



Uned 3, Pecyn 3

13

Diagramau

Pry Cop a Grisïau



Enw:

Cefndir

Beth yw'r gwaith?

Yn y pecyn diwethaf, gwelsom berthnasau cylchol o'r ffurf $x_{n+1} = g(x_n)$. Mae diagramau pry cop a diagramau grisiau yn **darlunio** beth sy'n cymryd lle wrth ddefnyddio perthnasau cylchol o'r math yma i ddarganfod gwreiddiau hafaliadau.

Beth sydd ei angen cyn cychwyn?

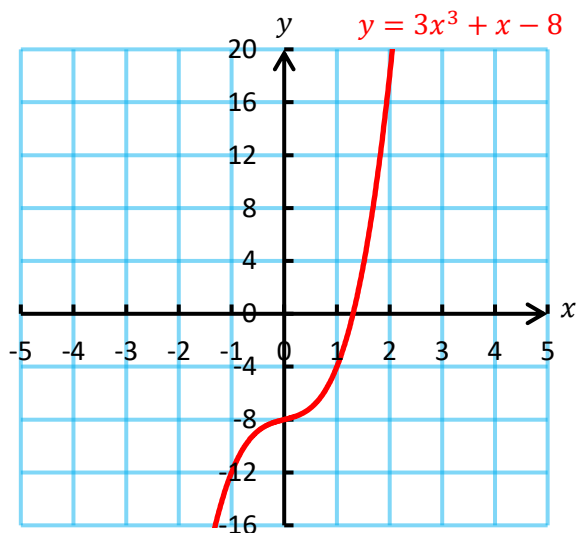
Gwaith TGAU: Amnewid, talgrynnu, newid testun.
Lefel A Uned 1: Braslunio cromliniau.
Lefel A Uned 3: Gwreiddiau hafaliadau (pecyn 2).

I ble mae'n arwain?

Lefel A Uned 3: Dull Newton-Raphson o ddatrys hafaliadau.
Cymwysiadau: Datrys problemau mewn cyd-destun sy'n golygu nad oes modd cael datrysiad dadansodol o hafaliad.

Theori

Ystyriwch unrhyw ffwythiant $y = f(x)$, er enghraifft $y = 3x^3 + x - 8$.



Theori ac
Enghraifft

Gwreiddiau'r hafaliad yw ble mae'r gromlin yn torri'r echelin- x , sef ble mae $f(x) = 0$. Ar gyfer yr enghraifft uchod, gwelwn fod un gwreiddyn yn bodoli, rhwng 1 a 2. Yn y pecyn gynt, gwelsom fod hi'n bosib ceisio datrys $f(x) = 0$ trwy ail-drefnu'r hafaliad i'r ffurf $x = g(x)$, ac yna ffurfio **perthynas gylchol** o'r ffurf $x_{n+1} = g(x_n)$.

Enghraifft: $y = 3x^3 + x - 8$.

$$\begin{aligned} \text{Ymgais 1:} \quad & 3x^3 + x - 8 = 0 \\ & x = 8 - 3x^3 \\ & x_{n+1} = 8 - 3x_n^3 \end{aligned}$$

$$\begin{aligned} \text{Ymgais 2:} \quad & 3x^3 + x - 8 = 0 \\ & 3x^3 = 8 - x \\ & x^3 = \frac{8-x}{3} \\ & x = \sqrt[3]{\frac{8-x}{3}} \\ & x_{n+1} = \sqrt[3]{\frac{8-x_n}{3}} \end{aligned}$$

Gwelwn o'r graff ar y dudalen flaenorol bod gwreiddyn yr hafaliad $y = 3x^3 + x - 8$ yn agos at $x = 1$. Felly, gallwn ddewis cychwyn yn $x_0 = 1$ a defnyddio'r berthynas gylchol i geisio darganfod y gwreiddyn.

$$x_1 = 8 - 3 \times x_0^3$$

$$x_1 = 5$$

$$x_2 = -367$$

$$x_3 = 148292597$$

$$x_4 = -9.783171512 \times 10^{24}$$

$$x_5 = 2.809055093 \times 10^{75}$$

$$x_6 = \text{MATH ERROR}$$

Dargyfeirio, dim yn gallu ffeindio gwreiddyn.

