GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – C1 (LEGACY)
0973-01

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

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Mathematics C1 May 2018

Solutions and Mark Scheme

1. (*a*) (i) Gradient of $AB(DC)$ = $\frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -2$, gradient of $DC = -2$,
(or equivalent, at least one correct) A1

Gradient of AB = gradient of $DC \Rightarrow AB$ and DC are parallel
(c.a.o.) A1

(ii) A correct method for finding the equation of AB using candidate's gradient for AB M1

Equation of AB: $y - 7 = -2\,[x - (-2)]$ (or equivalent)
(f.t. candidate's gradient for AB) A1

(*b*) (i) Gradient $L = {}^1/_2$ B1

${}^1/_2 \times -2 = -1 \Rightarrow L$ is perpendicular to AB (o.e.)
(f.t. candidate's derived gradients for AB and L) B1

(ii) An attempt to solve equations of AB and L simultaneously M1

$x = -1$, $y = 5$ (convincing) A1

(iii) A correct method for finding the coordinates of the mid-point of AB M1

Mid-point of AB has coordinates (0, 3) A1

A correct method for finding the length of EF M1

$EF = \sqrt{5}$ (f.t. the candidate's derived coordinates of F) A1

(*c*) $ABCD$ is a trapezium B1

2. $\sqrt{500} = 10\sqrt{5}$ B1

$(\sqrt{12} \times \sqrt{15}) = 6\sqrt{5}$ B1

$\frac{7\sqrt{60}}{\sqrt{3}} = 14\sqrt{5}$ B1

$\sqrt{500} + (\sqrt{12} \times \sqrt{15}) - \frac{7\sqrt{60}}{\sqrt{3}} = 2\sqrt{5}$ (c.a.o.) B1

3. (*a*) y-coordinate of $P = -1$ B1

$\frac{dy}{dx} = 2x - 6$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 2$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = -2$ (c.a.o.) A1

Gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - (-1) = {}^{1}/_{2}(x - 2)$

Equation of normal to C at P: $y = {}^{1}/_{2}x - 2$ (convincing) A1

(*b*) $x^2 - 6x + 7 = {}^{1}/_{2}x - 2$ M1

An attempt to collect terms, form and solve the quadratic equation in x either by correct use of the quadratic formula or by writing the equation in the form $(ax + b)(cx + d) = 0$, with $a \times c$ = candidate's coefficient of x^2 and $b \times d$ = candidate's constant m1

$2x^2 - 13x + 18 = 0 \Rightarrow (2x - 9)(x - 2) = 0$ (or equivalent)

$\Rightarrow x = {}^{9}/_{2}, (x = 2)$ (c.a.o.) A1

At Q, $x = {}^{9}/_{2}, y = {}^{1}/_{4}$ (c.a.o.) A1

4. (*a*) $a = 4$ B1

$b = 5$ B1

$c = -169$ B1

(*b*) $4(x + 5)^2 = 169$ (f.t. candidate's values for *a*, *b*, *c*) M1

$(x + 5) = (\pm)\frac{13}{2}$ (f.t. candidate's values for *a*, *b*, *c*) m1

$x = \frac{3}{2}, -\frac{23}{2}$ (both values) (c.a.o.) A1

5. (*a*) $\left(1 - \frac{x}{2}\right)^7 = 1 - \frac{7}{2}x + \frac{21}{4}x^2 - \frac{35}{8}x^3 + \ldots$ B1 B1 B1 B1

(– 1 for further incorrect simplification)

(*b*) ${}^{n}C_2 \times 4^k = 3360$ (k = 1, **2**) M1

Either $16n^2 - 16n - 6720 = 0$ or $n^2 - n - 420 = 0$ or $n(n - 1) = 420$ A1

$n = 21$ (c.a.o.) A1

6. Finding critical values $x = -2$, $x = {}^{2}/_{9}$ B1

A statement (mathematical or otherwise) to the effect that

$x < -2$ or $x > {}^{2}/_{9}$ (or equivalent)

(f.t. candidate's derived critical values) B2

Deduct 1 mark for each of the following errors

the use of non-strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = 9(x + \delta x)^2 - 7(x + \delta x) - 8$ B1

