Yr Adran Fathemateg

10

Mesur

Siapiau 3

Haen Uwch

Fersiwn
3.0
08/2025

Enw:

Cynnwys

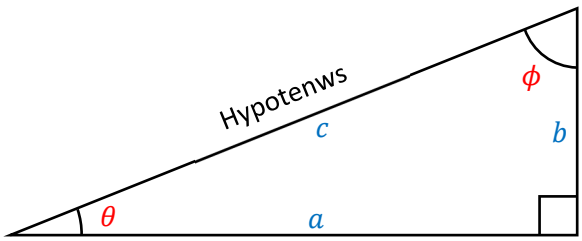
Pennod	Mathemateg	Rhif y Dudalen
Trigonometreg Trionglau Ongl Sgwâr	Labelu trionglau. Cyfrifo hyd ochr. Cyfrifo maint ongl.	3
Trawsffurfiadau: Helaethiad	Ffactor graddfa bositif. Ffactor graddfa ffracsiynol. Canol helaethiad. Darganfod canol helaethiad. Ffactor graddfa negatif.	11
Perimedr ac Arwynebedd Siapiau Cyfansawdd	Siapiau cyfansawdd. Hyd arc ac arwynebedd sector.	17

Trigonometreg

Trionglau Ongl Sgwâr

Mae gan unrhyw driongl ongl sgwâr:

- Un ongl sy'n ongl sgwâr, neu'n 90°;
- Dwy ongl lem θ a ϕ;
- Hypotenws c, sydd o hyd gyferbyn yr ongl sgwâr;
- Dwy ochr a a b sy'n fyrrach na'r hypotenws.



Ym mlwyddyn 9, cawsom gyflwyniad i Theorem Pythagoras, sy'n cysylltu'r hydoedd a, b ac c:

$$c^2 = a^2 + b^2$$

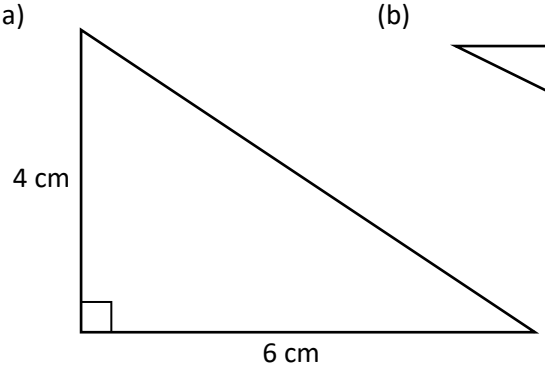
O gael hyd unrhyw ddau o ochrau'r triongl, mae'n bosib defnyddio Theorem Pythagoras i gyfrifo hyd y drydedd ochr.

Ymarfer 1

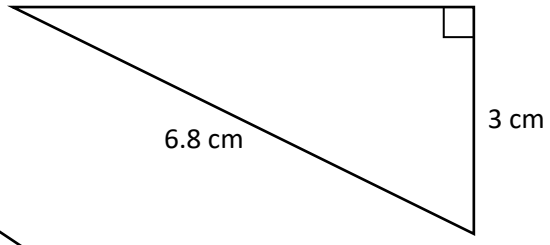
Adolygu 3

Defnyddiwch Theorem Pythagoras i gyfrifo hyd y drydedd ochr yn y trionglau ongl sgwâr canlynol. Talgrynnwch eich atebion i ddau le degol.

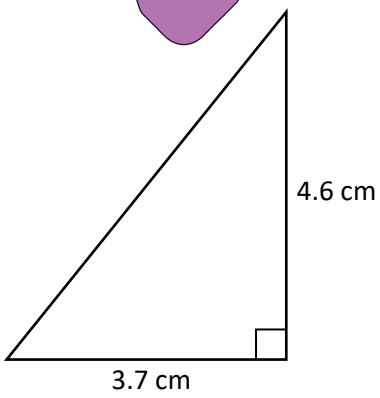
(a)



(b)



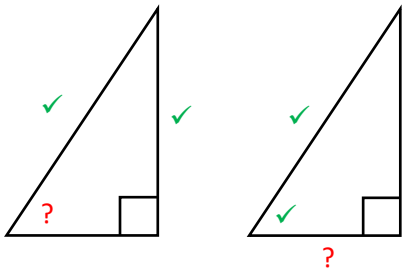
(c)



Beth yw trigonometreg?

Mae **trigonometreg** yn cael ei ddefnyddio ar gyfer:

- Cyfrifo maint un o'r onglau lem mewn triongl ongl sgwâr, o gael hyd unrhyw ddwy o'r ochrau;
- Cyfrifo hyd un o'r ochrau mewn triongl ongl sgwâr, o gael hyd un o'r ochrau eraill a maint un o'r onglau lem.



Sut?

Mae trigonometreg yn defnyddio'r berthynas rhwng meintiau'r onglau a hydoedd yr ochrau mewn unrhyw driongl ongl sgwâr.

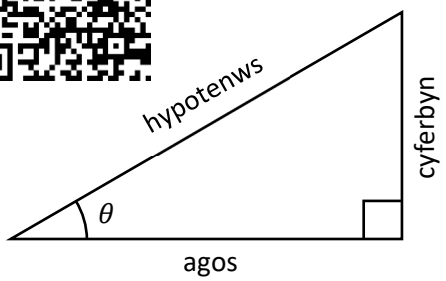
Ymarfer 2

Defnyddio

Lluniwch unrhyw dri thriongl ongl sgwâr ble mae maint un o'r onglau'n 30°.
Mesurwch hyd yr hypotenws a hyd yr ochr sydd gyferbyn yr ongl 30°. Beth rydych yn sylwi?

Labelu ochrau triongl ongl sgwâr

Gadewch i θ gynrychioli maint un o'r onglau lem mewn triongl ongl sgwâr. Rydym yn dilyn y confensiwn canlynol er mwyn labelu ochrau'r triongl.



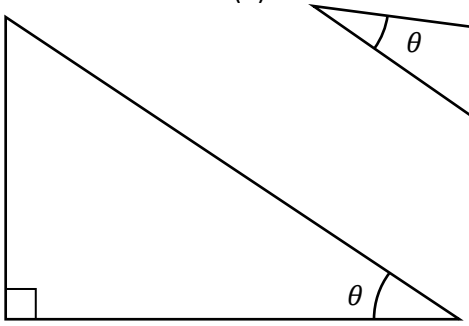
- Y **cyferbyn** yw'r ochr sydd gyferbyn yr ongl θ.
- Yr **hypotenws** yw'r ochr sydd gyferbyn yr ongl sgwâr.
- Yr **agos** yw'r ochr sydd ar ôl (mae'n agos at yr ongl θ).

Ymarfer 3

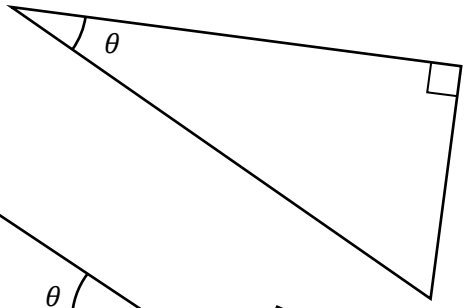
Sgil 3

Labelwch y trionglau canlynol efo'r geiriau "cyferbyn", "hypotenws" ag "agos".

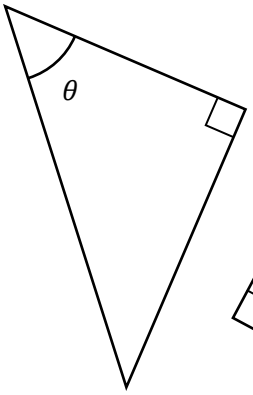
(a)



(b)

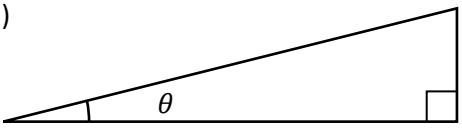


(c)

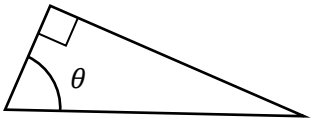


(ch)

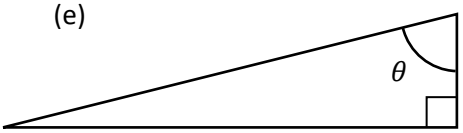
(d)



(dd)



(e)



Sin, Cos, Tan

Ar gyfer ongl benodol θ, mae **sin**, **cos** a **tan** yr ongl wastad yr un peth, ac yn cael ei ddiffinio fel yma.

$$\sin\theta = \frac{\text{cyferbyn}}{\text{hypotenws}} \qquad \cos\theta = \frac{\text{agos}}{\text{hypotenws}} \qquad \tan\theta = \frac{\text{cyferbyn}}{\text{agos}}$$

Sut i gofio'r fformiwlâu...

Ym mathemateg, rydym yn addoli'r brenin SOHCAHTOA.

$$S_H^O \quad C_H^A \quad T_A^O$$

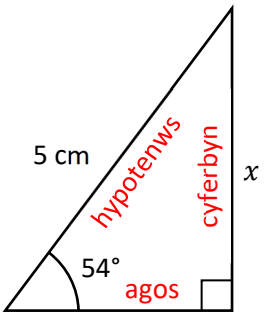
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Cyfieithu opposite ⇨ cyferbyn

$$\sin\theta = \frac{\text{cyferbyn}}{\text{hypotenws}} \qquad \cos\theta = \frac{\text{agos}}{\text{hypotenws}} \qquad \tan\theta = \frac{\text{cyferbyn}}{\text{agos}}$$

Ffeindio hydoedd efo trigonometreg

Ystyriwch y triongl ongl sgwâr sy'n cael ei ddangos ar y dde.
Gadewch i ni ddefnyddio trigonometreg er mwyn cyfrifo hyd yr ochr x.



I gychwyn, rhaid labelu'r triongl efo'r geiriau "cyferbyn", "agos" a "hypotenws".

Gwelwn ein bod eisiau cyfrifo hyd y **cyferbyn** (x), ac yn gwybod hyd yr **hypotenws** (5 cm).
Y gymhareb trigonometrig sy'n cynnwys y geiriau **cyferbyn** a **hypotenws** yw sin,
felly rhaid defnyddio'r gymhareb

$$\sin\theta = \frac{\text{cyferbyn}}{\text{hypotenws}}$$

yn y cwestiwn yma. Wrth amnewid gwerthoedd i mewn i'r gymhareb, cawn

$$\sin 54° = \frac{x}{5}.$$

x ar **dop** y ffracsiwn yn arwain at swm **lluosi** yn yr ateb.

Wrth luosi bob ochr yr hafaliad efo 5, cawn

$$x = 5 \times \sin 54°.$$

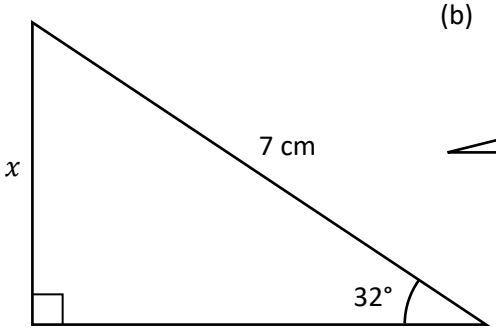
Wrth deipio'r swm yma ar gyfrifiannell, gallwn ddarganfod bod $x = 4.05$ cm, yn gywir i ddau le degol.

Ymarfer 4

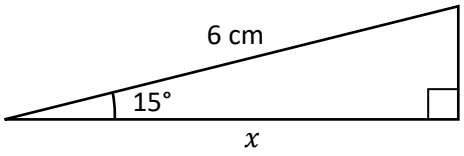
3

Ar gyfer y trionglau ongl sgwâr ganlynol, cyfrifwch hyd yr ochr sy'n cael ei labelu gyda'r llythyren x.