$$x_1 = \sqrt[3]{\frac{8-x_0}{3}}$$

$$x_1 = 1.326352403$$

$$x_2 = 1.305411212$$

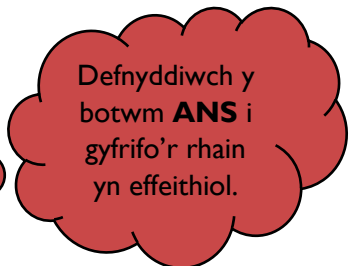
$$x_3 = 1.3067752$$

$$x_4 = 1.306686444$$

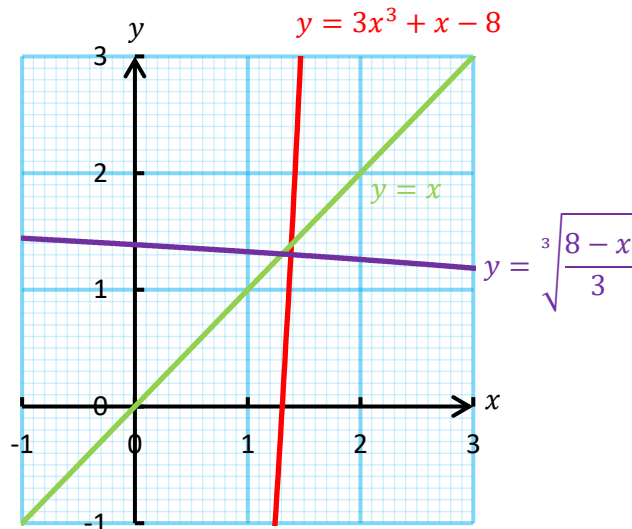
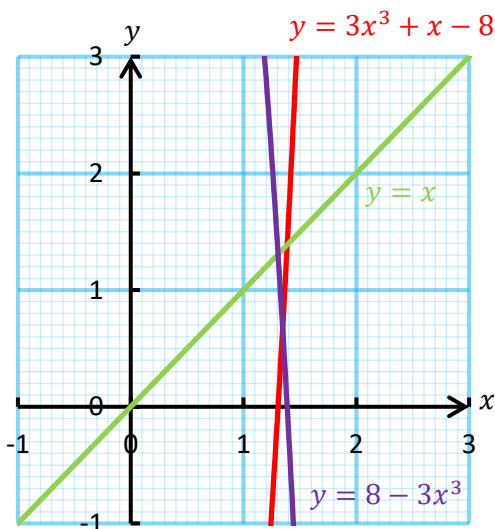
$$x_5 = 1.30669222$$

$$x_6 = 1.306691844 \quad \text{ac yn y blaen...}$$

Cydyfeirio at 1.306691867.



I ddarlunio beth sy'n digwydd uchod, gallwn blotio $y = f(x)$, $y = x$ ag $y = g(x)$ ar yr un diagram.



Ym mhob achos, gwelwn fod gwreiddyn $y = 3x^3 + x - 8$ yn hafal i'r gwerth ar gyfer x ble mae'r llinell $y = x$ yn croestorri'r gromlin $y = g(x)$. Mae hyn yn gwirio bod datrys $f(x) = 0$ yr un peth â datrys $x = g(x)$.

Gallwn ddefnyddio'r rhifau sy'n dod o'r berthynas gylchol $x_{n+1} = g(x_n)$ i ffurfio naill ai **diagram pry cop** neu **ddiagram grisiau**.