Subtracting y from above to find δy M1

$\delta y = 18x\delta x + 9(\delta x)^2 - 7\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 18x - 7$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = k \times (-1) \times x^{-2} + 14 \times \frac{1}{2} \times x^{-1/2}$ B1, B1

Attempting to substitute $x = 9$ in candidate's expression for $\frac{dy}{dx}$ and putting expression equal to 2 M1

(The M1 is only awarded if at least one B1 has been awarded)

$k = 27$ (c.a.o.) A1

8. (*a*) Denoting $8x^3 + 7x^2 - 13x + 10$ by $f(x)$,

(i) **Either:** showing that $f(-2) = 0$

Or: trying to find $f(r)$ for at least two values of r M1

$f(-2) = 0 \Rightarrow x = -2$ is a root of $8x^3 + 7x^2 - 13x + 10 = 0$ A1

(ii) $f(x) = (x + 2)(8x^2 + mx + n)$ with one of m, n correct M1

$f(x) = (x + 2)(8x^2 - 9x + 5)$ A1

An expression for $b^2 - 4ac$ for the quadratic $8x^2 - 9x + 5 = 0$ with at least two of a, b or c correct

(f.t. candidate's derived quadratic expression) M1

$b^2 - 4ac = -79$ or $b^2 - 4ac < 0$

(f.t. candidate's derived quadratic expression) A1

$b^2 - 4ac < 0 \Rightarrow 8x^2 - 9x + 5 = 0$ has no real roots $\Rightarrow x = -2$ is the only real root of $8x^3 + 7x^2 - 13x + 10 = 0$

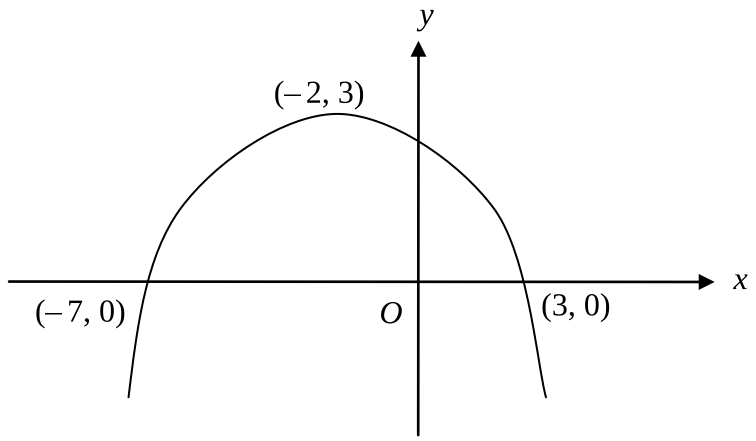
(f.t. candidate's derived **negative** value for $b^2 - 4ac$) A1

(*b*) Denoting $x^3 - 80$ by $g(x)$,

Use of $g(a) = 45$ M1

$a^3 - 80 = 45 \Rightarrow a = 5$ A1

9. (*a*)



Concave down curve with x-coordinate of maximum = – 2	B1
y-coordinate of maximum = 3	B1
Both points of intersection with x-axis	B1

(*b*)

$a = -2$	B1
$a = 3$	B1

10. (*a*) Height of box = $\frac{6000}{3x^2}$

(or an equivalent expression for the height) B1

$L = 4 \times (x + 3x + \frac{6000}{3x^2})$

(f.t. candidate's derived expression for height of box in terms of x) M1

$L = 16x + \frac{8000}{x^2}$ (convincing) A1

(*b*) $\frac{dL}{dx} = 16 - \frac{16000}{x^3}$ (o.e.) B1

Putting derived $\frac{dL}{dx} = 0$ M1

$x = 10$ (f.t. candidate's $\frac{dL}{dx}$ provided x is positive) A1

Stationary value of L at $x = 10$ is 240

(f.t. candidate's derived positive value for x) A1

A correct method for finding nature of the stationary point yielding a minimum value (provided x is positive) B1