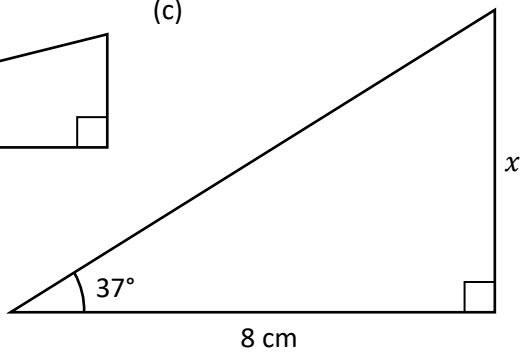
(a)



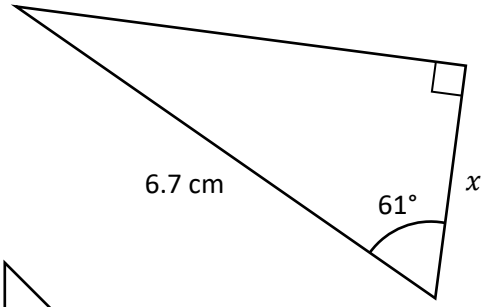
(b)



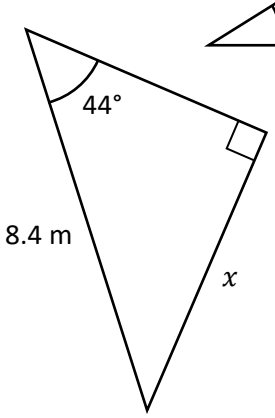
(c)



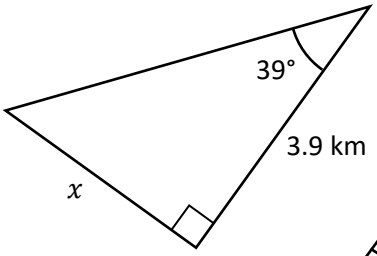
(ch)



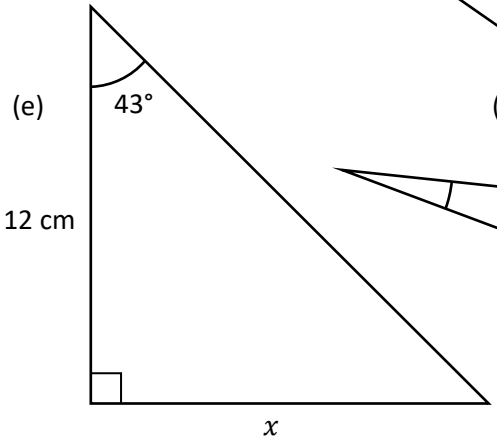
(d)



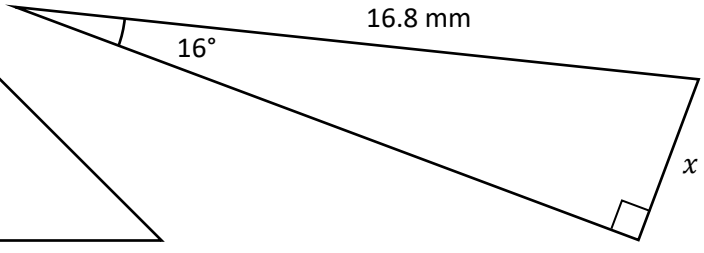
(dd)



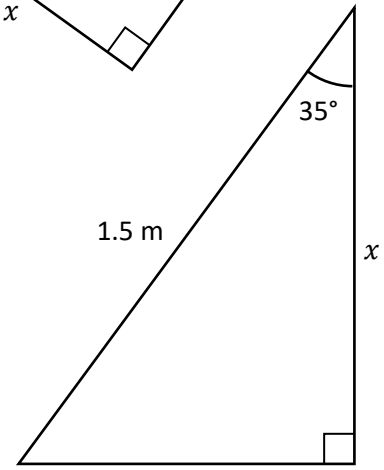
(e)



(f)

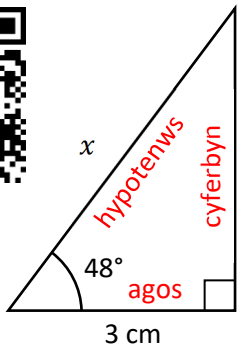


(ff)



Enghraifft

Ystyriwch y triongl ongl sgwâr sy'n cael ei ddangos ar y dde.
Gadewch i ni ddefnyddio trigonometreg er mwyn cyfrifo hyd yr ochr x.



I gychwyn, rhaid labelu'r triongl efo'r geiriau "cyferbyn", "agos" a "hypotenws".

Gwelwn ein bod yn gwybod hyd yr **agos** (3 cm), ac eisiau cyfrifo hyd yr **hypotenws** (x).
Y gymhareb trigonometrig sy'n cynnwys y geiriau **agos** a **hypotenws** yw cos,
felly rhaid defnyddio'r gymhareb

$$\cos\theta = \frac{\text{agos}}{\text{hypotenws}}$$

yn y cwestiwn yma. Wrth amnewid gwerthoedd i mewn i'r gymhareb, cawn

$$\cos 48° = \frac{3}{x}.$$

x ar **waelod** y ffracsiwn yn arwain at swm **rhannu** yn yr ateb.

Wrth luosi bob ochr yr hafaliad efo x, cawn

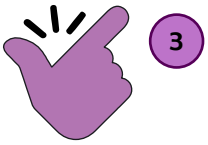
$$x \times \cos 48° = 3.$$

Wrth rannu bob ochr yr hafaliad efo $\cos 48°$, cawn

$$x = 3 \div \cos 48°.$$

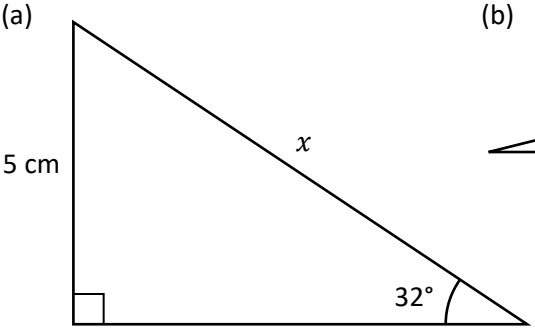
Wrth deipio'r swm yma ar gyfrifiannell, gallwn ddarganfod bod $x = 4.48$ cm, yn gywir i ddau le degol.

Ymarfer 5

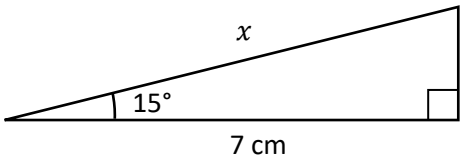


Ar gyfer y trionglau ongl sgwâr ganlynol, cyfrifwch hyd yr ochr sy'n cael ei labelu gyda'r llythyren x.

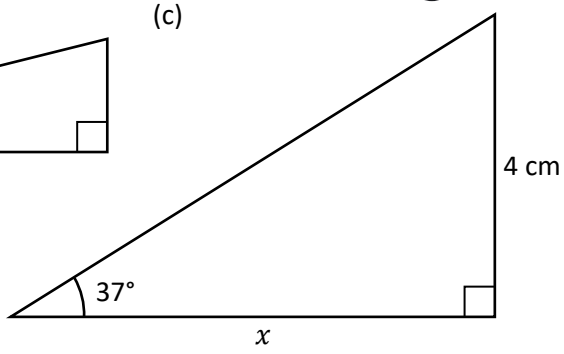
(a)



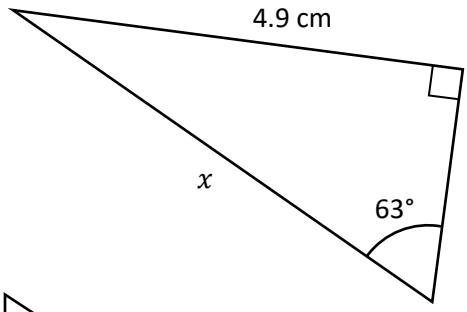
(b)



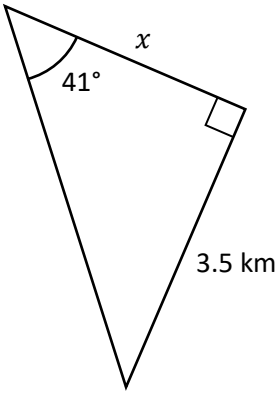
(c)



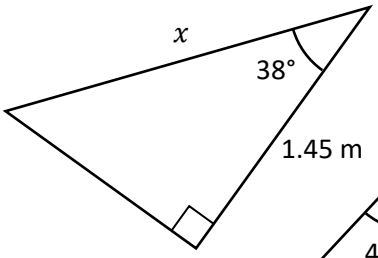
(ch)



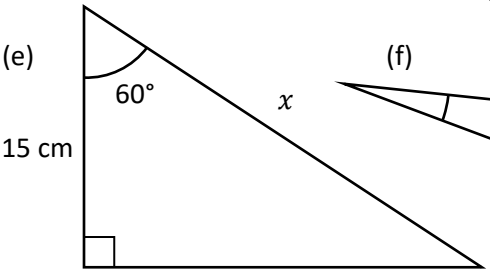
(d)



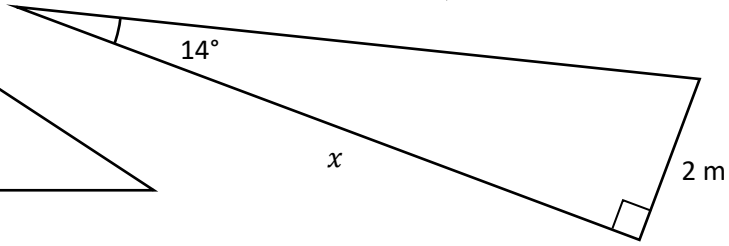
(dd)



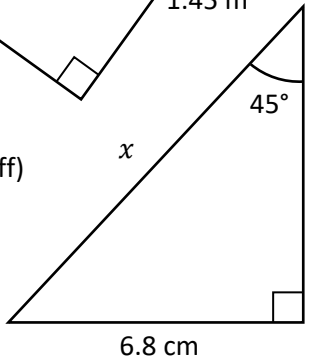
(e)



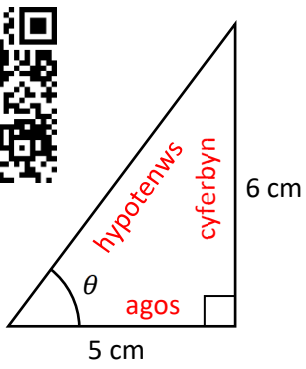
(f)



(ff)



Ffeindio onglau efo trigonometreg



Ystyriwch y triongl ongl sgwâr sy'n cael ei ddangos ar y dde.
Gadewch i ni ddefnyddio trigonometreg er mwyn cyfrifo maint yr ongl θ.

I gychwyn, rhaid labelu'r triongl efo'r geiriau "cyferbyn", "agos" a "hypotenws".

Gwelwn ein bod yn gwybod hyd y **cyferbyn** (6 cm) a hyd yr **agos** (5 cm).
Y gymhareb trigonometrig sy'n cynnwys y geiriau **cyferbyn** ag **agos** yw tan, felly rhaid defnyddio'r gymhareb

$$\tan\theta = \frac{\text{cyferbyn}}{\text{agos}}$$

yn y cwestiwn yma. Wrth amnewid gwerthoedd i mewn i'r gymhareb, cawn

$$\tan\theta = \frac{6}{5}.$$

Rhaid defnyddio'r botwm SHIFT er mwyn defnyddio'r ffwythiant gwrthdro.

Er mwyn darganfod maint θ rhaid defnyddio ffwythiant gwrthdro tan:

$$\theta = \tan^{-1}\left(\frac{6}{5}\right).$$

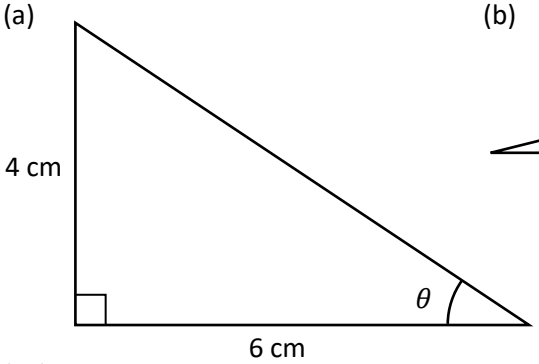
Wrth deipio'r swm yma ar gyfrifiannell, gallwn ddarganfod bod $\theta = 50.19°$, yn gywir i ddau le degol.

Ymarfer 6

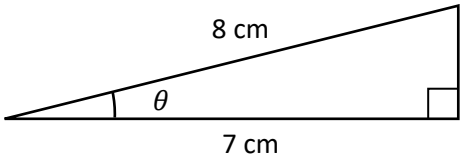
3

Ar gyfer y trionglau ongl sgwâr ganlynol, cyfrifwch faint yr ongl θ.

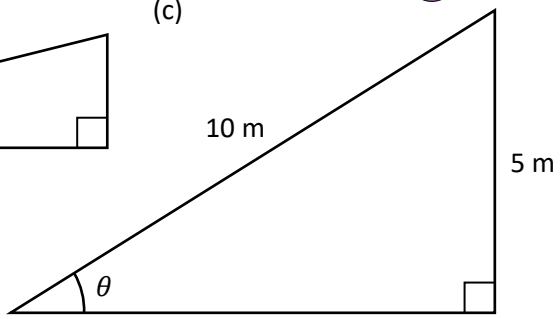
(a)



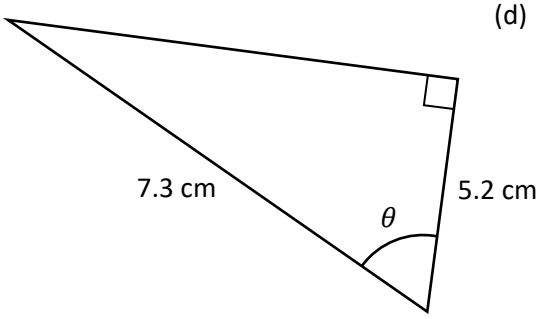
(b)



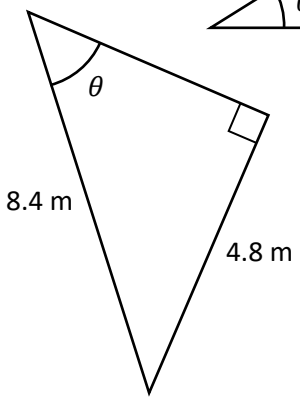
(c)



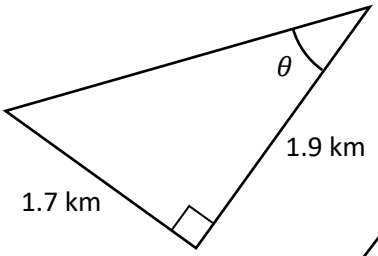
(ch)