Diagram pry cop

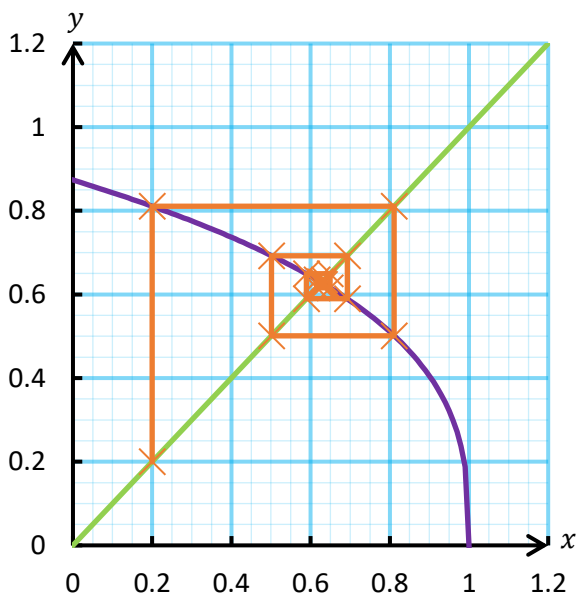
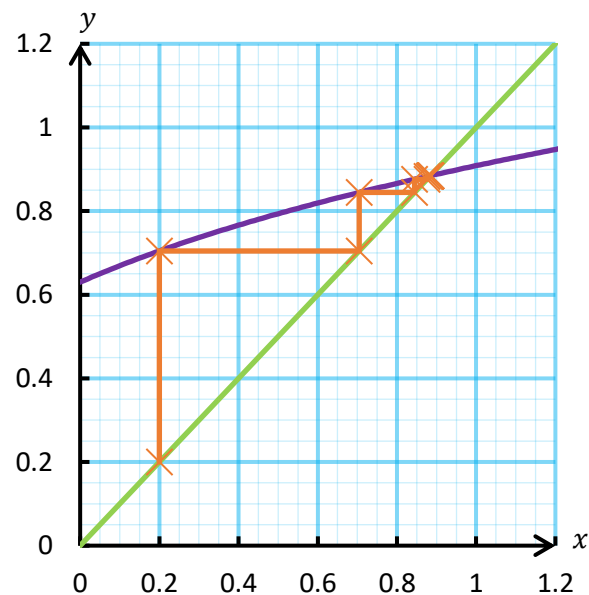


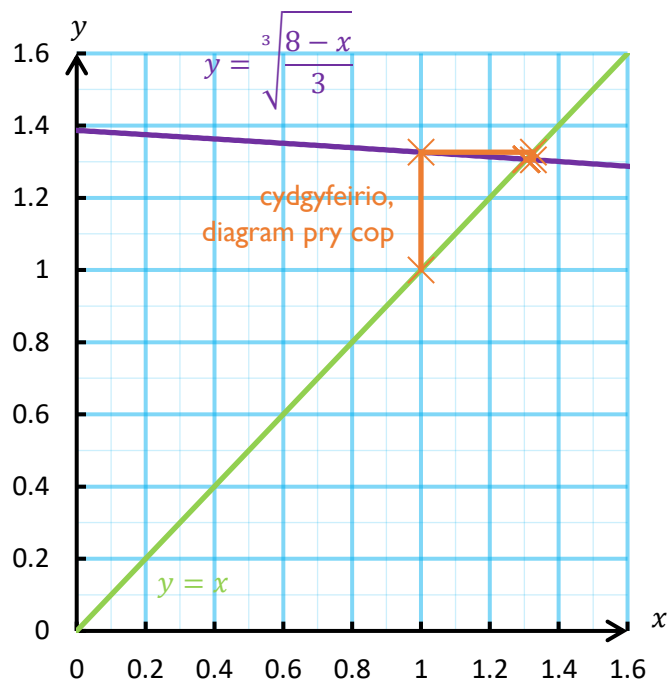
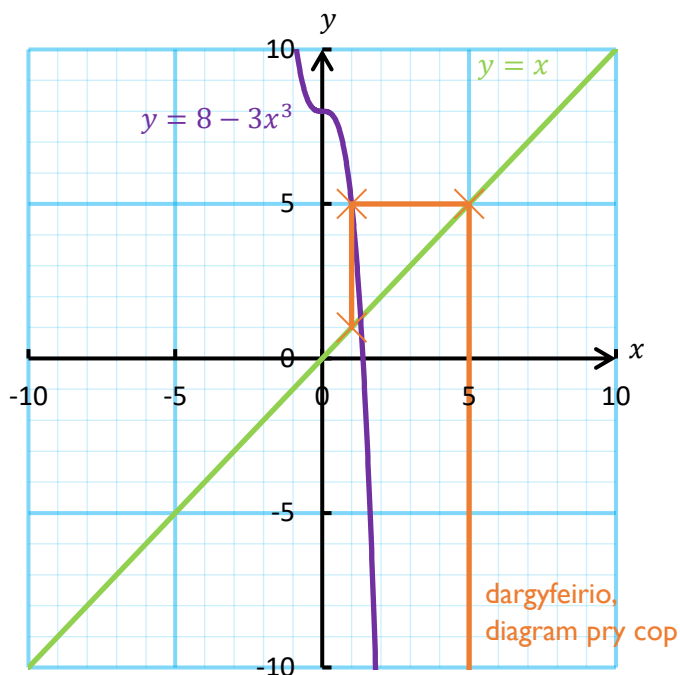
Diagram grisiau