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – C2 (LEGACY)
0974-01

INTRODUCTION

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Mathematics C2 May 2018

Solutions and Mark Scheme

1. (*a*)

1	0·6989700043		
1·75	0·9777236053		
2·5	1·146128036		
3·25	1·267171728		
4	1·361727836	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0·75$ M1

$I \approx \frac{0·75}{2} \times \{0·6989700043 + 1·361727836 + 2(0·9777236053 + 1·146128036 + 1·267171728)\}$

$I \approx 8·842744579 \times 0·75 \div 2$

$I \approx 3·316029217$

$I \approx 3·316$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

Special case for candidates who put $h = 0·6$

1	0·6989700043		
1·6	0·9344984512		
2·2	1·086359831		
2·8	1·198657087		
3·4	1·28780173		
4	1·361727836	(all values correct)	B1

Correct formula with $h = 0·6$ M1

$I \approx \frac{0·6}{2} \times \{0·6989700043 + 1·361727836 + 2(0·9344984512 + 1·086359831 + 1·198657087 + 1·28780173))$

$I \approx 11·07533204 \times 0·6 \div 2$

$I \approx 3·322599612$

$I \approx 3·323$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*) $\int_1^4 \log_{10} \sqrt{(6x - 1)}\, dx \approx 1·658$ (f.t. candidate's answer to (*a*)) B1

2. (*a*) $10\sin^2\theta + 3\sin\theta = 4(1 - \sin^2\theta) - 2$

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$14\sin^2\theta + 3\sin\theta - 2 = 0 \Rightarrow (2\sin\theta + 1)(7\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2},\ \sin\theta = \frac{2}{7}$ (c.a.o.) A1

$\theta = 210°, 330°$ B1 B1

$\theta = 16{\cdot}6°, 163{\cdot}4°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) Correct use of $\frac{\sin\phi}{\cos\phi} = \tan\phi$ (o.e.) M1

$\tan\phi = \frac{5}{3}$ A1

$\phi = 59°, 239°$ (f.t $\tan\phi = a$) B1

3. (*a*) (i) $\frac{1}{2} \times x \times (2x - 1) \times \sin 30° = 11{\cdot}25$

(substituting the correct values and expressions in the correct places in the area formula) M1

$2x^2 - x - 45 = 0$ A1

An attempt to solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c$ = candidate's coefficient of x^2 and $b \times d$ = candidate's constant m1

$(2x + 9)(x - 5) = 0 \Rightarrow x = 5$ (convincing) A1

(ii) $AC^2 = 5^2 + 9^2 - 2 \times 5 \times 9 \times \cos 30°$

(correct use of cos rule) M1

$AC = 5{\cdot}3$ cm (c.a.o.) A1

(*b*) $\frac{\sin XZY}{29} = \frac{\sin 17°}{16}$

(substituting the correct values in the correct places in the sin rule) M1

$XZY = 32°, 148°$ (at least one value) A1

Use of angle sum of a triangle = 180° M1

$YXZ = 131°, 15°$ (both values)

(f.t. candidate's values for XZY provided both M's awarded) A1

4. This is an A.P. with $a = P$, $d = -x$ (s.i.) B1

$\frac{24}{2} \times [2 \times P + 23 \times (-x)] = 3900$ B1

$P + 7 \times (-x) = 185$ B1

An attempt to solve candidate's derived equations simultaneously M1

$P = 220$, $x = 5$ (c.a.o.) A1

Alternative mark scheme

$\frac{24}{2} \times [2a + 23d] = 3900$ (B1)

$a + 7d = 185$ (B1)

An attempt to solve candidate's derived equations simultaneously (M1)

$a = 220$, $d = -5$ (c.a.o.) (A1)

Interpretation $\therefore P = 220$, $x = 5$ (c.a.o.) (B1)

5. (*a*)

$S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) (i)

$a + ar^2 = 340$ B1

$ar^3 + ar^5 = 73{\cdot}44$ B1

A correct method for solving the candidate's equations simultaneously e,g

multiplying the first equation by r^3 and subtracting

or eliminating a and $(1 + r^2)$ M1

$340r^3 = 73{\cdot}44$ (o.e.) A1

$r = 0{\cdot}6$ (convincing) A1

(ii)