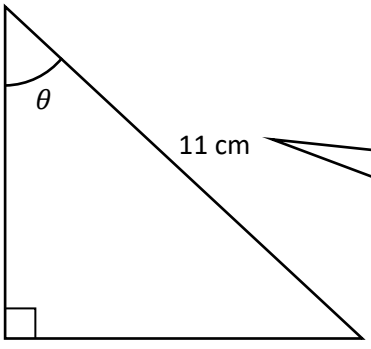
(d)



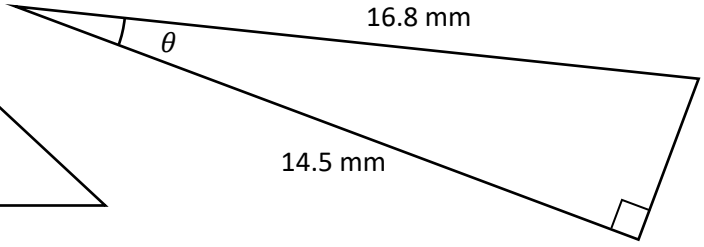
(dd)



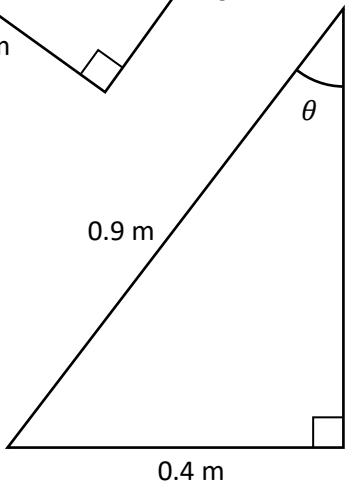
(e)



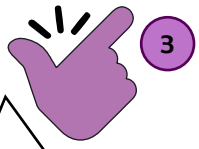
(f)



(ff)

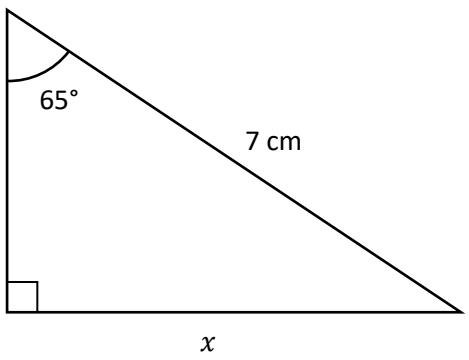


Ymarfer 7

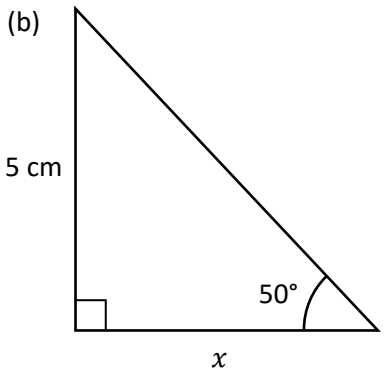


Ar gyfer y trionglau ongl sgwâr ganlynol, cyfrifwch hyd yr ochr x, neu faint yr ongl θ.
Rhowch eich atebion yn gywir i ddau le degol.

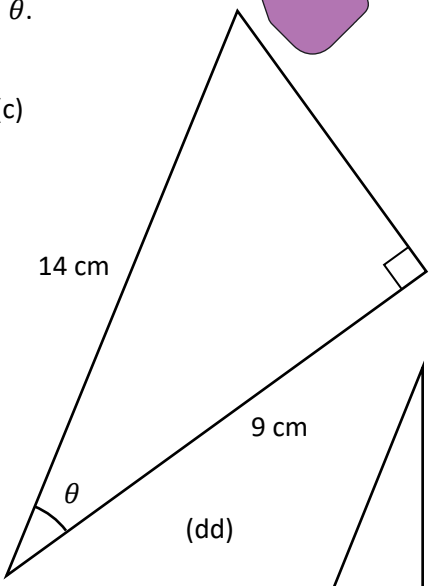
(a)



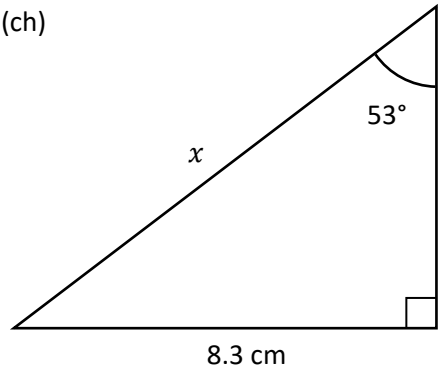
(b)



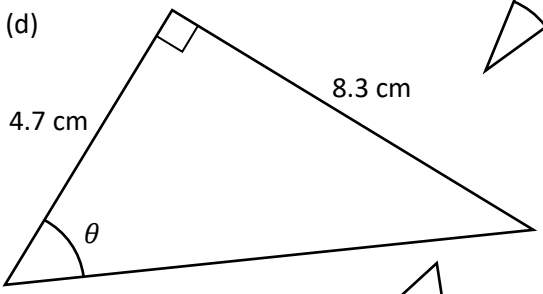
(c)



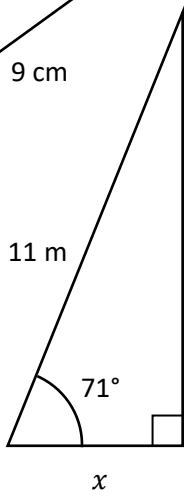
(ch)



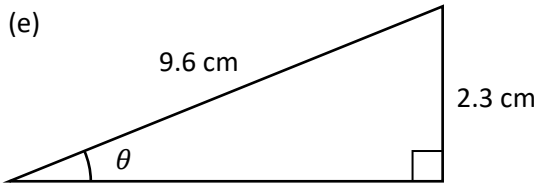
(d)



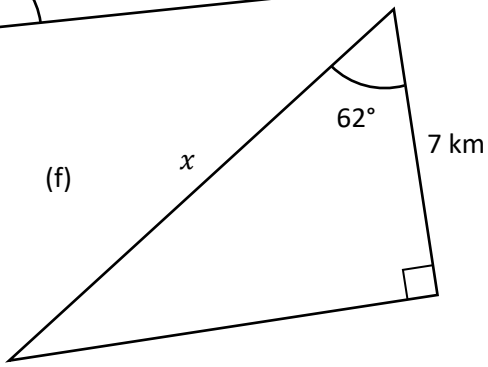
(dd)



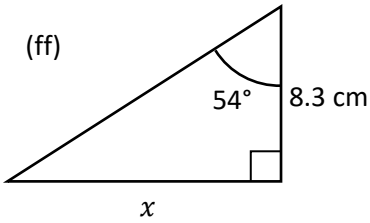
(e)



(f)



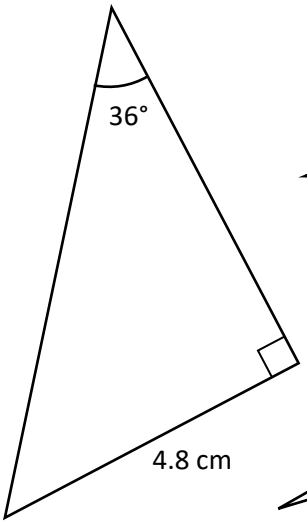
(ff)



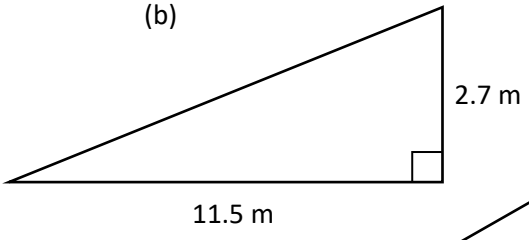
Ymarfer 8

Ar gyfer y trionglau ongl sgwâr ganlynol, darganfyddwch faint **pob** ongl goll a hyd **bob** ochr coll.

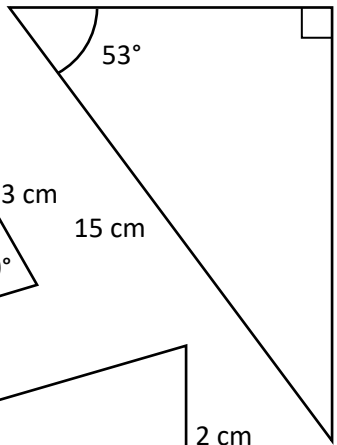
(a)



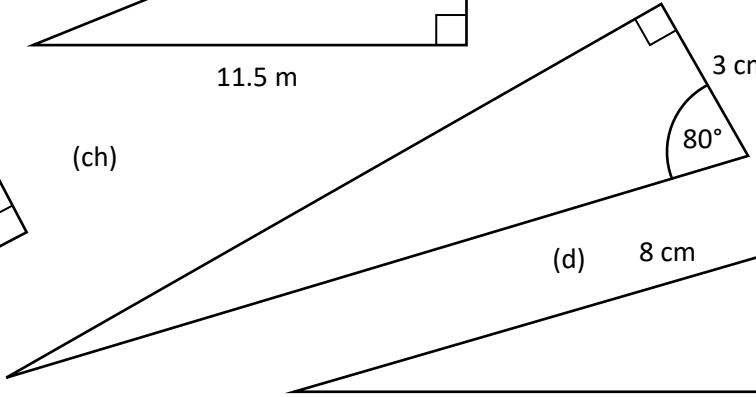
(b)



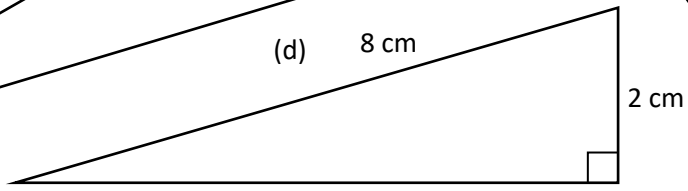
(c)



(ch)



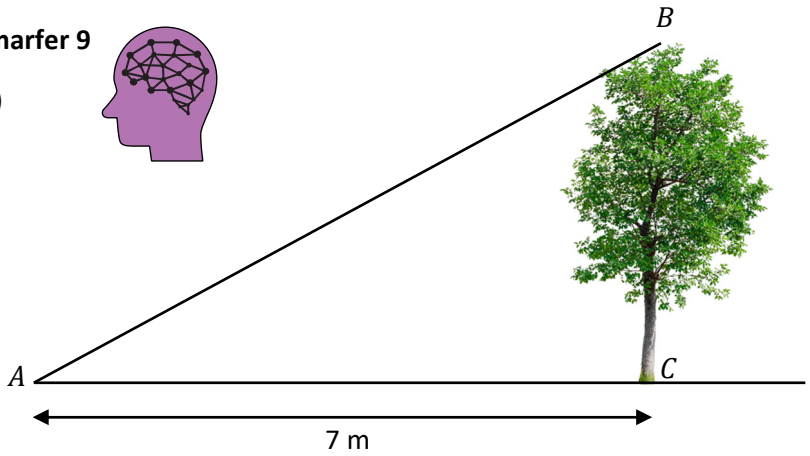
(d)



Ymarfer 9

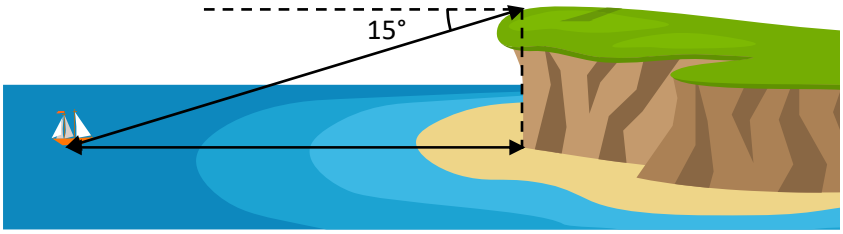
Defnyddio 3

(a)



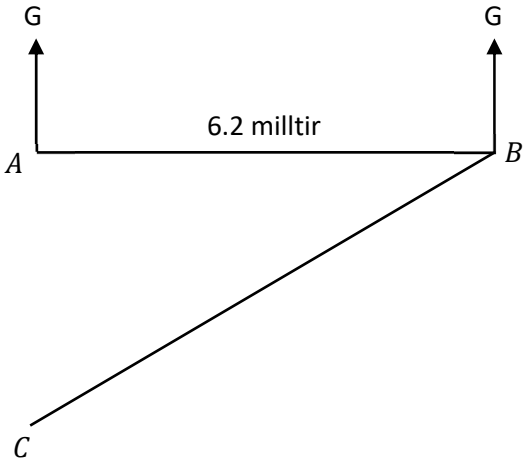
Uchder coeden fertigol yw 3.2 metr. Pellter llorweddol pwynt A o waelod y goeden yw 7 metr.
Cyfrifwch ongl godiad pen ucha'r goeden o'r pwynt A.

(b)



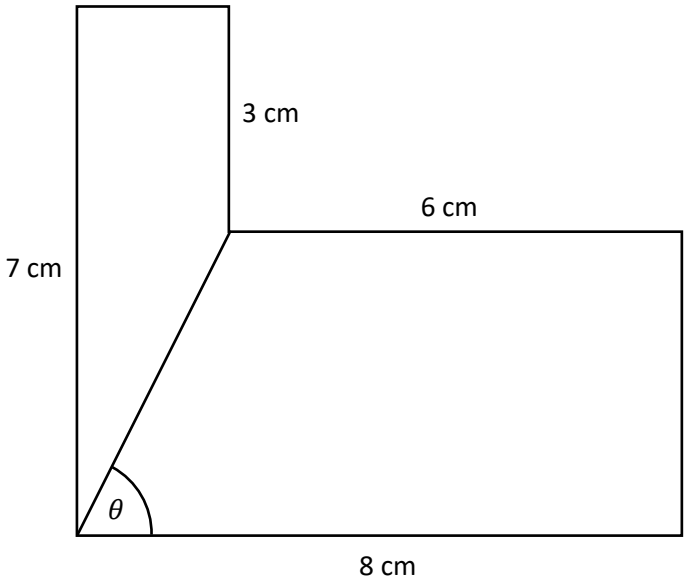
O ben uchaf clogwyn fertigol, ongl ostwng (*angle of depression*) cwch hwylio yw 15°. Os yw'r cwch hwylio 200 m o waelod y clogwyn, cyfrifwch uchder y clogwyn uwchlaw lefel y môr.

