



The Mathematics Department

11

The End of

Year 11

Name:

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Trial and Improvement

If we cannot solve an equation by algebraic methods, then we can attempt to solve the equation using **trial and improvement**.

In this method, we start with a sensible guess for the solution (a **trial**), and then **improve** this guess, repeating until we find the solution to a **specified degree of accuracy**.

Example

Let us attempt to solve the equation

$$x^3 + 4x - 29 = 0$$



As we have not seen an equation of this type before (and therefore we do not have an algebraic method of solving the equation), let us use the trial and improvement method to solve the equation. We will find the solution correct to the nearest unit and allow ourselves to use a calculator to perform the calculations.

To begin, let us **guess** that $x = 5$ is the solution to the equation. To see whether this is correct, we need to substitute $x = 5$ into the left-hand side of the equation, and check to see whether we obtain zero.

Buttons to press (Casio fx-83GT CW)	Calculator screen

The sum gives the answer 116, which is too **too high** (116 is greater than zero). Therefore, we must trial a number that is less than 5. Let us try $x = 3$.

Buttons to press	Calculator screen

The answer is still too high (10 is greater than 0), so we must trial an even smaller number, say $x = 2$.

Buttons to press	Calculator screen

This time, we see that the answer is **too low** (-13 is less than 0), so we can say that the solution to the equation is between $x = 2$ and $x = 3$. Because we have decided to solve the equation to the nearest unit, we must now decide whether the true solution is closer to $x = 2$ or to $x = 3$. It *appears* that the solution is closer to $x = 3$ (because 10 is closer to zero than -13), however this is not sufficient evidence to **prove** that the solution (to the nearest unit) is $x = 3$. Instead, we must consider the number that is **half way** between 2 and 3 (namely 2.5) and substitute this number into the left-hand side of the equation.

Buttons to press	Calculator screen

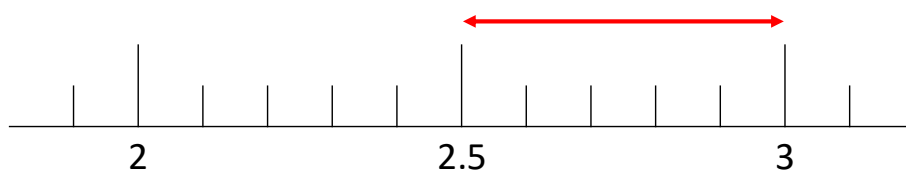
Because -3.375 is less than zero, we can say that the true solution to the equation lies between $x = 2.5$ and $x = 3$. Because every number in this range would round off (to the nearest unit) to be 3, we can now state, without doubt, that the solution to the equation, to the nearest unit, is $x = 3$.

Here's how to set out the solution in the form of a table.

Trial	The value of $x^3 + 4x - 29$	Too high / Too low?
5	116	Too high
3	10	Too high
2	-13	Too low
2.5	-3.375	Too low

The solution to the equation is between $x = 2.5$ and $x = 3$ so, to the nearest unit, the solution to the equation is $x = 3$.

We can also show the location of the true solution of the equation on a number line.



Exercise 1

The following equations have a solution between $x = 0$ and $x = 10$.

Use the trial and improvement method to find the solution correct to the nearest unit.

- (a) $x^3 + 8x - 15 = 0$
- (b) $x^3 - 8x - 15 = 0$
- (c) $x^3 - 2x - 15 = 0$
- (d) $x^2 + 7x - 100 = 0$
- (e) $x^2 + 2x - 50 = 0$
- (f) $2x^2 - 11x - 80 = 0$
- (g) $2x^3 - 2x - 15 = 0$
- (h) $0.2x^3 - 10x + 4 = 0$
- (i) $x^4 - 60x^2 - 14 = 0$



Trial and improvement to one decimal place

Let us return to the previous example of solving the equation $x^3 + 4x - 29 = 0$. We know, from previous work, that the solution to the nearest unit is $x = 3$. Let us now find a more detailed solution, to one decimal place.



From the table on the top of this page, we know that substituting $x = 2.5$ into the equation gives an answer that is too low (-3.375 is less than zero). Let us therefore try to substitute a value that is greater than 2.5, let's say $x = 2.7$.

Buttons to press	Calculator screen

As 1.483 is greater than zero, we must trial a number that is less than 2.7, let's say $x = 2.6$.

