections	I am happy with...	I need to revise...



Reflection Sheet

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test	Correct in the test?
I can recognise the opposite , the adjacent and the hypotenuse in a right-angled triangle.			1, 2	
I can calculate the lengths of edges in a right-angled triangle using trigonometry .			1	
I can calculate the angles of a right-angled triangle using trigonometry .			2	
I can calculate the lengths of edges in a right-angled triangle using Pythagoras' Theorem .				
I can enlarge shapes using a scale factor that is a positive whole number .			3	
I can enlarge shapes using a scale factor that is fractional .			5	
I know how to use the centre of enlargement whilst enlarging shapes.			4, 5	
Given a shape and its enlargement, I can find the scale factor and the centre of enlargement .			6	
I can enlarge shapes using a scale factor that is negative .			11	
I can calculate the perimeter of a composite shape .			7, 8	
I can calculate the area of a composite shape .			9, 10	
I can calculate the length of an arc in a circle.			12	
I can calculate the area of a sector in a circle.			12	