(c)



Mae llong yn gadael porthladd A ac yn hwylio 6.2 milltir ar gyfeiriant 090° i bwynt B. Yna mae'n troi ac yn hwylio ar gyfeiriant 224° nes iddi gyrraedd pwynt C, sydd i'r de o borthladd A. Cyfrifwch y pellter rhwng y pwynt C a'r porthladd A.

(ch)

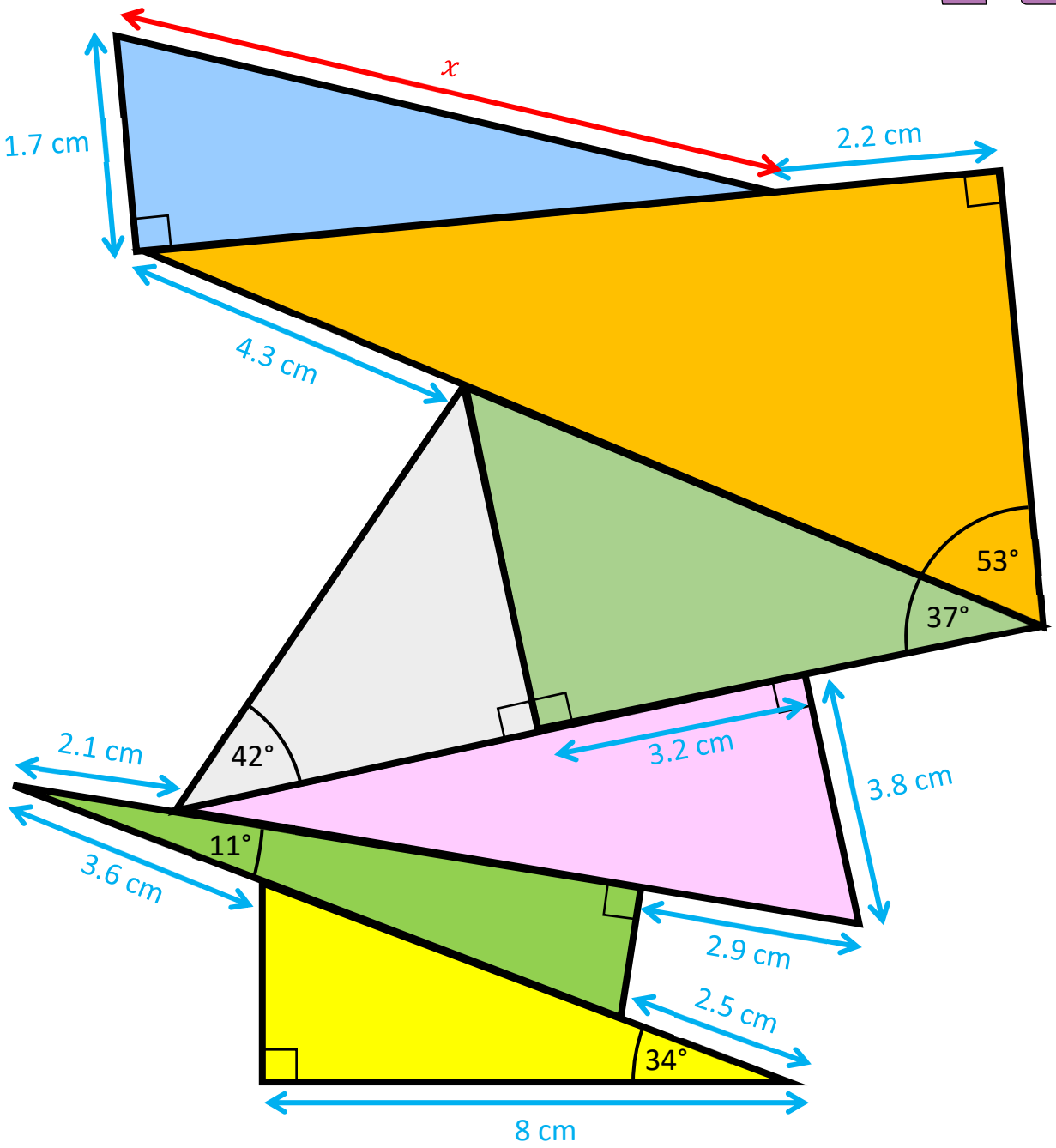


Mae'r diagram yn dangos hecsagon afreolaidd wedi'i hollti i mewn i ddau drapesiwm. Mae ochrau'r hecsagon naill ai'n fertigol neu'n llorweddol. Cyfrifwch faint yr ongl θ.

Mae'r gair trigonometreg yn dod o'r Groeg: "*trigon*" yn golygu triongl a "*metreg*" yn golygu mesur.

Ymarfer 10: Pentwr o Drionglau

Defnyddiwch Drigonometreg a Theorem Pythagoras i ddarganfod yr hyd x.



Gwerthuso

Geirfa Allweddol	Cywiriadau	Rwyf yn hapus efo...	Rwyf angen adolygu...

Helaethiad

Mae **helaethiad** yn un o'r pedwar trawsffurfiad.

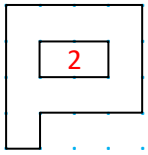
Blwyddyn 7	Blwyddyn 8	Blwyddyn 9	Blwyddyn 10
Trawsfudiad	Cylchdro	Adlewyrchiad	**Helaethiad**

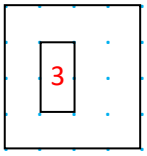
Pan fo siâp yn cael ei helaethu, mae **maint** y siâp yn newid. Y **ffactor graddfa** sy'n rheoli sut mae'r maint yn newid. Er enghraifft, os yw'r ffactor graddfa'n 2, yna bydd maint y siâp yn dyblu. Os yw'r ffactor graddfa'n $\frac{1}{2}$, yna bydd maint y siâp yn haneru.

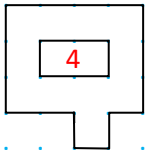
Sgil

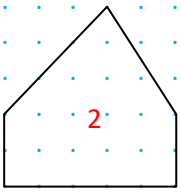
Ymarfer 11

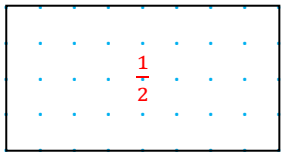
Helaethwch y siapiau canlynol gan ddefnyddio'r ffactor graddfa yng nghanol bob siâp. **2**

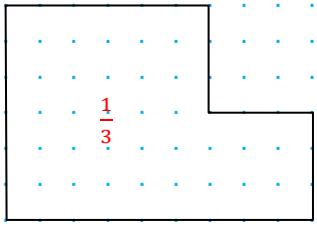






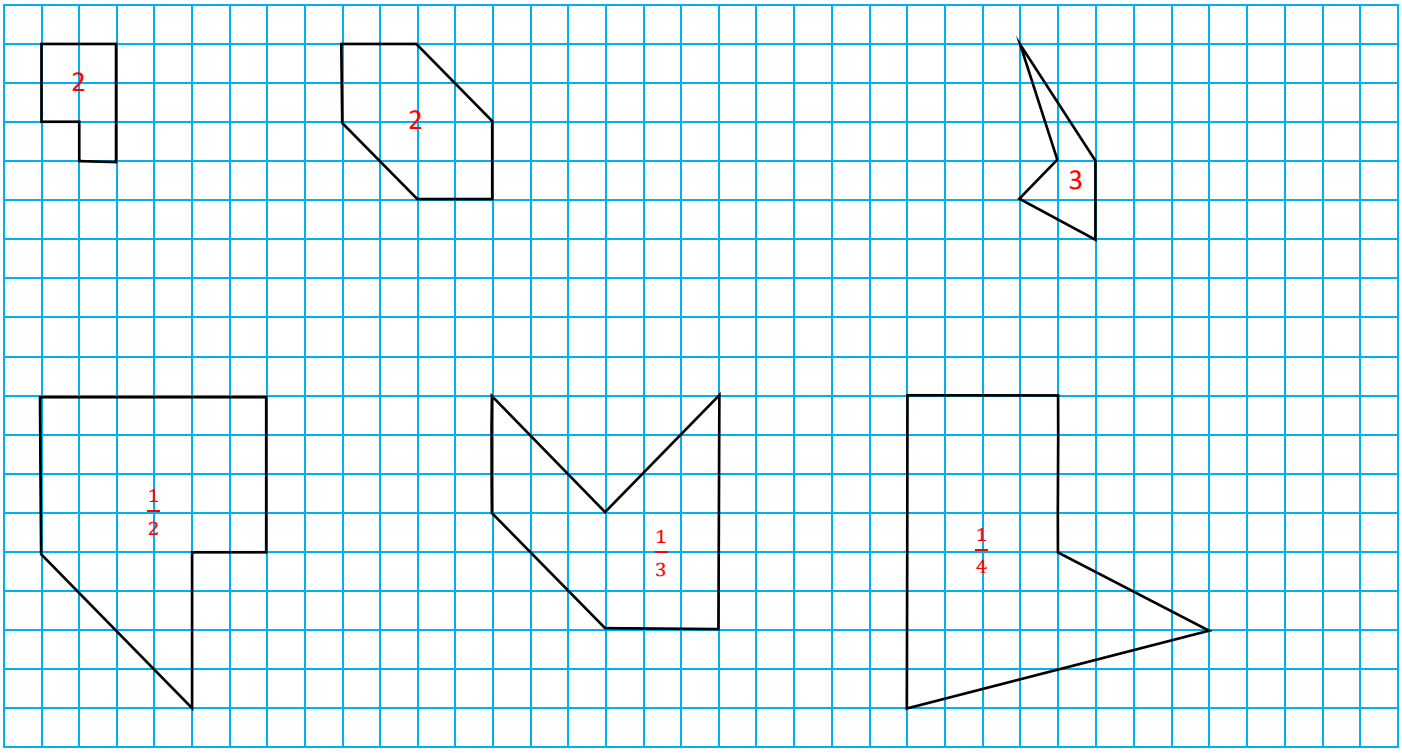






Ymarfer 12

Helaethwch y siapiau canlynol gan ddefnyddio'r ffactor graddfa yng nghanol bob siâp.

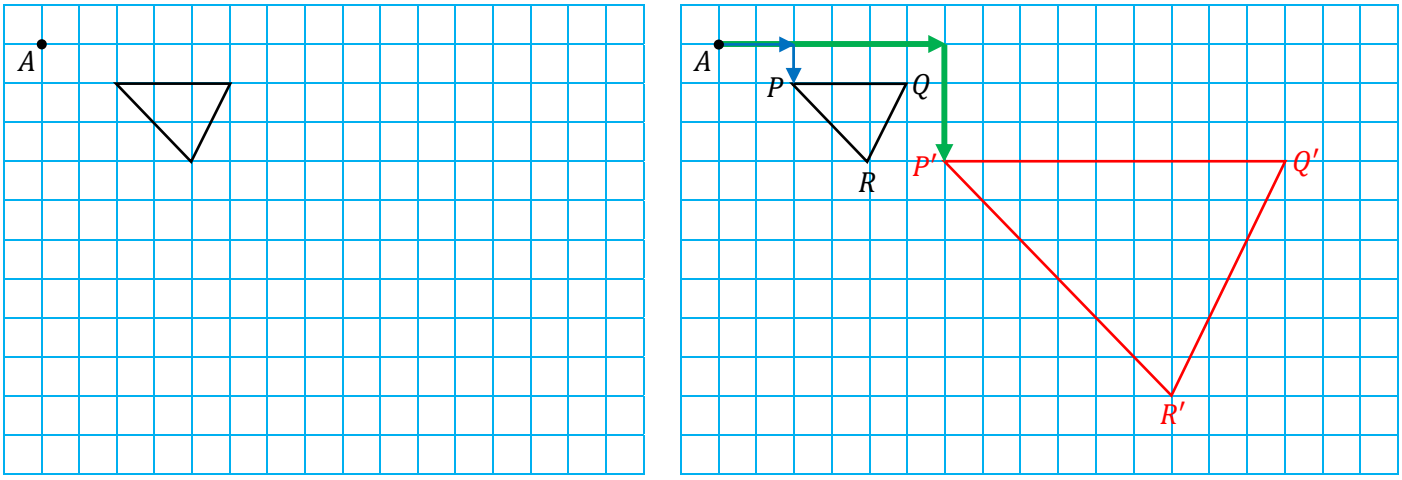


Canol Helaethiad

Os yw cwestiwn yn nodi pwynt fel **canol yr helaethiad**, yna mae'n rhaid i'r helaethiad ymddangos mewn lleoliad penodol (ni chaiff ymddangos yn rhywle fel yn ymarferion 11 ag 12).