$a + a \times 0{\cdot}6^2 = 340 \Rightarrow a = 250$ B1

$S_\infty = \frac{250}{1 - 0{\cdot}6}$ (correct use of formula for S_∞, f.t. candidate's derived value for a) M1

$S_\infty = 625$ (f.t. candidate's derived value for a) A1

6. (*a*) $\frac{x^{4/3}}{4/3} - 4 \times \frac{x^{-5/2}}{-5/2} + c$ (–1 if no constant term present) B1, B1

(*b*) Use of integration to find the area under the curve M1

$\int 25\mathrm{d}x = 25x$ $\int x^2\,\mathrm{d}x = (1/3)x^3$ (both correct) A1

Correct method of substitution of candidate's limits m1

$[25x - (1/3)x^3]_3^5 = (125 - 125/3) - (75 - (27/3)) = 52/3$

Use of a correct method to find the area of the triangle M1

Use of correct limits and trying to find the total area by adding the area of the triangle to the area under the curve m1

Shaded area = 64 + 52/3 = 244/3 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1

$xy = a^p \times a^q = a^{p+q}$ (the laws of indices) B1

$\log_a xy = p + q$ (the relationship between log and power)

$\log_a xy = p + q = \log_a x + \log_a y$ (convincing) B1

(*b*) $\log_a(11x^2 + 16x + 5) - \log_a(4x^2 + 1) = \log_a \left(\frac{11x^2 + 16x + 5}{4x^2 + 1}\right)$

(subtraction law) B1

$3 \log_a 2 = \log_a 2^3$ (power law) B1

$\frac{11x^2 + 16x + 5}{4x^2 + 1} = 2^3$ (removing logs) M1

An attempt to collect terms, form and solve quadratic equation with three terms in *x*, either by using the quadratic formula or by getting the expression into the form

$(ax + b)(cx + d)$, with $a \times c$ = candidate's coefficient of x^2 and

$b \times d$ = candidate's constant m1

$21x^2 - 16x + 3 = 0 \Rightarrow (7x - 3)(3x - 1) = 0 \Rightarrow x = {}^3/_7, {}^1/_3$

(both values, c.a.o.) A1

Note: Answer only with no working earns 0 marks

8.	(*a*)	(i)	$r^2 = (6-2)^2 + (1-(-1))^2$	B1
			Equation of C_1: $(x-2)^2 + (y-(-1))^2 = 20$	
			(f.t. candidate's derived value for r^2)	M1
			Equation of C_1: $x^2 + y^2 - 4x + 2y - 15 = 0$	
			(convincing)	A1
		(ii)	A correct method for finding Q	M1
			$Q(-2, -3)$	A1
		(iii)	Gradient $AP = \frac{\text{inc in } y}{\text{inc in } x}$	M1
			Gradient $AP = \frac{1-(-1)}{6-2} = \frac{1}{2}$ (o.e.)	A1
			Use of $m_{tan} \times m_{rad} = -1$	M1
			Equation of tangent is:	
			$y - 1 = -2(x-6)$ (f.t. candidate's gradient for AP)	A1
	(*b*)		Distance between centres of C_1 and $C_2 = 10$	B1
			Use of the fact that the shortest distance between the circles = distance between centres – sum of the radii	M1
			Shortest distance between the circles $= 10 - \sqrt{8} - \sqrt{20} = 2{\cdot}7$	
			(f.t. candidate's radius for C_1 and their distance between centres, provided the answer is positive)	A1
9.	(*a*)		$r + r + r\theta = 27$	B1
			$\frac{r^2\theta}{2} = 45$	B1
	(*b*)		A correct method for eliminating θ	M1
			$2r^2 - 27r + 90 = 0$ (convincing)	A1
	(c)		An attempt to solve given quadratic equation in r, either by using the quadratic formula or by getting the expression into the form $(ar + b)(cr + d)$, with $a \times c = 2$ and $b \times d = 90$	M1
			$(r-6)(2r-15) = 0 \Rightarrow r = 6,\ r = 7{\cdot}5$ (c.a.o.)	A1
			$\theta = 2{\cdot}5,\ \theta = 1{\cdot}6$ (f.t. r = 6, 7.5 using one of their equations in (*a*))	A1