Buttons to press	Calculator screen

Now -1.024 is less than zero, so we can say that the solution lies between $x = 2.6$ and $x = 2.7$. As before, we must now decide whether the solution lies closer to $x = 2.6$ or to $x = 2.7$. To do this, we must consider the number that is **half way** between 2.6 and 2.7 (namely 2.65) and substitute this number into the left-hand side of the equation.

Buttons to press	Calculator screen

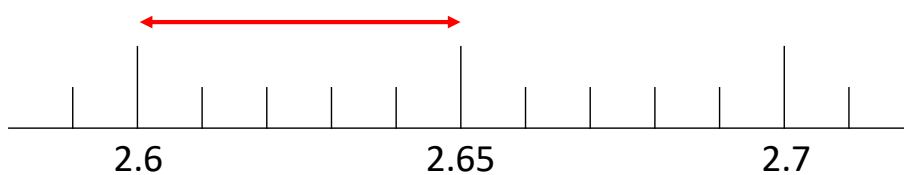
Because 0.209625 is greater than zero, we can say that the true solution to the equation lies between $x = 2.6$ and $x = 2.65$. Because every number in this range would round off (to one decimal place) to be 2.6, we can now state, without doubt, that the solution to the equation, correct to one decimal place, is $x = 2.6$.

Here's how to set out the solution to this question in the form of a table (continuing the previous work).

Trial	The value of $x^3 + 4x - 29$	Too high / Too low?
5	116	Too high
3	10	Too high
2	-13	Too low
2.5	-3.375	Too low
2.7	1.483	Too high
2.6	-1.024	Too low
2.65	0.209625	Too high

The solution to the equation is between $x = 2.6$ and $x = 2.65$ so, to one decimal place, the solution to the equation is $x = 2.6$.

We can also show the location of the true solution of the equation on a number line.



Exercise 2

The following equations have a solution between $x = 0$ and $x = 10$. Use the trial and improvement method to find the solution correct to one decimal place.



- (a) $x^3 + 8x - 15 = 0$
- (b) $x^3 - 7x - 120 = 0$
- (c) $x^3 - 3x - 93 = 0$
- (d) $x^3 + 3x = 15$
- (e) $x^2 + x - 79 = 0$
- (f) $x^2 + 3x = 39$
- (g) $2x^3 - 2x - 11 = 0$
- (h) $2x(x + 5) = 110$
- (i) $x^4 - 43x^2 - 25 = 0$

Trial and improvement to two decimal places


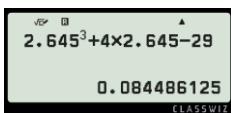

Let us again return to the example of solving the equation $x^3 + 4x - 29 = 0$. We know, from previous work, that the solution to one decimal place is $x = 2.6$. Let us now find a more detailed solution, to two decimal places.



From the table on the top of this page, we see that substituting $x = 2.65$ into the equation gives an answer that is too high (0.209625 is greater than zero). Let us therefore try to substitute a value that is smaller than 2.65, let's say $x = 2.64$.

Buttons to press	Calculator screen


Now -0.040256 is less than zero, so we can say that the solution lies between $x = 2.64$ and $x = 2.65$. As before, we must now decide whether the solution is closer to $x = 2.64$ or to $x = 2.65$. To do this, we must consider the number that is **half way** between 2.64 and 2.65 (namely 2.645) and substitute this number into the left-hand side of the equation.

Buttons to press	Calculator screen
	
	

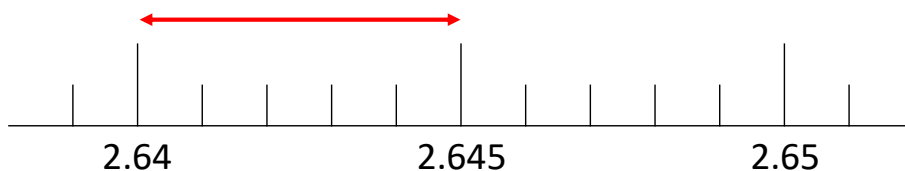
Because 0.084486125 is greater than zero, we can say that the true solution to the equation lies between $x = 2.64$ and $x = 2.645$. Because every number in this range would round off (to two decimal places) to be 2.64, we can now state, without doubt, that the solution to the equation, correct to two decimal places, is $x = 2.64$.

Here's how to set out the solution to this question in the form of a table (continuing the previous work).