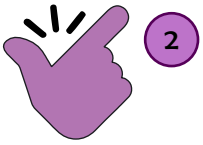
Enghraifft

Helaethwch y triongl isod gan ddefnyddio ffactor graddfa 3 a'r pwynt A fel canol yr helaethiad.

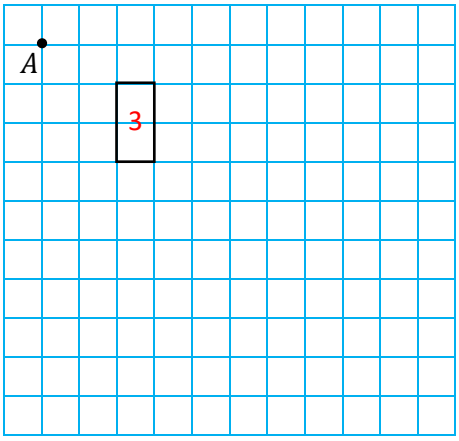


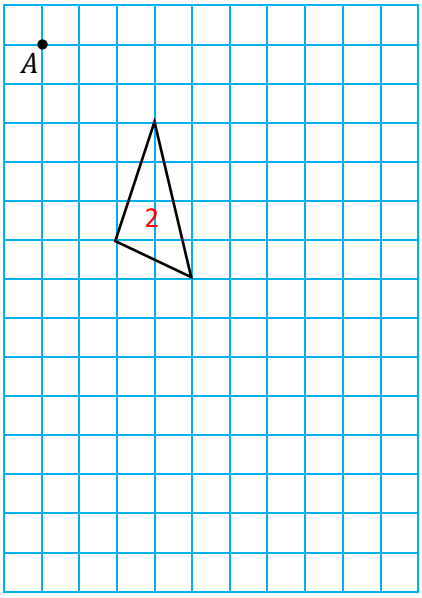
I fynd o'r pwynt A i fertig P y triongl gwreiddiol, rhaid mynd 2 uned i'r dde ag 1 uned i lawr. Gan fod y ffactor graddfa'n 3, i fynd o'r pwynt A i fertig P' y triongl newydd, rhaid mynd $2 \times 3 = 6$ uned i'r dde a $1 \times 3 = 3$ uned i lawr. Gellid ailadrodd hyn efo'r fertigau eraill (Q ag R), neu gellid cychwyn yn y fertig P' a llunio triongl sydd tair gwaith yn fwy.

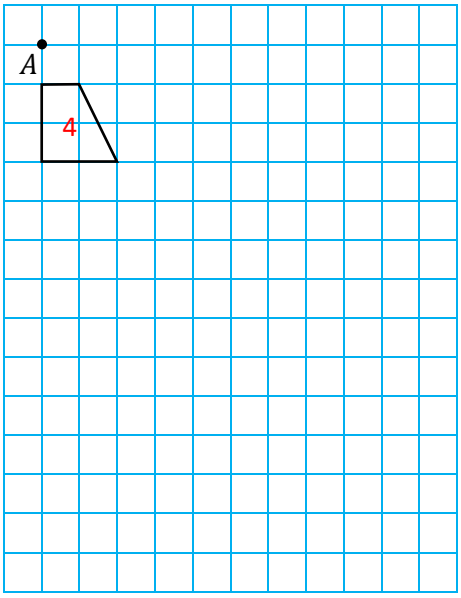
Ymarfer 13

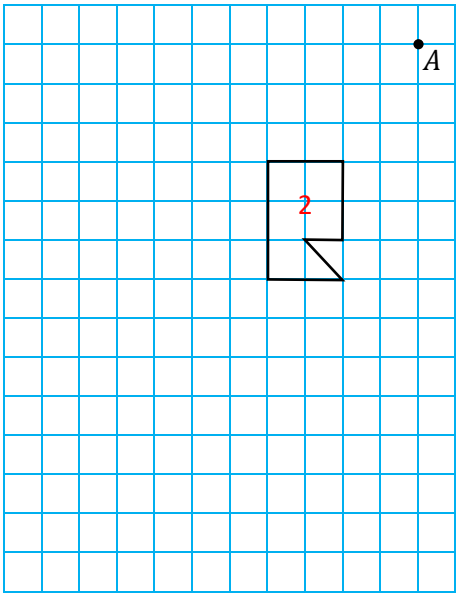


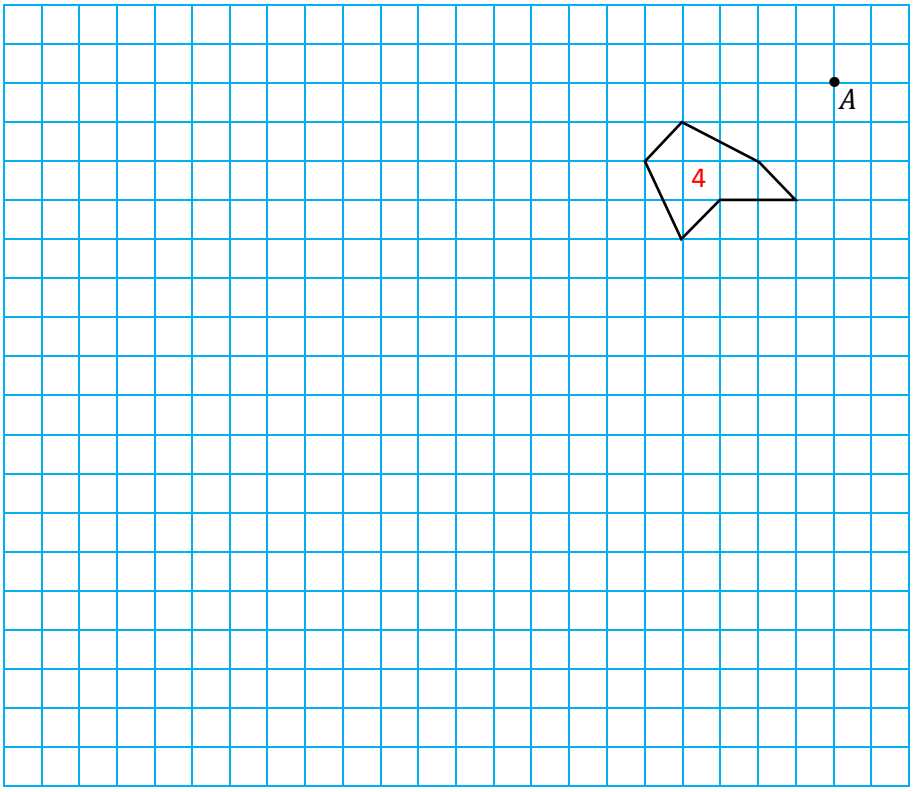
Helaethwch y siapiau canlynol gan ddefnyddio'r pwynt A fel canol yr helaethiad a'r rhif yng nghanol y siâp fel y ffactor graddfa.

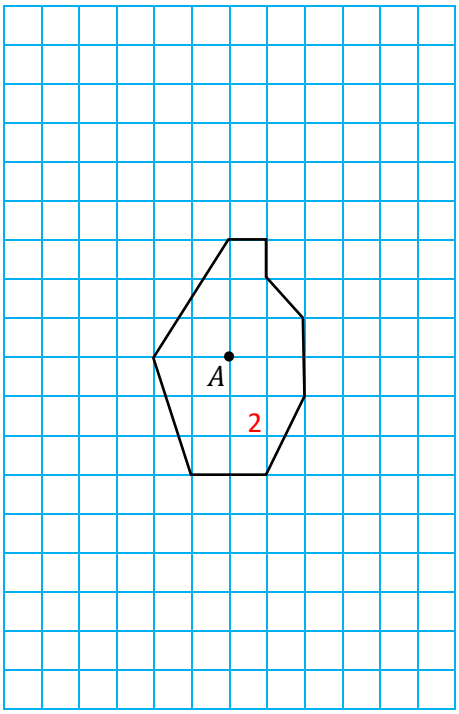


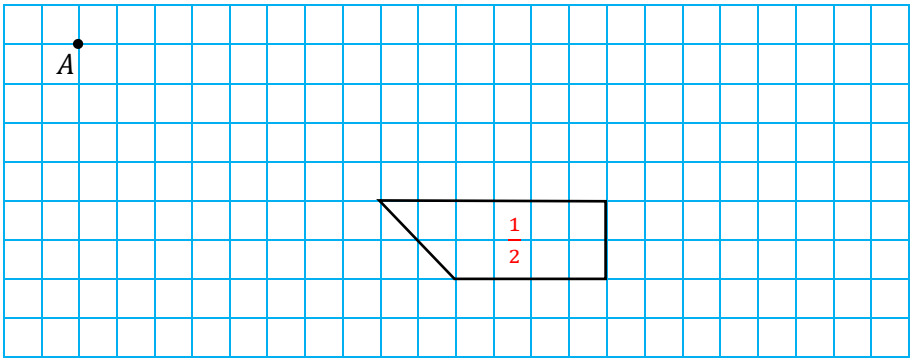




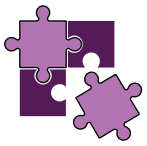




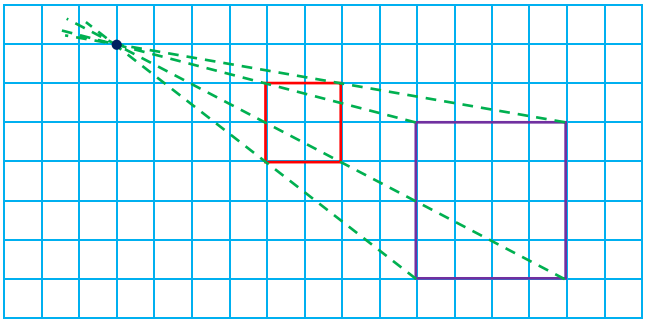




Darganfod canol helaethiad



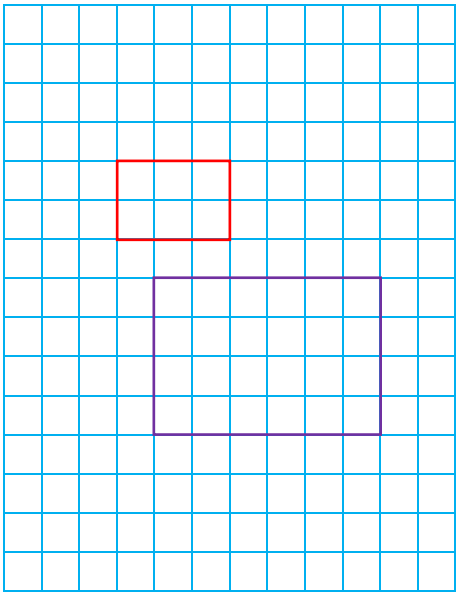
O gael siâp a'i helaethiad, cysylltwch y fertigau cyfatebol ac ymestynnwch y llinellau er mwyn darganfod ble mae canol yr helaethiad.

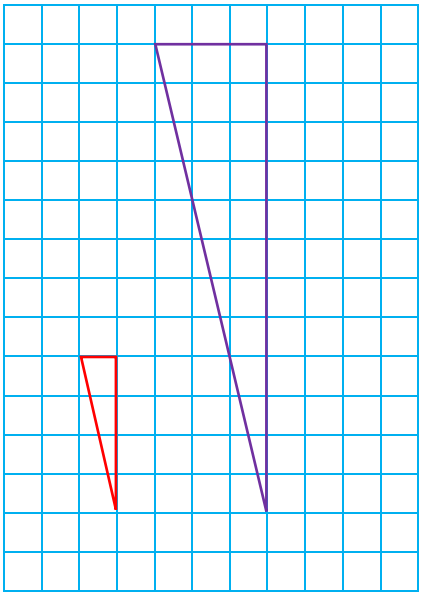


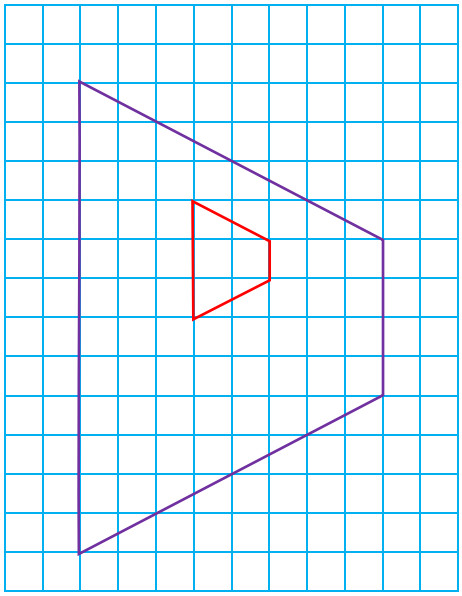
Gallwch ffeindio'r ffactor graddfa trwy gymharu maint y siapiau.