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – C3 (LEGACY)
0975-01

INTRODUCTION

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Mathematics C3 May 2018

Solutions and Mark Scheme

1. (*a*)

0	1		
0·25	1·015747709		
0·5	1·133148453		
0·75	1·524817911		
1	2·718281828	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}25$ M1

$I \approx \frac{0{\cdot}25}{3} \times \{1 + 2{\cdot}718281828 + 4(1{\cdot}015747709 + 1{\cdot}524817911) + 2(1{\cdot}133148453)\}$

$I \approx 16{\cdot}14684121 \times 0{\cdot}25 \div 3$

$I \approx 1{\cdot}345570101$

$I \approx 1{\cdot}34557$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$\int_0^1 e^{x^3 - 1}\,dx = \frac{1}{e} \times \int_0^1 e^{x^3}\,dx$ (o.e.) M1

$\int_0^1 e^{x^3 - 1}\,dx = 0{\cdot}495$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $3(1 + \cot^2\theta) + 6\cot\theta = 8 - 5\cot^2\theta$

(correct use of $\text{cosec}^2\theta = 1 + \cot^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cot\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cot\theta + b)(c\cot\theta + d)$, with $a \times c$ = candidate's coefficient of $\cot^2\theta$ and $b \times d$ = candidate's constant m1

$8\cot^2\theta + 6\cot\theta - 5 = 0 \Rightarrow (2\cot\theta - 1)(4\cot\theta + 5) = 0$

$\Rightarrow \cot\theta = \frac{1}{2}, \cot\theta = -\frac{5}{4}$

$\Rightarrow \tan\theta = 2, \tan\theta = -\frac{4}{5}$ (c.a.o.) A1

$\theta = 63{\cdot}43°, 243{\cdot}43°$ B1

$\theta = 141{\cdot}34°, 321{\cdot}34°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\tan\theta$ = +, –, f.t. for 3 marks, $\tan\theta$ = –, –, f.t. for 2 marks
$\tan\theta$ = +, +, f.t. for 1 mark

(*b*) $\sec\phi \geq 1, \tan\phi \geq 0$ and thus $\sec\phi + 2\tan\phi$ cannot be less than 1 E1

3. $\frac{d}{dx}(x^5) = 5x^4$, $\frac{d}{dx}(17) = 0$ B1

$\frac{d}{dx}(4xy^2) = (4x)(2y)\frac{dy}{dx} + 4y^2$ B1

$\frac{d}{dx}(-2y^3) = -6y^2\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{4y^2 + 5x^4}{6y^2 - 8xy}$ (o.e.) (c.a.o.) B1

4. (*a*) candidate's *x*-derivative = $\frac{1}{t}$ B1

candidate's *y*-derivative = $16t^3 - 6t$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = 16t^4 - 6t^2$ (c.a.o.) A1

(*b*) (i) $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 64t^3 - 12t$ B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div$ candidate's *x*-derivative M1

$\frac{d^2y}{dx^2} = 64t^4 - 12t^2$ (c.a.o.) A1

(ii) An attempt to solve $64t^4 - 12t^2 - 1 = 0$ as a quadratic in t^2 M1

$(4t^2 - 1)(16t^2 + 1) = 0 \Rightarrow t^2 = {}^1/_4$ $(t^2 = -{}^1/_{16})$ A1

$t = {}^1/_2$ (c.a.o.) A1

5. (*a*) $\frac{d}{dx}[(2x - 5)\,e^{2x}] = e^{2x} \times f(x) + (2x - 5) \times g(x)$

$(f(x) \neq 1,\ g(x) \neq 1)$ M1

$\frac{d}{dx}[(2x - 5)\,e^{2x}] = e^{2x} \times 2 + (2x - 5) \times 2e^{2x}$ A1

$e^{2x} \times 2 + (2x - 5) \times 2e^{2x} + 12 = 0$ (o.e.)