Trial	The value of $x^3 + 4x - 29$	Too high / Too low?
5	116	Too high
3	10	Too high
2	-13	Too low
2.5	-3.375	Too low
2.7	1.483	Too high
2.6	-1.024	Too low
2.65	0.209625	Too high
2.64	-0.040256	Too low
2.645	0.084486125	Too high

 The solution to the equation is between $x = 2.64$ and $x = 2.645$ so, to two decimal places, the solution to the equation is $x = 2.64$.

We can also show the location of the true solution of the equation on a number line.

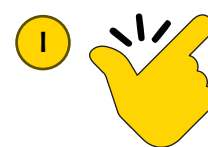


Exercise 3

The following equations have a solution between $x = 0$ and $x = 10$.

Use the trial and improvement method to find the solution correct to two decimal places.

- (a) $x^3 + 8x - 15 = 0$
- (b) $x^3 - 5x - 120 = 0$
- (c) $x^3 - 2x - 65 = 0$
- (d) $x^3 + 7x = 24$
- (e) $x^2 + x - 43 = 0$
- (f) $x^2 + 6x = 24$
- (g) $2x^3 - 3x - 25 = 0$
- (h) $x(24 + x) = 18$
- (i) $x^4 - x^2 - 48 = 0$



Using the 'Table Mode'

Some scientific calculators have a 'table mode' that can be used in the solution of trial and improvement questions. The advantage of using the *table mode* is to eliminate the need for any guessing as it can show us the answer to many trials at once.



Let us now use the *table mode* to solve the example from earlier in the chapter, namely to solve the equation $x^3 + 4x - 29 = 0$ correct to two decimal places.

Buttons to press (Casio fx-83GT CW)	Calculator screen	Notes
		Enter the Table Mode and start to define the function.
		Finish defining the function.
		Substitute different numbers into the function.
		Continue to enter numbers to solve correct to two decimal places. (You can use the arrows to see the answers in full.)

Using the calculator's *table mode*, this is what we would have to write as the solution to this question.

Trial	The value of $x^3 + 4x - 29$	Too high / Too low?
2	-13	Too low
3	10	Too high
2.6	-1.024	Too low
2.7	1.483	Too high
2.64	-0.040256	Too low
2.65	0.209625	Too high
2.645	0.084486125	Too high

The solution to the equation lies between $x = 2.64$ and $x = 2.645$ so, to two decimal places, the solution to the equation is $x = 2.64$.

Exercise 4

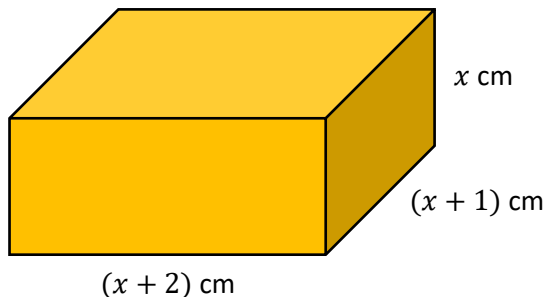
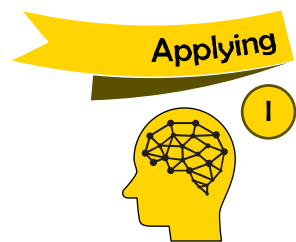
The following equations have a solution between $x = 0$ and $x = 10$. Using the trial and improvement method, and the *table mode* on your calculator, find the solution to the required degree of accuracy.

- (a) $x^2 - 4x - 3 = 0$ to the nearest unit
- (b) $x^3 + 6x - 31 = 0$ to one decimal place
- (c) $x^3 + 8x - 15 = 0$ to two decimal places
- (d) $x^2 - \sqrt{9x} - 1 = 0$ to the nearest whole number
- (e) $x^5 - 63x - 17 = 0$ to one decimal place
- (f) $x^3 - 6x - 19 = 0$ to two decimal places



Exercise 5

(a) The length of a cuboid is $(x + 2)$ cm, its width is $(x + 1)$ cm and its height is x cm. The volume of the cuboid is 140 cm^3 .