Mae angen plotio llinellau i fynd o (x_0, x_0) i (x_0, x_1) i (x_1, x_1) i (x_1, x_2) i (x_2, x_2) i (x_2, x_3) i ...

Mae'r llinellau yma'n naill ai'n **cydgyfeirio** i wreiddyn $y = f(x)$ neu'n **dargyfeirio**.

Ar gyfer yr enghraifft ar y dudalen flaenorol, dyma beth sy'n digwydd.



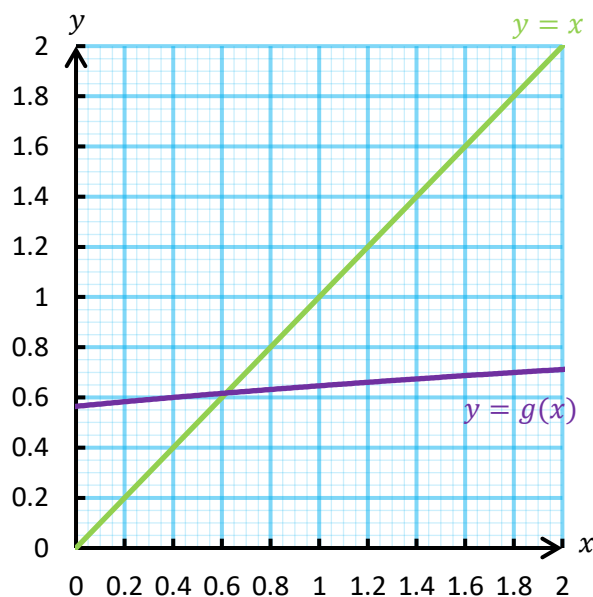
Ymarferion

Ymarfer 1

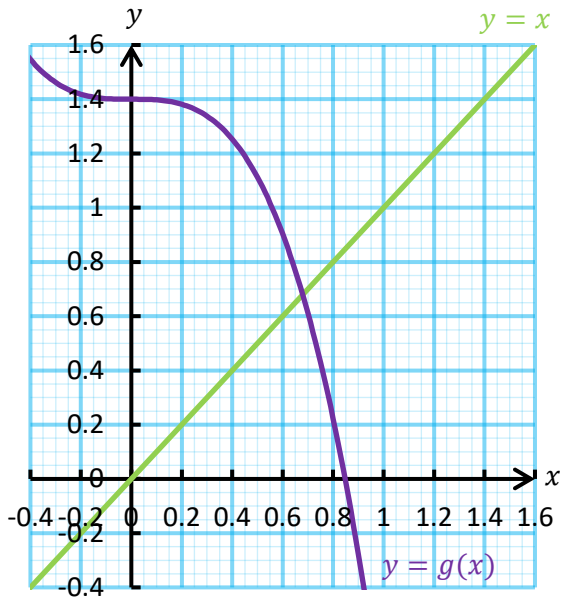
Mae'r graffiau isod yn dangos llinell $y = x$ a ffwythiant $y = g(x)$. Trwy gychwyn yn y gwerth a nodir ar gyfer x_0 , ychwanegwch linellau i'r graff i ffurfio naill ai diagram pry cop neu ddiagram grisiau. Nodwch o dan bob graff os yw'r llinellau'n **cydgyfeirio** i wreiddyn $x = g(x)$ neu'n **dargyfeirio**.