(f.t. one slip in coefficients) A1

$e^{2x} \times (x - 2) + 3 = 0$ (convincing) A1

(*b*) $x_0 = 2$

$x_1 = 1{\cdot}945053083$ (x_1 correct, at least 4 places after the point) B1

$x_2 = 1{\cdot}938670473$

$x_3 = 1{\cdot}93788257$

$x_4 = 1{\cdot}937784608 = 1{\cdot}9378$ (x_4 correct to 4 decimal places) B1

Let $g(x) = e^{2x} \times (x - 2) + 3$

An attempt to check values or signs of $g(x)$ at $x = 1{\cdot}93775$

$x = 1{\cdot}93785$ M1

$g(1{\cdot}93775) = -8{\cdot}73 \times 10^{-4}$, $g(1{\cdot}93785) = 3{\cdot}35 \times 10^{-3}$ A1

Change of sign $\Rightarrow \alpha = 1{\cdot}9378$ (correct to four decimal places) A1

6.	(*a*)	(i)	$\frac{dy}{dx} = \frac{f(x)}{8 + 7x - 4x^3}$	$(f(x) \neq 1)$	M1
			$\frac{dy}{dx} = \frac{7 - 12x^2}{8 + 7x - 4x^3}$		A1
		(ii)	$\frac{dy}{dx} = \frac{1}{3} \times (5 - 9x^2)^{-2/3} \times g(x)$	$(g(x) \neq 1)$	M1
			$\frac{dy}{dx} = (5 - 9x^2)^{-2/3} \times (-6x)$		A1
		(iii)	$\frac{dy}{dx} = \frac{(4 - 3\cos x) \times m\cos x - (2 + 5\sin x) \times k\sin x}{(4 - 3\cos x)^2}$		
				(m = **5**, -5 k = **3**, -3)	M1
			$\frac{dy}{dx} = \frac{(4 - 3\cos x) \times 5\cos x - (2 + 5\sin x) \times 3\sin x}{(4 - 3\cos x)^2}$		A1
			$\frac{dy}{dx} = \frac{20\cos x - 6\sin x - 15}{(4 - 3\cos x)^2}$	(c.a.o.)	A1
	(*b*)		$5x = \tan y \Rightarrow 5 = \sec^2 y \times \frac{dy}{dx}$		B1
			$5 = (1 + \tan^2 y) \times \frac{dy}{dx}$		B1
			$5 = [1 + (5x)^2] \times \frac{dy}{dx}$		B1
			$\frac{dy}{dx} = \frac{5}{1 + 25x^2}$		B1

7. (*a*) (i) $\displaystyle\int \frac{5}{e^{3-4x}}\,dx = \int 5\,e^{4x-3}\,dx = k \times 5 \times e^{4x-3} + c$ ($k = 1, 4, -{}^1/_4$, $\mathbf{{}^1/_4}$) M1

$\displaystyle\int \frac{5}{e^{3-4x}}\,dx = {}^5/_4 \times e^{4x-3} + c$ (o.e.) A1

(ii) $\displaystyle\int \frac{6}{9x-4}\,dx = k \times 6 \times \ln|9x-4| + c$ ($k = 1$, $\mathbf{{}^1/_9}$, 9) M1

$\displaystyle\int \frac{6}{9x-4}\,dx = \frac{2}{3} \times \ln|9x-4| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) (i) $\displaystyle\int \cos\left(2x - \frac{\pi}{6}\right)dx = k \times \sin\left(2x - \frac{\pi}{6}\right)$

($k = 1, 2$, $\mathbf{{}^1/_2}$, $-{}^1/_2$) M1

$\displaystyle\int \cos\left(2x - \frac{\pi}{6}\right)dx = {}^1/_2 \times \sin\left(2x - \frac{\pi}{6}\right)$ A1

A correct method for substitution of the correct limits in an expression of the form $m \times \sin\left(2x - \frac{\pi}{6}\right)$ M1

$\displaystyle\int_{\pi/3}^{\pi/2} \cos\left(2x - \frac{\pi}{6}\right) dx = -\frac{1}{4}$