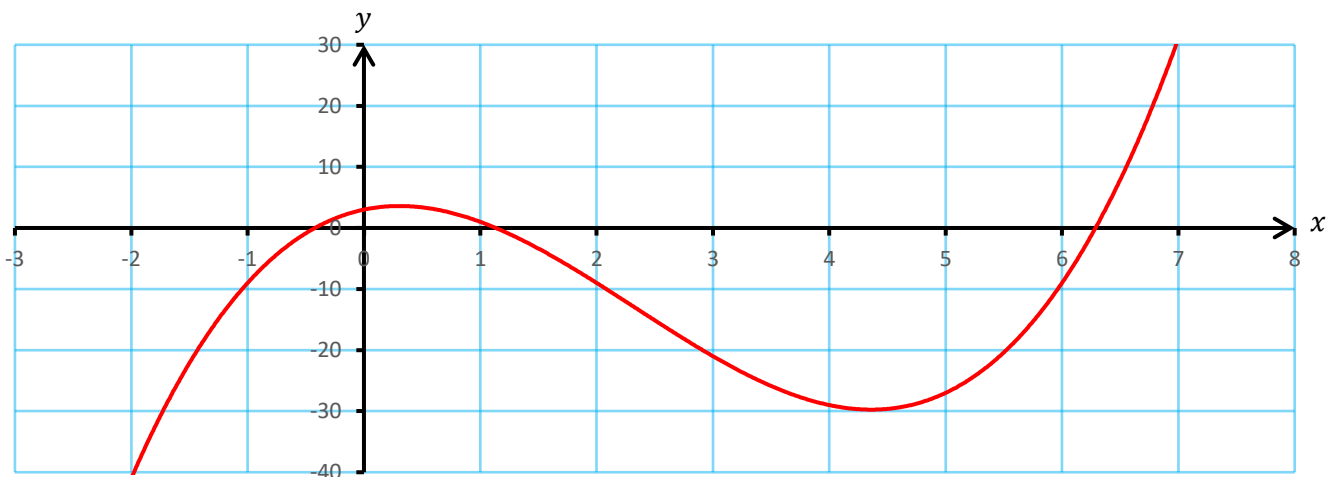


Calculate the length of the cuboid correct to one decimal place.

(b) Find the whole number that satisfies the equation $x^3 + 4x = 240$.

(c) Huw enters a number into his calculator, before adding the cube of the same number. The calculator screen shows 190.893. Find the number that Huw initially entered into his calculator. Show your method.

(d) Here is a graph of the equation $y = x^3 - 7x^2 + 4x + 3$.



Use the trial and improvement method to find, correct to one decimal place, each of the solutions to the equation $x^3 - 7x^2 + 4x + 3 = 0$.

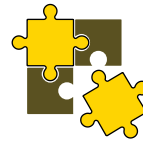
Evaluation

Key words	Corrections	I am happy with...	I need to revise...



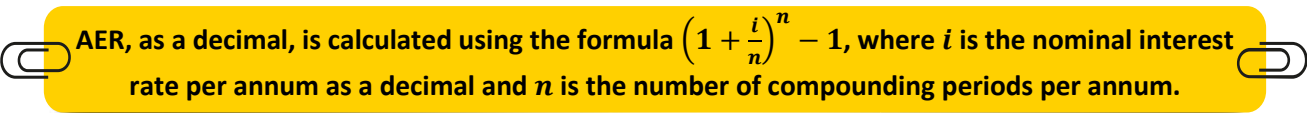
AER, APR

When **borrowing** or **investing** money, it is important to consider the **interest rate** that is used to calculate the **interest** that is added to the loan or investment. Another factor is important however – **how frequently** interest is added. This makes it difficult to directly compare interest rates if the periods for adding interest are different. For example, consider that you want to invest a sum of money. Which option is best for you: an interest rate of 4% paid every six months, or an interest rate of 2% paid every quarter? In order to compare interest rates of this type fairly, we use the special percentages **AER** and **APR**.




AER = Annual Equivalent Rate

AER is used to note the percentage of interest earned in a period of **one year**. It allows you to compare different accounts that pay interest at different times, e.g. every month, every quarter, every six months. The following method for calculating AER is given on page 2 of the examination paper.



AER, as a decimal, is calculated using the formula $\left(1 + \frac{i}{n}\right)^n - 1$, where i is the nominal interest rate per annum as a decimal and n is the number of compounding periods per annum.

Note that sometimes the 'nominal interest rate' is referred to as the 'gross interest rate'.

Exercise 6

Complete the following table. (The first row has been completed for you.)



Skill

H

Interest rate	Number of compounding periods per annum	Nominal interest rate per annum
3%	4	12%
4%	2	
2.5%		7.5%
	12	6%

Example

Calculate the AER for the following two savings accounts: one with an interest rate of 4% paid every six months, and another with an interest rate of 2% paid every quarter.