(a) $x_0 = 0.2$

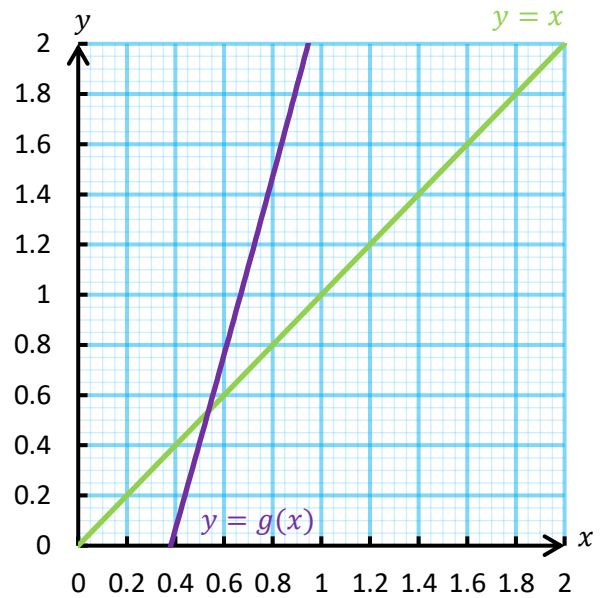
(b) $x_0 = 1.6$



(c) $x_0 = 0.6$

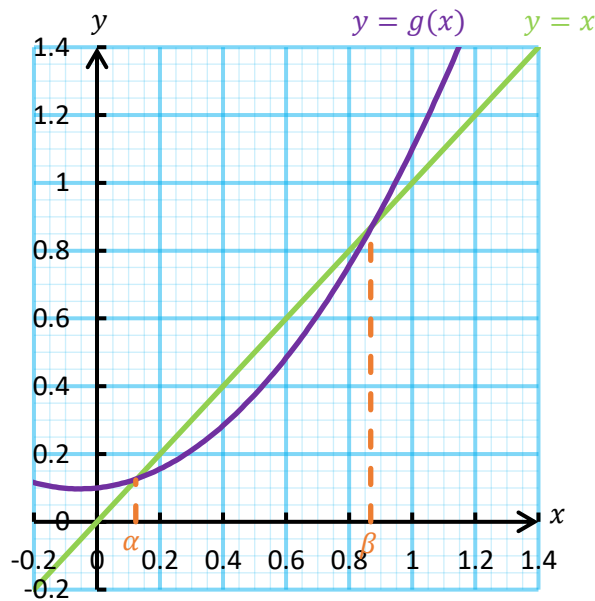


(ch) $x_0 = 0.6$



Ymarfer 2

Mae'r graff isod yn dangos braslun o'r hafaliad $y = g(x)$ a'r llinell $y = x$. Mae'r hafaliad $x = g(x)$ efo gwreiddiau $x = \alpha$ ac $x = \beta$ fel sy'n cael eu dangos ar y graff.



Mae perthynas gylchol o'r ffurf $x_{n+1} = g(x_n)$ yn cael ei ddefnyddio i geisio darganfod gwreiddiau o'r hafaliad $x = g(x)$. Disgrifiwch beth sy'n digwydd i'r iteriad mewn achos ble mae'r gwerth cychwynol x_0

(a) rhwng 0 ag α

(b) rhwng α a β

(c) yn fwy na β

Ymarfer 3

Mae Siwan yn dymuno datrys yr hafaliad ciwbig $y = 2x^3 + 2x - 3$.

(a) (i) Ail-drefnwch yr hafaliad $2x^3 + 2x - 3 = 0$ i'r ffurf $x = g(x)$, gan wneud yr x cyntaf (yr un **coch**) yn destun i'r hafaliad.