Ymarfer 14

Darganfyddwch y ffactor graddfa a chanol yr helaethiad ar gyfer yr helaethiadau canlynol. 2



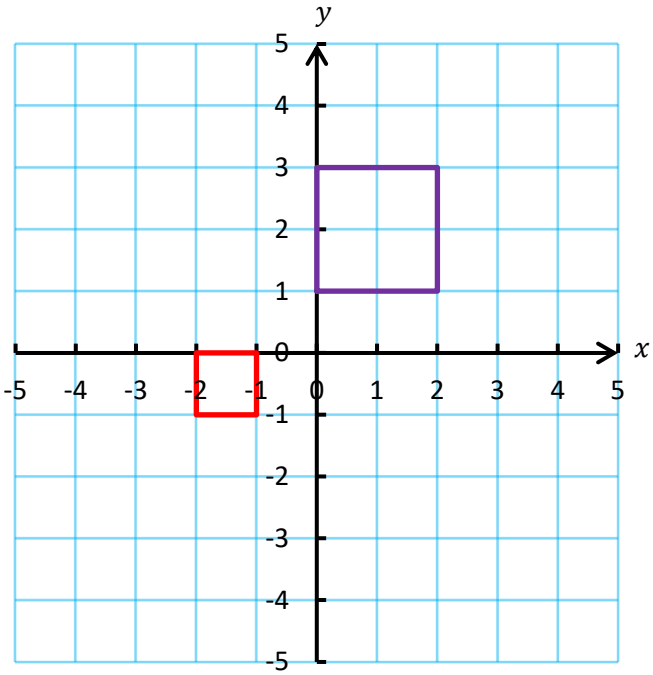




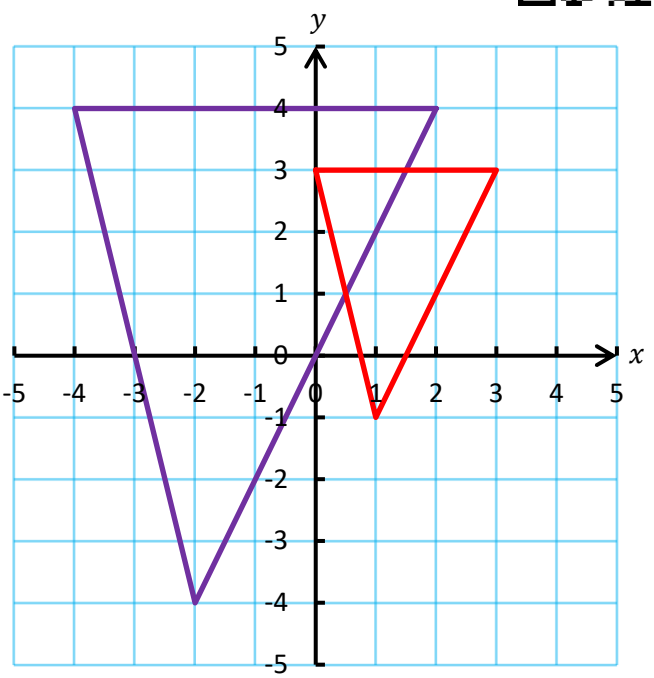
Ymarfer 15

Darganfyddwch y ffactor graddfa a chanol yr helaethiad ar gyfer yr helaethiadau canlynol.

(a)



(b)



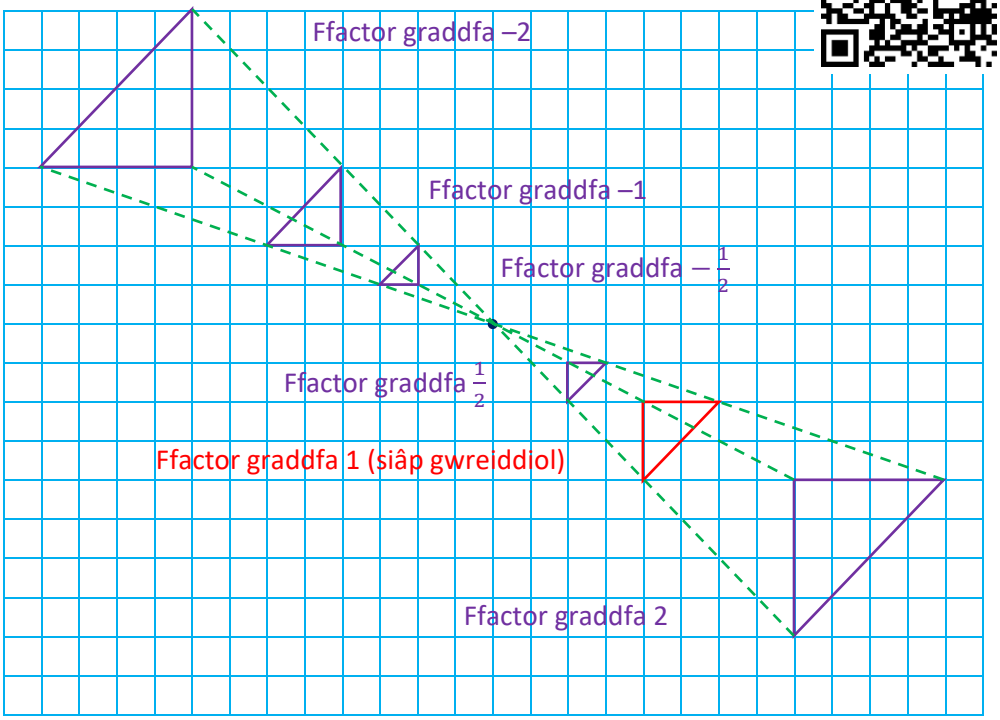
Ffactor Graddfa Negatif

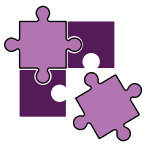
Mae ffactor graddfa **negatif** yn golygu gweithio o ganol yr helaethiad i'r cyfeiriad dirgroes (*opposite*).

Er enghraifft, ystyriwch y diagram ar y dde. I fynd o ganol yr helaethiad i fertig top chwith y triongl gwreiddiol, rhaid mynd 4 uned i'r dde a 2 uned i lawr.

Gyda ffactor graddfa 2, rhaid mynd 4 × 2 = 8 uned i'r dde a 2 × 2 = 4 uned i lawr.

Gyda ffactor graddfa –2, rhaid mynd 4 × 2 = 8 uned i'r **chwith** a 2 × 2 = 4 uned i **fyny**.

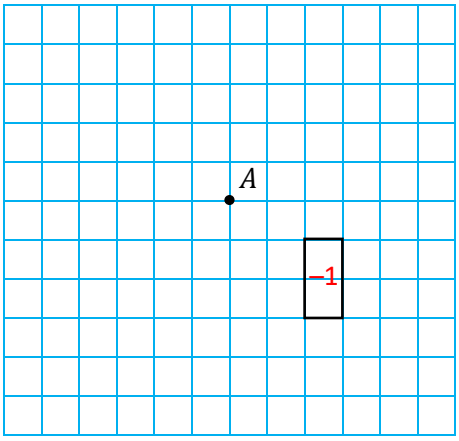


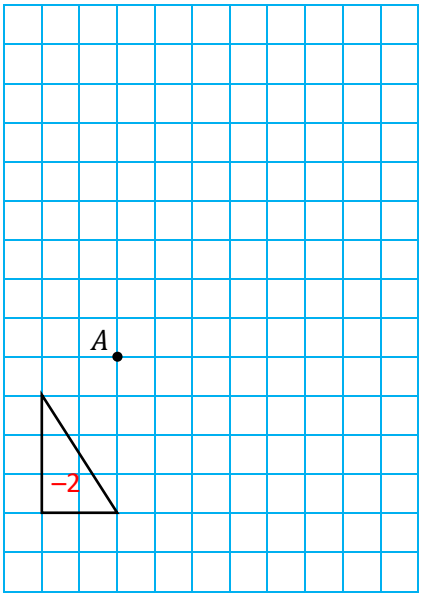


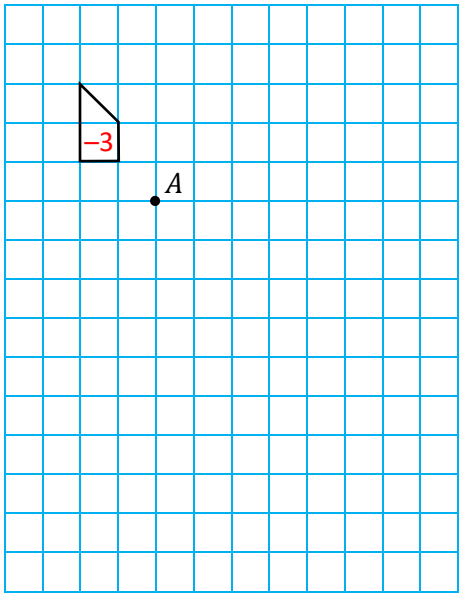
Ymarfer 16

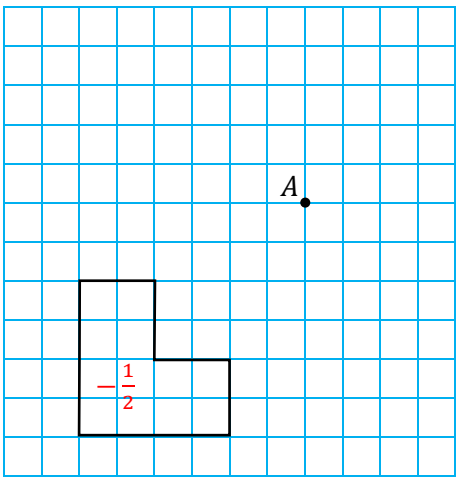
2

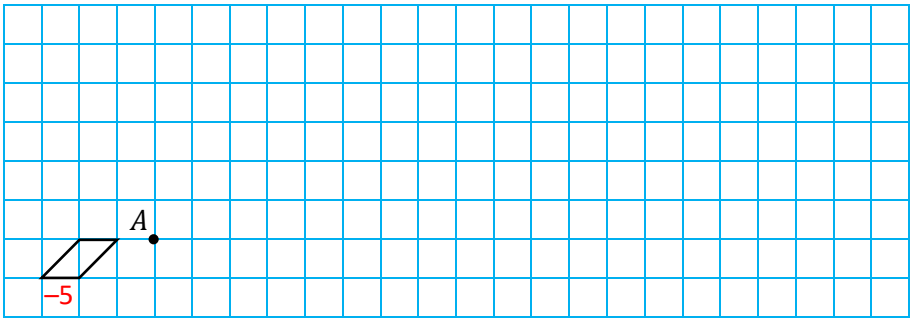
Helaethwch y siapiau canlynol gan ddefnyddio'r pwynt A fel canol yr helaethiad a'r rhif yng nghanol y siâp fel y ffactor graddfa.







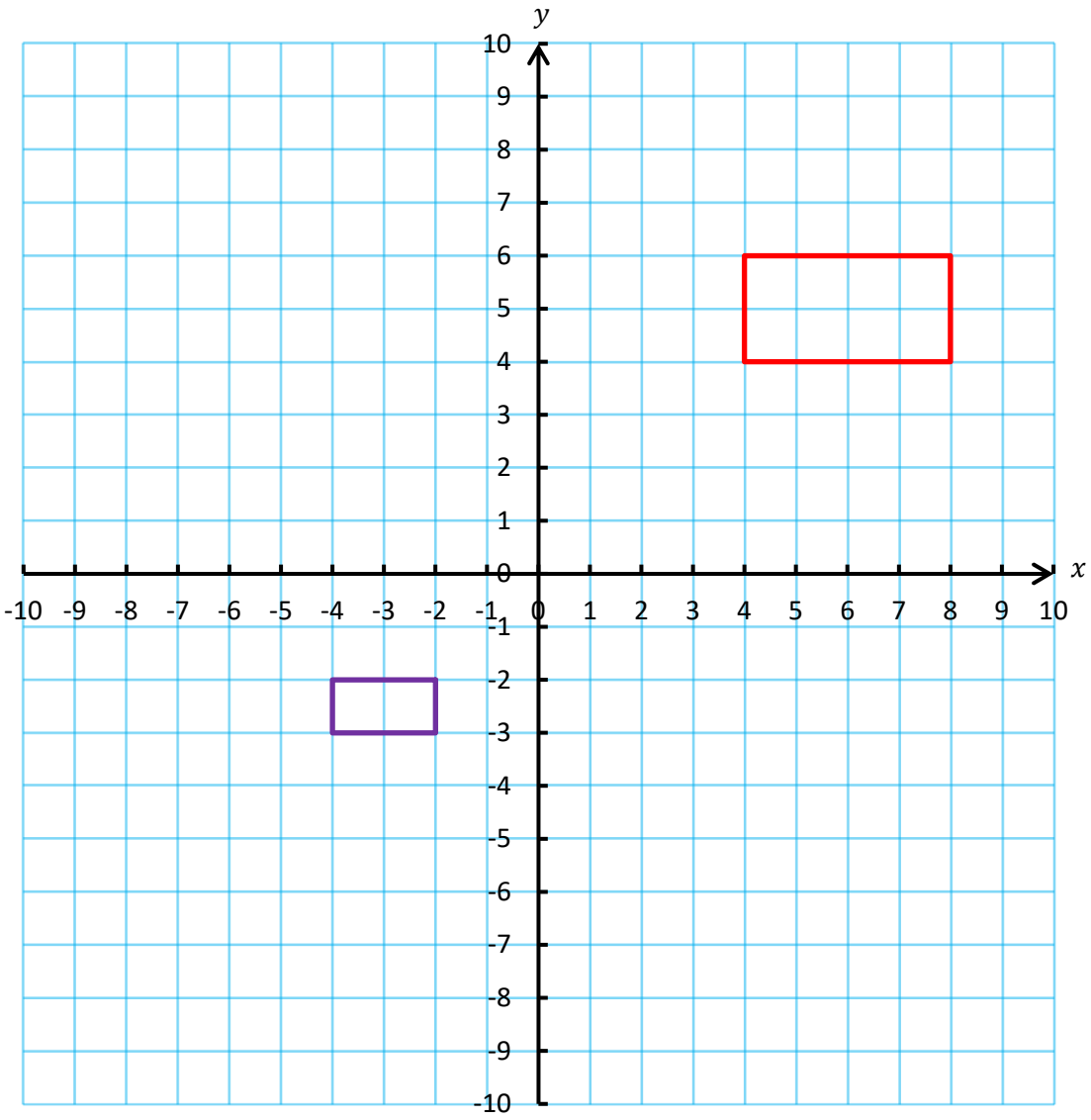




Ymarfer 17 2

Mae'r petryal mwyaf yn cael ei drawsffurfio i'r petryal lleiaf. Cyfesurynnau canol yr helaethiad yw (0, 0). Cwblhewch y frawddeg ganlynol i ddisgrifio'r trawsffurfiad hwn yn llawn.