Interest rate 4% paid every six months

There are 2 compounding periods during the year.
The nominal interest rate per annum is $4\% \times 2 = 8\%$.
As a decimal, this is 0.08.

$$\text{AER} = \left(1 + \frac{0.08}{2}\right)^2 - 1$$

$$\text{AER} = 0.0816$$

As a percentage, the AER is 8.16%.

Interest rate 2% paid every quarter

There are 4 compounding periods during the year.
The nominal interest rate per annum is $2\% \times 4 = 8\%$.
As a decimal, this is 0.08.

$$\text{AER} = \left(1 + \frac{0.08}{4}\right)^4 - 1$$

$$\text{AER} = 0.08243216$$

As a percentage, the AER is 8.24%, correct to 2 decimal places.



By comparing the two AER values, it is possible to see that the second account (2% interest paid every quarter) is the better option, as the AER is higher.

Exercise 7

Calculate the AER for the following savings accounts.

- (a) An interest rate of 5% paid every six months.
- (b) An interest rate of 3% paid every quarter.
- (c) An interest rate of 7% paid every 4 months.
- (d) An interest rate of 2% paid every month.
- (e) An interest rate of 8.4% paid every quarter.
- (f) An interest rate of 0.25% paid every quarter.



Example

Susan intends to invest £2,500 into a savings account for one year. HSBC bank offer a nominal interest rate of 3% a year, with interest paid every quarter.

- (a) Calculate the AER for HSBC's account.
- (b) If Susan decides to invest her money with HSBC for one year, how much money will be in her account at the end of the year?

Answer: (a) With interest paid every quarter, there are 4 compounding periods during the year.

$$AER = \left(1 + \frac{0.03}{4}\right)^4 - 1$$

$$AER = 0.03033919066 \dots$$

AER = 3.03%, to 2 decimal places.

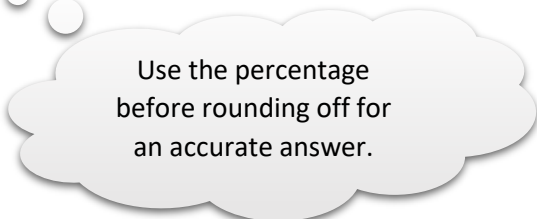
(b) Method 1: Use the AER.

$$£2,500 \times 103.033919066\% = £2,575.85, \text{ to the nearest penny.}$$

Method 2: Use the nominal interest rate.

3% a year so $3 \div 4 = 0.75\%$ a quarter.

$$£2,500 \times 100.75\%^4 = £2,575.85, \text{ to the nearest penny.}$$



Exercise 8

(a) Dave intends to invest £4,000 into a savings account for one year. Barclays bank offer a nominal interest rate of 2% a year, with interest to be paid every quarter.

- (i) Calculate the AER for Barclays' account.
- (ii) If Dave decides to invest the money with Barclays for one year, how much money will be in his account at the end of the year?

(b) Victoria intends to invest £2,500 into a savings account for one year. HSBC bank offer a nominal interest rate of 5% a year, with interest to be paid every month.

- (i) Calculate the AER for HSBC's account.
- (ii) If Victoria decides to invest the money with HSBC for one year, how much money will be in her account at the end of the year?

(c) Which is better: investing money into an account that offers AER at a rate of 4%, or investing money into an account that offers an interest rate of 1% paid every three months?

(d) Always / sometimes / never: AER is always greater than the nominal interest rate.



Example

(a) Morgan invests £400 with Barclays bank at an AER of 2.4%. How much money will Morgan have in the bank after 3 years?

(b) Four years ago, Mari invested a sum of money into HSBC bank at an AER of 4.5%. The money is now worth £4,000. What is the minimum amount of money that Mari had to invest in order to accomplish this?

Answer: (a) $£400 \times 102.4\%^3 = £429.50$, to the nearest penny. (b) $? \times 104.5\%^4 = £4,000$
 $? = £4,000 \div 104.5\%^4$
 $? = £3,354.25$, to the nearest penny.



Exercise 9



(a) Ffion invests £800 into Lloyds bank at an AER of 3.1%. How much money will Ffion have in the bank after 5 years?

(b) Three years ago, Jac invested a sum of money into Santander bank at an AER of 2.3%. The money is now worth £1,400. What is the minimum amount of money that Jac had to invest in order to accomplish this?

(c) Meical invests £6,500 into Halifax bank at an AER of 1.7%. How much money will Meical have in the bank after 2 years?