(ii) Ail-drefnwch yr hafaliad $2x^3 + 2x - 3 = 0$ i'r ffurf $x = g(x)$, gan wneud yr ail x (yr un **gwyrdd**) yn destun i'r hafaliad.

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(b) Ffurfiwch berthynas gylchol $x_{n+1} = g(x_n)$ o'r hafaliadau o ran (a) y cwestiwn.

(i)

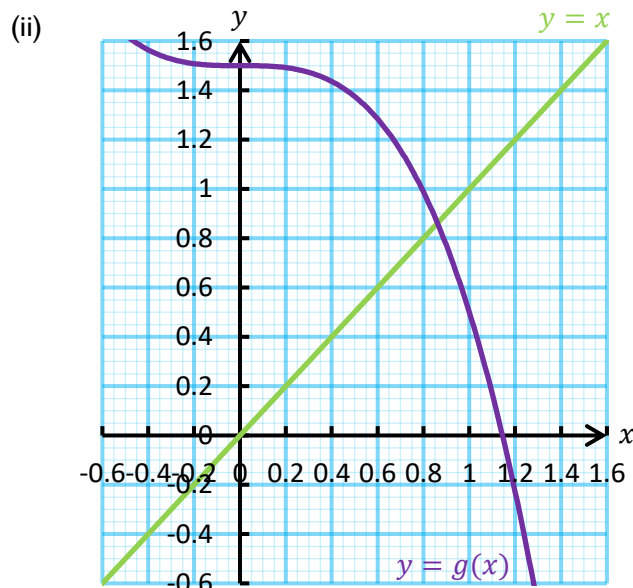
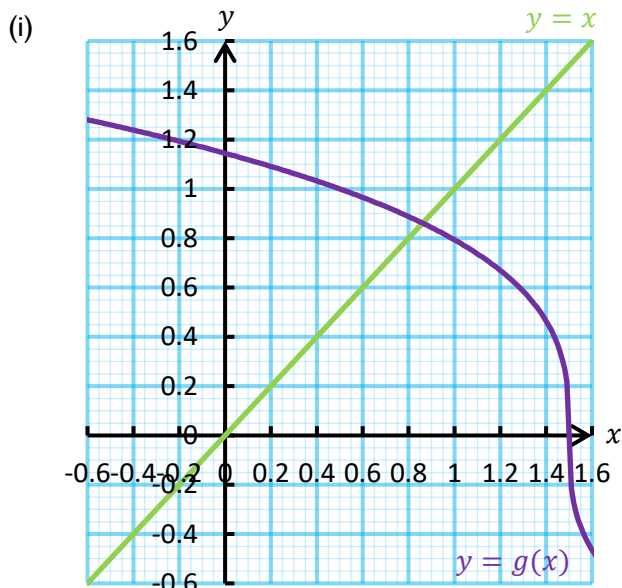
(ii)

(c) Gan gychwyn yn $x_0 = 0.5$, darganfyddwch werthoedd x_1, x_2, x_3, x_4, x_5 ym mhob achos.

(i) $x_1 =$	(ii) $x_1 =$
$x_2 =$	$x_2 =$
$x_3 =$	$x_3 =$
$x_4 =$	$x_4 =$
$x_5 =$	$x_5 =$

(ch) Pa un o'r dilyniannau uchod sy'n cydgyfeirio, a pha un sy'n dargyfeirio?

(d) Defnyddiwch y diagramau isod i lunio diagram pry cop ar gyfer bob iteriad.



(dd) Ar gyfer yr iteriad sy'n cydgyfeirio, ysgrifennwch werth x_5 yn gywir i un lle degol. Profwch mai hwn yw gwerth gwreiddyn hafaliad gwreiddiol Siwan yn gywir i un lle degol.