Mae trawsffurfiad y petryal mwyaf i'r petryal lleiaf yn helaethiad â ffactor graddfa _____ a chanol (0, 0).



Gwerthuso

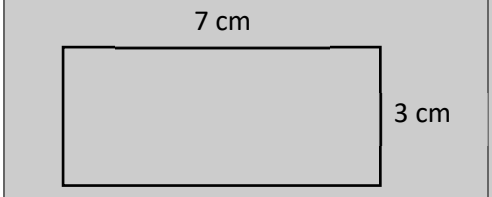
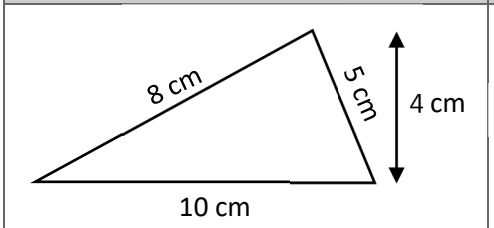
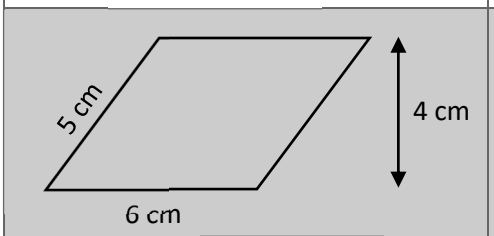
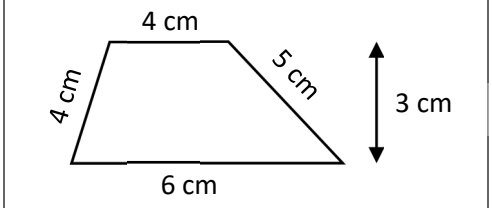
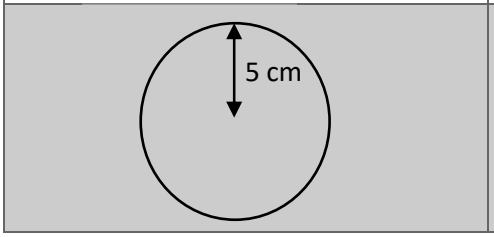
Geirfa Allweddol	Cywiriadau	Rwyf yn hapus efo...	Rwyf angen adolygu...

Siapiau Cyfansawdd

Adolygu 13

Ymarfer 18

Cwblhewch y tabl canlynol.

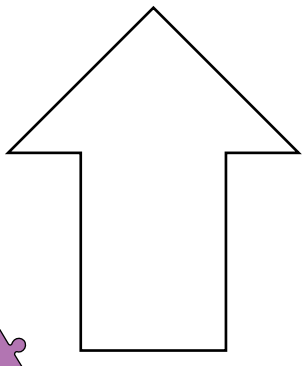
Siâp	Enw'r Siâp	Fformiwla i ffeindio arwynebedd y siâp	Cyfrifo arwynebedd y siâp
			
			
			
			
			

Ymarfer 19

Cyfrifwch berimedr bob un o'r siapiau yn y tabl uchod.

Siapiau Cyfansawdd

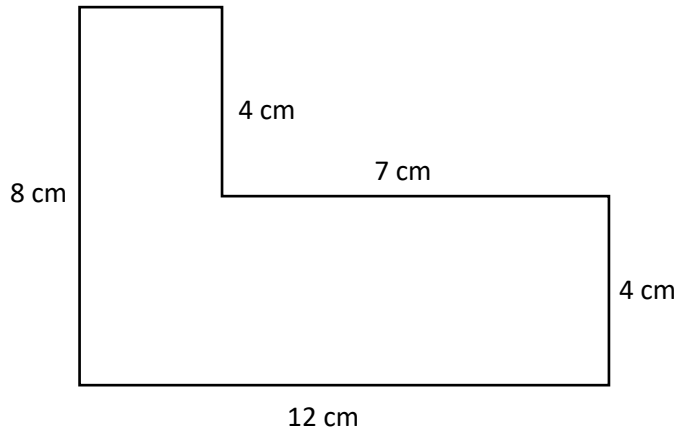
Mae **siâp cyfansawdd** yn siâp y mae'n bosib ei hollti i mewn i siapiau mwy syml, fel y rhai yn y tabl uchod. Er enghraifft, mae'r siâp ar y dde yn siâp cyfansawdd – mae'n bosib ei hollti i mewn i betryal (ar y gwaelod) a thriongl (ar y top). Byddai'n bosib cyfrifo arwynebedd y siâp cyfansawdd trwy adio arwynebedd y petryal a'r triongl.



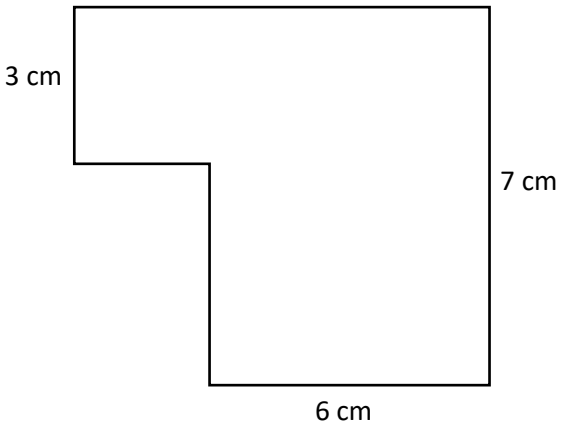
Ymarfer 20

Cyfrifwch arwynebedd bob un o'r siapiau cyfansawdd canlynol.

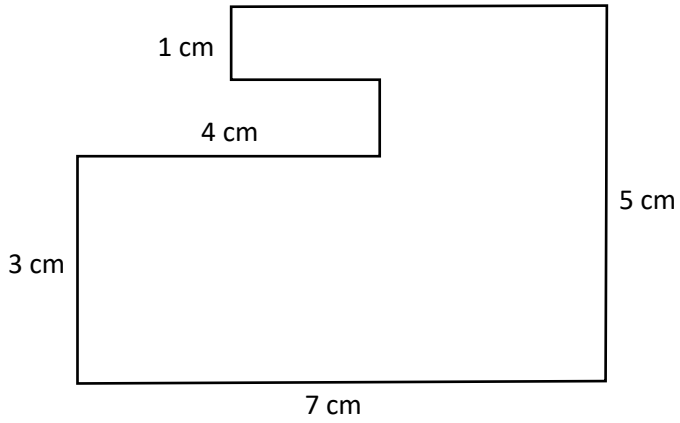
(a)



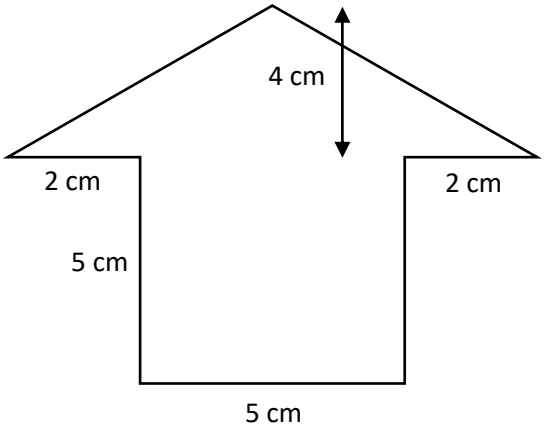
(b)



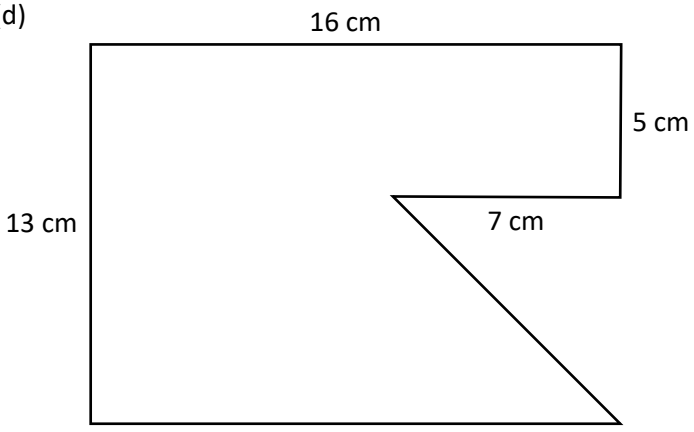
(c)



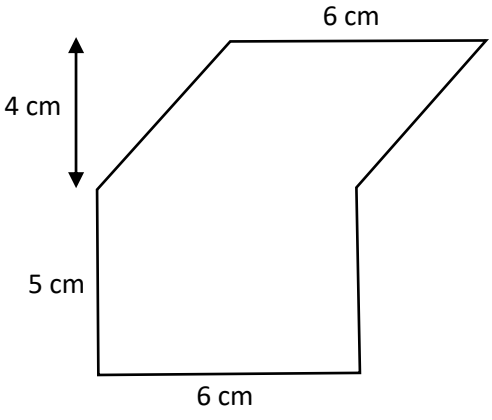
(ch)



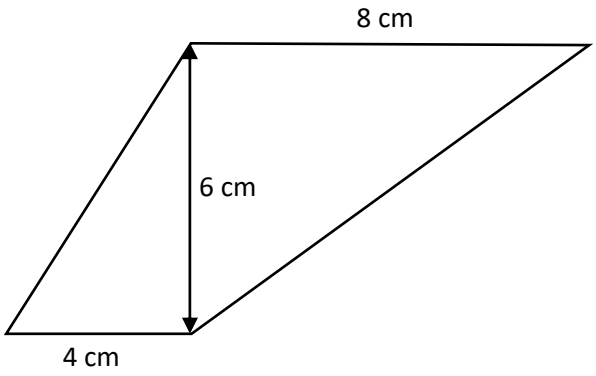
(d)



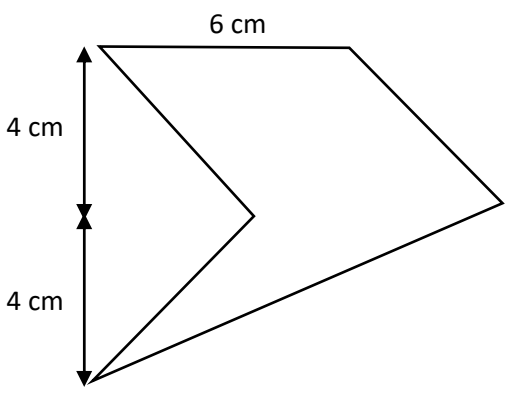
(dd)



(e)



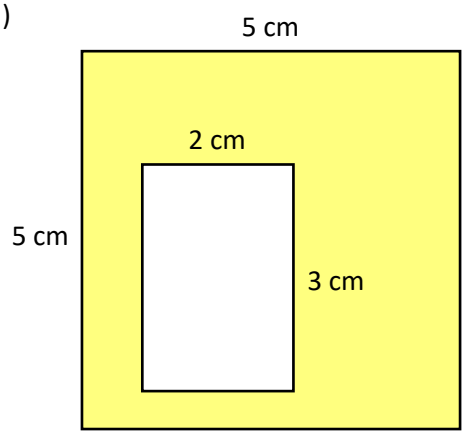
(f)



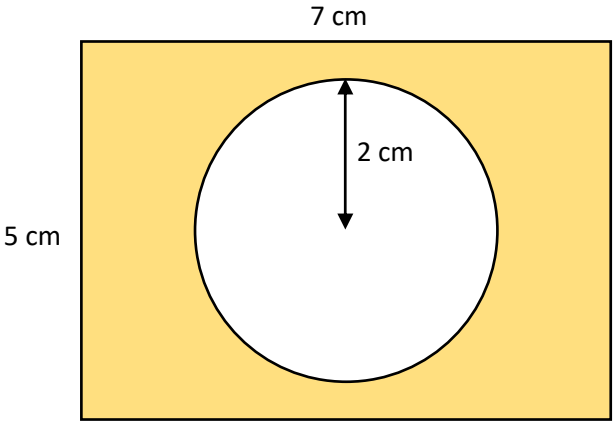
Ymarfer 21 13

Cyfrifwch arwynebedd y darn lliw.