(d) Nine years ago, Catrin invested a sum of money into Barclays bank at an AER of 6.25%. The money is now worth £20,000. What is the minimum amount of money that Catrin had to invest in order to accomplish this?

(e) Megan has £5,000 to invest in HSBC bank at an AER of 6.4%. In how many years will Megan's money be worth more than £7,000?



Alternative method of calculating AER

As well as the method shown on page 2 of a GCSE examination paper, it is possible to use the following method for calculating AER.



$$\text{AER} = \frac{\text{Interest accrued over one year}}{\text{Initial value}} \times 100\%$$

Example

Calculate the AER for a savings account that offers an interest rate of 4% paid every quarter.

Answer: Imagine that we decide to invest £1,000 into this savings account. After one year, the money will be worth $£1,000 \times 104\%^4 = £1,169.86$ (to the nearest penny), so the interest accrued over one year is £169.86 (to the nearest penny). So, the AER is $\frac{169.86}{1000} \times 100\% = 16.99\%$, to 2 decimal places.

(The previous method gives the same answer, as $(1 + \frac{0.16}{4})^4 - 1 = 0.16985856 = 16.99\%$, to 2 decimal places.)

Exercise 10



Use the alternative method of calculating AER to calculate the AER for the following savings accounts.

- (a) An interest rate of 5% paid every six months.
- (b) An interest rate of 3% paid every quarter.
- (c) An interest rate of 7% paid every 4 months.
- (d) An interest rate of 2% paid every month.
- (e) An interest rate of 8.4% paid every quarter.
- (f) An interest rate of 0.25% paid every quarter.



$$\text{APR} = \text{Annual Percentage Rate}$$



APR is used to compare accounts where there is a charge for the account, or these are additional costs associated with the account.

For a savings account,

$$\text{APR} = \frac{\text{Interest accrued over one year} - \text{costs}}{\text{Initial value}} \times 100\%$$

For a borrowing account,

$$\text{APR} = \frac{\text{Interest accrued over one year} + \text{costs}}{\text{Initial value}} \times 100\%$$



In most cases, there are **no costs** associated with a **savings** account, so the AER and APR rates are equal to each other. This explains why we see AER rates advertised alongside savings accounts.

In most cases, there **are** costs associated with a **borrowing** account, so we must use the APR rate. This explains why we see APR rates advertised alongside borrowing accounts such as mortgages, credit cards and loans from the bank.

Example

Huw intends to borrow £4,800 from the company *Loans4U*. The company offers an interest rate of 4% a month, and charges an annual fee of £150 to use the account.

- (a) How much interest will this loan accrue over a period of one year?
 (b) Calculate the APR for this loan.

Answer: (a) There are 12 compounding periods during the year. $£4,800 \times 104\%^{12} = £7,684.95$, to the nearest penny.
 So, $£7,684.95 - £4,800 = £2,884.95$ of interest is accrued during the year.

$$\begin{aligned} \text{(b) APR} &= \frac{\text{Interest accrued over one year} + \text{costs}}{\text{Initial value}} \times 100\% \\ \text{APR} &= \frac{2884.95 + 150}{4800} \times 100\% \\ \text{APR} &= 63.2\%, \text{ to one decimal place.} \end{aligned}$$



Applying

H

Exercise 11

(a) Lisa intends to borrow £7,000 from the company *BestLoans*. The company offers an interest rate of 2% a month, and charges an annual fee of £200 to use the account.

- (i) How much interest will this loan accrue over a period of one year?
 (ii) Calculate the APR for this loan.

(b) Deiniol intends to borrow £24,000 from the company *LoanKing*. The company offers an interest rate of 3% every six months, and charges a monthly fee of £15 for using the account.

- (i) How much interest will this loan accrue over a period of one year?
 (ii) Calculate the APR for this loan.

(c) Sophie intends to borrow £154,000 from the company *MorgaisGorau*. The company offers an interest rate of 0.4% a month, and charges an annual fee of £300 for using the account.

- (i) How much interest will this loan accrue over a period of one year?
 (ii) Calculate the APR for this loan.



Example

Calculate the AER or APR for each of the following situations.

Situation 1: Savings account with no costs.

Method 1: Use the formula $AER = \left(1 + \frac{i}{n}\right)^n - 1$

Nominal interest rate per annum $3\% \times 4 = 12\%$.

$$AER = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$AER = 0.12550881 \dots$$

AER = 12.55% to 2 decimal places.