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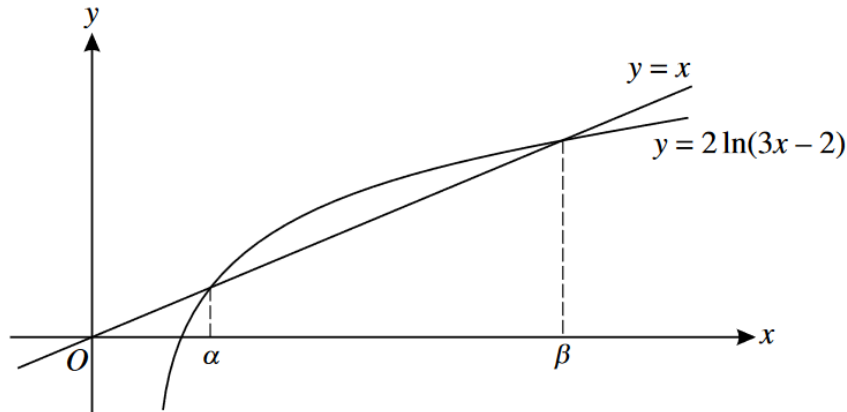
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(OCR Further Pure Mathematics 2 [4726] Haf 2010)

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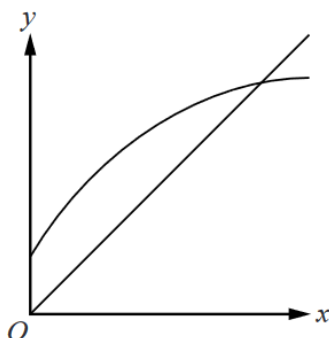
The line $y = x$ and the curve $y = 2 \ln(3x - 2)$ meet where $x = \alpha$ and $x = \beta$, as shown in the diagram.

- (i) Use the iteration $x_{n+1} = 2 \ln(3x_n - 2)$, with initial value $x_1 = 5.25$, to find the value of β correct to 2 decimal places. Show all your working. [2]
- (ii) With the help of a ‘staircase’ diagram, explain why this iteration will not converge to α , whatever value of x_1 (other than α) is used. [3]

(OCR Further Pure Mathematics 2 [4726] Haf 2012)

- 4 It is given that the equation $x^4 - 2x - 1 = 0$ has only one positive root, α , and $1.3 < \alpha < 1.5$.

(i)



The diagram shows a sketch of $y = x$ and $y = \sqrt[4]{2x+1}$ for $x \geq 0$. Use the iteration $x_{n+1} = \sqrt[4]{2x_n + 1}$ with $x_1 = 1.35$ to find x_2 and x_3 , correct to 4 decimal places. On the copy of the diagram show how the iteration converges to α . [3]

- (ii) For the same equation, the iteration $x_{n+1} = \frac{1}{2}(x_n^4 - 1)$ with $x_1 = 1.35$ gives $x_2 = 1.1608$ and $x_3 = 0.4077$, correct to 4 decimal places. Draw a sketch of $y = x$ and $y = \frac{1}{2}(x^4 - 1)$ for $x \geq 0$, and show how this iteration does not converge to α . [2]

(OCR Further Pure Mathematics 2 [4726/01] Haf 2016)

4 You are given the equation $(2x - 1)^2 - e^x = 0$.

(i) Verify that 0 is a root of the equation. [1]

There are also two other roots, α and β , where $0 < \alpha < \beta$.

(ii) The iterative formula $x_{r+1} = \ln(2x_r - 1)^2$ is to be used to find a root of the equation.

(a) Sketch the line $y = x$ and the curve $y = \ln(2x - 1)^2$ on the same axes, showing the roots 0, α and β . [3]

(b) By drawing a ‘staircase’ diagram on your sketch, starting with a value of x that is between α and β , show that this iteration does not converge to α . [1]

(c) Using this iterative formula with $x_1 = 3.75$, find the value of β correct to 3 decimal places. [3]



(OCR Further Pure Mathematics 2 [4726/01] Haf 2014)

9 The equation $10x - 8 \ln x = 28$ has a root α in the interval $[3, 4]$. The iteration $x_{n+1} = g(x_n)$, where $g(x) = 2.8 + 0.8 \ln x$ and $x_1 = 3.8$, is to be used to find α .

(i) Find the value of α correct to 5 decimal places. You should show the result of each step of the iteration to 6 decimal places. [4]

(ii) Illustrate this iteration by means of a sketch. [2]

A series of horizontal dotted lines for student work.