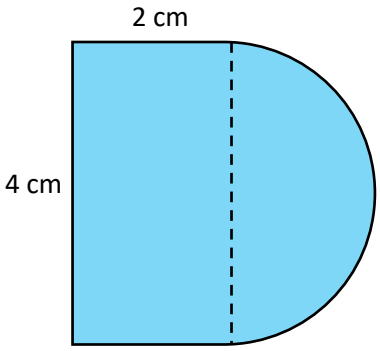
(a)



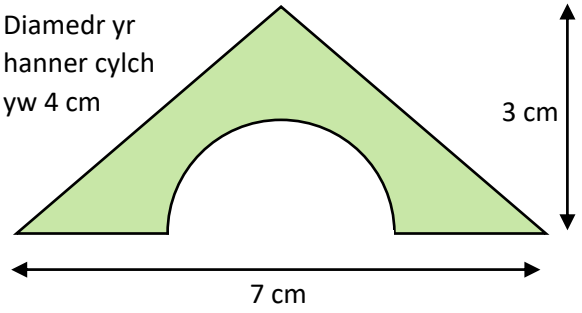
(b)



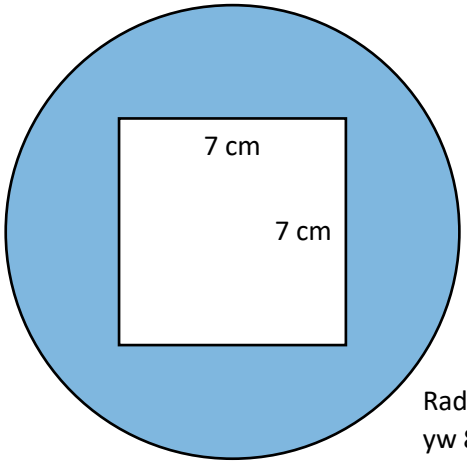
(c)



(ch)

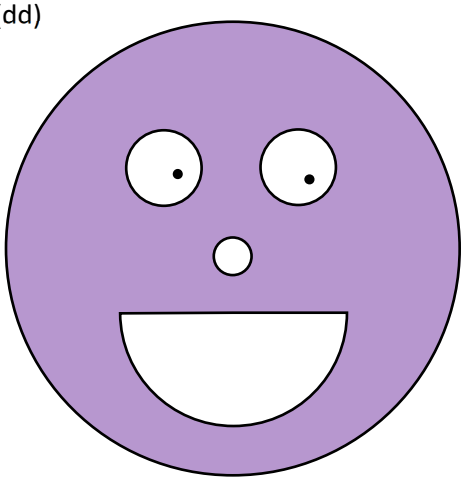


(d)



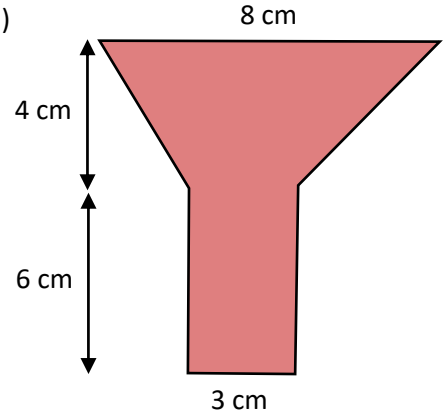
Radiws y cylch yw 8 cm

(dd)

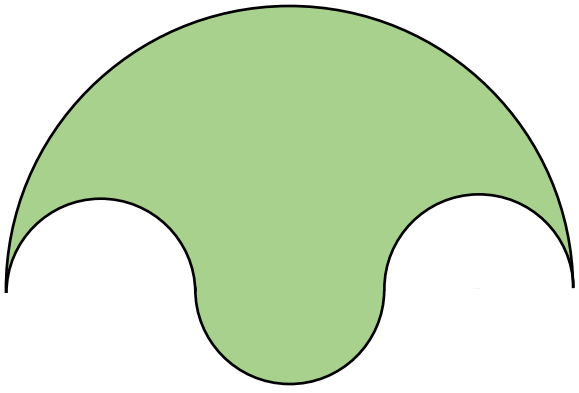


Cylch	Radiws
Wyneb	6 cm
Llygad	1 cm
Trwyn	0.5 cm
Ceg	3 cm

(e)



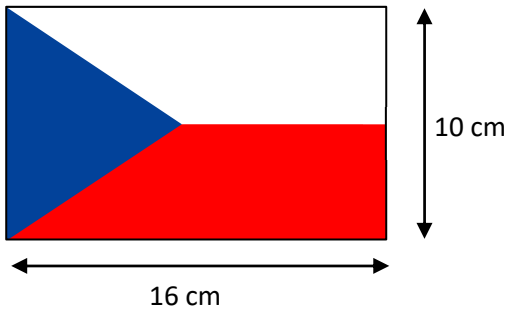
(f)



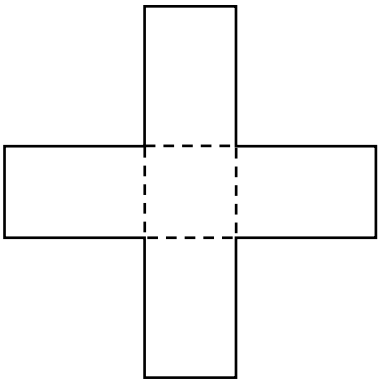
Cylch	Radiws
Mawr (1)	7.5 cm
Bach (3)	2.5 cm

Ymarfer 22

(a) Mae fflag Y Weriniaeth Tsiec yn cael ei ddangos ar y dde.
O wybod bod fertig y triongl glas yng nghanol y fflag, cyfrifwch arwynebedd y rhanbarth coch.



(b) Mae patrwm sydd â siâp croes yn cael ei wneud drwy drefnu pedwar petryal unfath (*identical*) o amgylch ochrau sgwâr, fel sy'n cael ei ddangos yn y diagram ar y dde. (Nid yw'r diagram wedi'i luniadu wrth raddfa.)

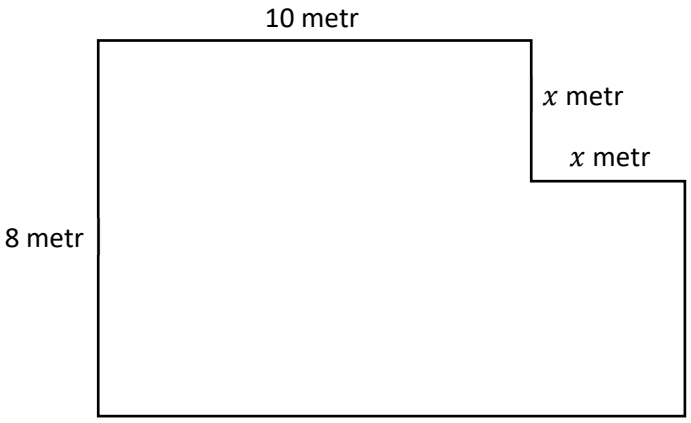


Arwynebedd y sgwâr yw 36 cm². Mae arwynebedd pob petryal unwaith a hanner cymaint ag (*one and a half times*) arwynebedd y sgwâr.

(i) Darganfyddwch arwynebedd y patrwm sydd â siâp croes.
(ii) Darganfyddwch berimedr y patrwm sydd â siâp croes.

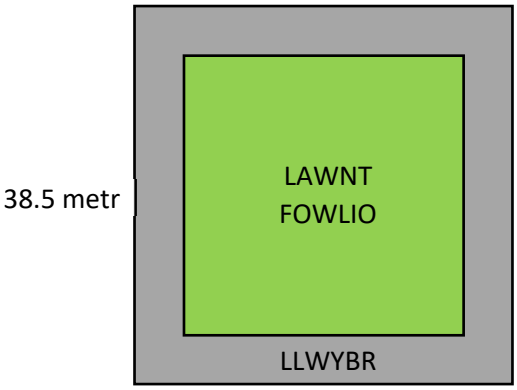
(c) Mae Gerry wedi braslunio cynllun llawr ystafell. Mae pob un o gorneli'r ystafell naill ai'n 90° neu'n 270°. Mae Gerry wedi anghofio rhai o'r mesuriadau ond mae hi'n gwybod bod dau o'r mesuriadau yr un fath, ac felly mae hi wedi labelu y ddau ohonyn nhw yn x metr.

Dangoswch mai arwynebedd y llawr, mewn metrau sgwâr, yw $80 + 8x - x^2$.



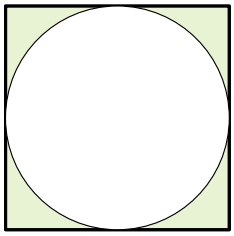
(ch) Penderfynodd y cyngor lleol osod llwybr concrit â'i led yn $1\frac{1}{2}$ metr o amgylch eu lawnt fowlio sgwâr (*square bowling green*), fel sy'n cael ei ddangos yn y diagram.

Cyfrifwch arwynebedd y llwybr concrit, mewn metrau sgwâr.



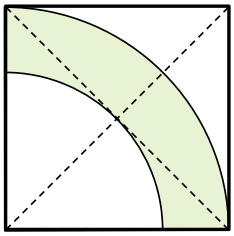
Sialens!

Pa ffracsiwn o'r sgwâr sydd wedi'i dywyllu?



Sialens 2!

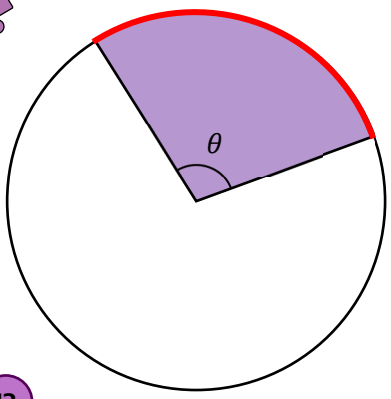
Pa ffracsiwn o'r sgwâr sydd wedi'i dywyllu?



Hyd Arc ac Arwynebedd Sector

Mae hyd arc cylch yn ffracsiwn o gylchedd y cylch, tra bod arwynebedd sector cylch yn ffracsiwn o arwynebedd y cylch.

Hyd Arc = $\frac{\theta}{360°} \times$ cylchedd y cylch Arwynebedd Sector = $\frac{\theta}{360°} \times$ arwynebedd y cylch



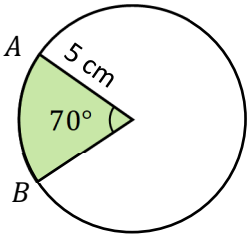
Hyd Arc = $\frac{\theta}{360°} \times \pi \times$ diamedr

Arwynebedd Sector = $\frac{\theta}{360°} \times \pi \times$ radiws²

Ymarfer 23

13

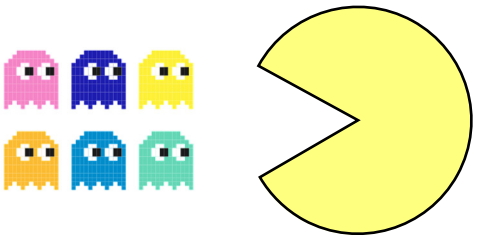
Ar gyfer y cylch a ddangosir ar y dde,



(a) Cyfrifwch hyd yr arc leiaf AB.
(b) Cyfrifwch arwynebedd y sector leiaf AB.
(c) Cyfrifwch hyd yr arc fwyaf AB.
(ch) Cyfrifwch arwynebedd y sector fwyaf AB.
(d) Pa ffracsiwn o'r cylch sydd wedi'i liwio'n wyrdd? Rhowch eich ateb ar ei ffurf symlaf.

Ymarfer 24

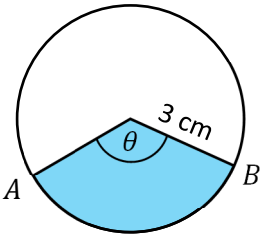
Siâp sector sydd gan Pacman, sef cymeriad o'r byd gemau cyfrifiadurol. Yr ongl yng nghanol y siâp yw 300°. Os yw radiws Pacman yn 6 cm,



(a) Beth yw arwynebedd Pacman?
(b) Beth yw perimedr Pacman?

Ymarfer 25

Hyd yr arc leiaf AB yn y diagram ar y dde yw 7 cm.



(a) Beth yw maint yr ongl θ?
(b) Cyfrifwch arwynebedd y sector las.

Gwerthuso

Geirfa Allweddol	Cywiriadau	Rwyf yn hapus efo...	Rwyf angen adolygu...

Mesur

Siapiau 3

Myfyrio

Enw: .....

Canran yn y prawf: .....

	Yn gwybod y gwaith?	Angen adolygu?	Cwestiwn yn y prawf	Yn gywir yn y prawf?
Rwy'n gallu adnabod **y cyferbyn, yr agos a'r hypotenws** mewn triongl ongl sgwâr.			1, 2	
Rwy'n gallu **cyfrifo hydoedd** mewn triongl ongl sgwâr gan ddefnyddio **trigonometreg**.			1	
Rwy'n gallu **cyfrifo onglau** mewn triongl ongl sgwâr gan ddefnyddio **trigonometreg**.			2	
Rwy'n gallu **cyfrifo hydoedd** mewn triongl ongl sgwâr gan ddefnyddio **Theorem Pythagoras**.				
Rwy'n gallu **helaethu** siâp gan ddefnyddio ffactor graddfa sy'n **gyfanrif positif**.			3	
Rwy'n gallu **helaethu** siâp gan ddefnyddio ffactor graddfa sy'n **ffracsiwn**.			5	
Rwy'n gwybod sut i ddefnyddio **canol helaethiad** wrth helaethu siâp.			4, 5	
O gael siâp a'i helaethiad, rwy'n gallu **darganfod y ffactor graddfa a chanol yr helaethiad**.			6	
Rwy'n gallu **helaethu** siâp gn ddefnyddio ffactor graddfa sy'n **negatif**.			11	
Rwy'n gallu cyfrifo **perimedr siâp cyfansawdd**.			7, 8	
Rwy'n gallu cyfrifo **arwynebedd siâp cyfansawdd**.			9, 10	
Rwy'n gallu cyfrifo **hyd arc cylch**.			12	
Rwy'n gallu cyfrifo **arwynebedd sector cylch**.			12	