Method 2: Use the formula

$$AER = \frac{\text{Interest accrued over one year}}{\text{Initial value}} \times 100\%$$

Value at the end of the year

$$= £2,500 \times 103\%^4$$

$$= £2,813.77 \text{ to the nearest penny.}$$

Interest accrued over one year

$$= £2,813.77 - £2,500$$

$$= £313.77$$

$$AER = \frac{£313.77}{£2,500} \times 100\%$$

AER = 12.55% to 2 decimal places.

Situation 2: Savings account with costs of £40 a year.

We must use the formula

$$APR = \frac{\text{Interest accrued over one year} - \text{costs}}{\text{Initial value}} \times 100\%$$

Value at the end of the year

$$= £2,500 \times 103\%^4$$

$$= £2,813.77 \text{ to the nearest penny.}$$

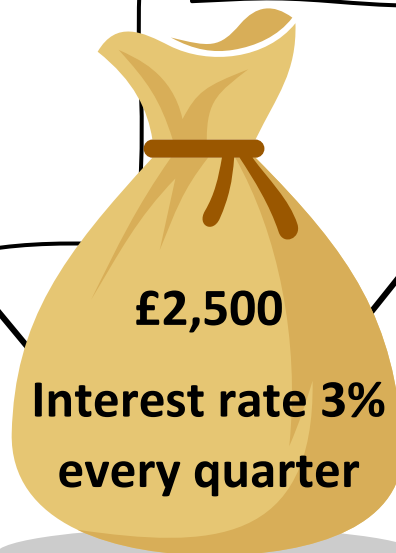
Interest accrued over one year

$$= £2,813.77 - £2,500$$

$$= £313.77$$

$$APR = \frac{£313.77 - £40}{£2,500} \times 100\%$$

APR = 10.95% to 2 decimal places.

**Situation 3: Borrowing account with no charges.**

The calculations are exactly the same as for situation 1. So, the AER is 12.55% to 2 decimal places.

Because there are no costs, the APR is also 12.55% to 2 decimal places.

Situation 4: Borrowing account with costs of £40 a year.

We must use the formula

$$APR = \frac{\text{Interest accrued over one year} + \text{costs}}{\text{Initial value}} \times 100\%$$

Loan at the end of the year

$$= £2,500 \times 103\%^4$$

$$= £2,813.77 \text{ to the nearest penny.}$$

Interest accrued over one year

$$= £2,813.77 - £2,500$$

$$= £313.77$$

$$APR = \frac{£313.77 + £40}{£2,500} \times 100\%$$

APR = 14.15% to 2 decimal places.

Exercise 12

Calculate the AER or APR for each of the following situations.

- (a) An investment of £1,500 into a savings account that offers an interest rate of 2% per quarter.
- (b) An investment of £2,400 into a savings account that offers an interest rate of 5% per year and annual costs of £50.
- (c) A loan of £3,500 from an account that offers an interest rate of 3.2% per quarter.
- (d) A loan of £15,000 from an account that offers an interest rate of 1.2% per month and annual costs of £150.
- (e) A loan of £140,000 from an account that offers an interest rate of 1.8% per quarter and quarterly costs of £50.
- (f) An investment of £250,000 into a savings account that offers an interest rate of 0.4% per month and monthly costs of £5.



Challenge! 

HSBC's website shows the following information for a personal loan of £10,000 taken over 12 months.

How much would you like to borrow?
Enter a value between 1,000 and 25,000

GBP 10,000

GBP 1,000 GBP 25,000

Over how many months?
Enter a value between 12 and 60

12

12 months 60 months

Representative example*

Total amount payable
GBP 10,340.09

Monthly repayment
GBP 861.67

Representative
6.4% APR

Interest rate p.a. fixed
6.4%

<https://www.hsbc.co.uk/loans/products/personal/> , 11/05/2025

6.4% of £10,000 is £640. Why is the total amount payable not £10,640? Investigate...

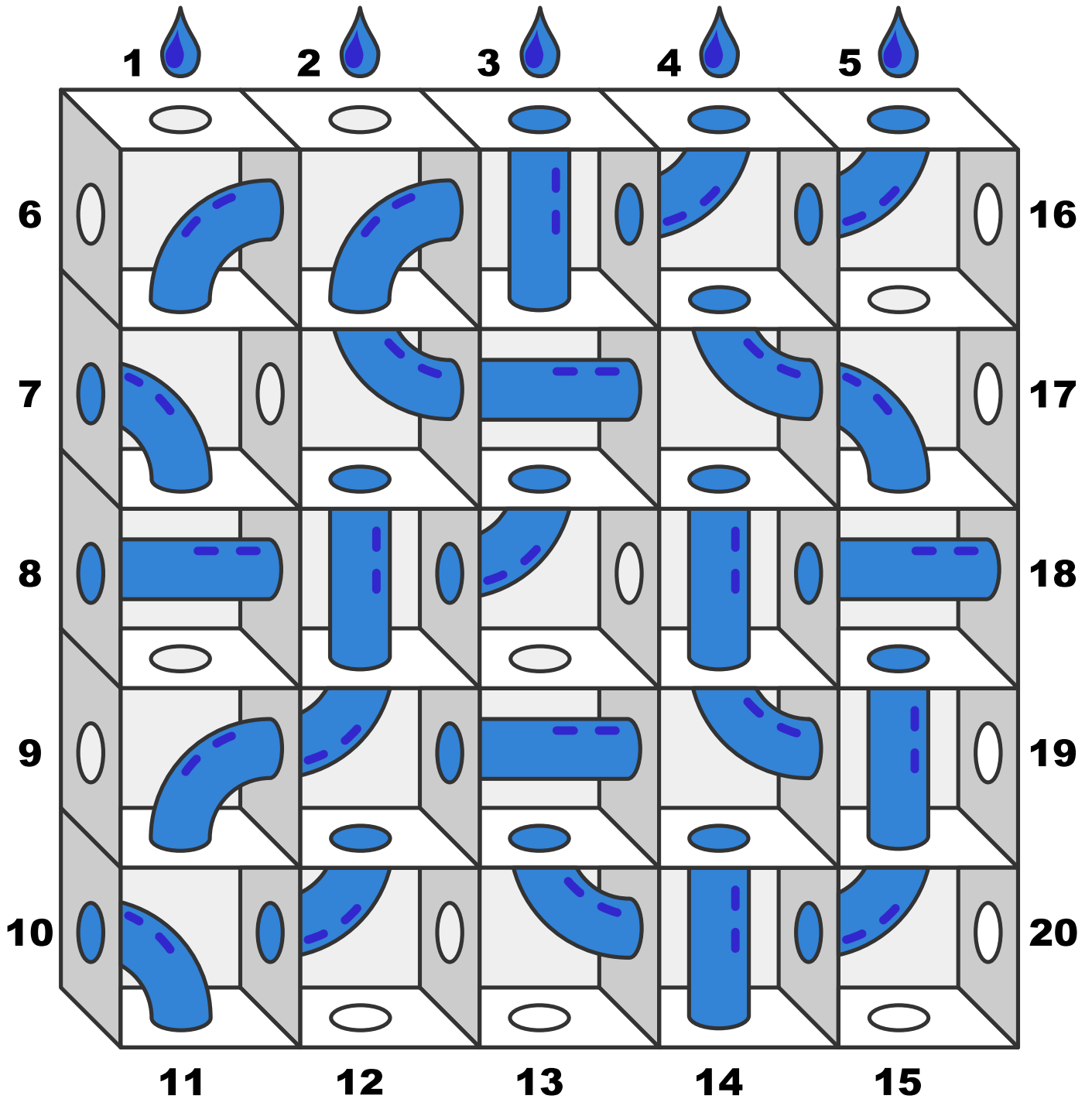


Key words	Corrections	I am happy with...	I need to revise...

Puzzle

The front of the tank below is solid and transparent.

Where will the liquid pour out if it is poured into hole 1?
 What about hole 2? Hole 3? Hole 4? Hole 5?







Reflection

Name:

Percentage in the test:

	I know this. 	I need to revise this. 	Question in the test:	Correct in the test?
<p>I can use the trial and improvement method to solve equations correct to the nearest unit; to one decimal place; or to two decimal places.</p>			1	
<p>I can use the trial and improvement method to solve equations, e.g. find the whole number that satisfies the equation $x^3 - 2x = 115$.</p>				
<p>I can calculate the AER for a savings account using the formula $\left(1 + \frac{i}{n}\right)^n - 1$.</p>			2	
<p>I can use the AER to calculate how much money is in a savings account, at the end or start of an investment.</p>			2, 3, 4	
<p>I can calculate the APR for savings or borrowing accounts where there is an additional charge associated with the account.</p>			5	