(f.t. only for solutions of $-\frac{1}{2}$, -1 from $k = 1$, $k = 2$, respectively) A1

Note: Answer only with no working shown earns 0 marks

(ii) $\cos(2x - \pi/6) < 0$ when $x \in (\pi/3, \pi/2)$ (o.e.)
and thus the integral will have a negative value E1

8. (*a*) Choice of θ, ϕ in **different quadrants** such that $\sin\theta = \sin\phi$ M1
$\sin 2\theta \neq \sin 2\phi$ (including correct evaluations) A1

(*b*) (i) $| x - 16 | = 5| x - 7 |$ B1
(ii) Trying to solve $x - 16 = 5(x - 7)$ M1
Trying to solve $x - 16 = -5(x - 7)$ M1
(f.t. candidate's values for *a*, *b* (*b* > 0), *c* for both M marks)
$x = 4·75$, $x = 8·5$ (c.a.o.) A1
F is either 11·25 km or 7·5 km from *T*
(f.t candidate's derived positive values for *x* if the first three marks have been awarded) A1

Alternative mark scheme for first three marks of (ii)
$(x - 16)^2 = 5^2 \times (x - 7)^2$ (attempting to square both sides) M1
$24x^2 - 318x + 969 = 0$
(o.e., at least 2 coefficients correct) M1
(f.t. candidate's values for *a*, *b* (*b* > 0), *c* for both M marks)
$x = 4·75$, $x = 8·5$ (c.a.o.) A1

9. (*a*) $y = 4 - \dfrac{7}{2 - 3x} \Rightarrow 4 \pm y = \pm\dfrac{7}{2 - 3x}$ (separating variables) M1
$2 - 3x = \pm\dfrac{7}{(4 \pm y)}$ m1
$x = \dfrac{1}{3}\left(2 - \dfrac{7}{(4 - y)}\right)$ (c.a.o.) A1
$f^{-1}(x) = \dfrac{1}{3}\left(2 - \dfrac{7}{(4 - x)}\right)$
(f.t. one slip in candidate's expression for *x*) A1

(*b*) $D(f^{-1}) = [0·5, 4)$
[0·5 B1
4) B1

10. (*a*) $D(fg) = (0, \infty)$ B1

(*b*) (i) $fg(x) = (5 - 3x)^2 + 2(5 - 3x) - 24$ B1
(ii) Putting expression for $fg(x)$ equal to 200, collecting terms and setting up a quadratic in *x* of the form $mx^2 + nx + p = 0$ M1
An attempt to solve quadratic equation in *x* either by using the quadratic formula or by getting the quadratic expression into the form $(ax + b)(cx + d)$, with $a \times c$ = candidate's coefficient of x^2 and $b \times d$ = candidate's constant m1
$9x^2 - 36x - 189 = 0 \Rightarrow x^2 - 4x - 21 = 0 \Rightarrow$
$(x + 3)(x - 7) = 0$ (o.e.) $\Rightarrow x = -3, 7$ (c.a.o.) A1
Reject $x = -3$, thus $x = 7$ (f.t. only for $x = -7, 3$) A1

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – C4 (LEGACY)
0976-01

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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Mathematics C4 June 2018

Solutions and Mark Scheme

1. (*a*) $f(x) \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ (correct form) M1

$3x^2 - 3x - 8 \equiv A(x-2)^2 + Bx(x-2) + Cx$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -2$, $C = -1$, $B = 5$ (all three coefficients correct) A2

(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)

(*b*) $\int f(x)\,\mathrm{d}x = -2\ln x + 5\ln(x-2) - (-1) \times (x-2)^{-1}$

(f.t. candidate's values for *A*, *B*, *C*)

(**at least one** the first and second terms) B1

(third term) B1

$\int_6^9 f(x)\,\mathrm{d}x = 1{\cdot}88$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2. (*a*) $2x - 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} - 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 0$

$\left(2x - 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ B1

$\left(-3x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y\right)$ B1

Either $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - 3y}{3y^2 + 3x}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}$ (o.e.) (c.a.o.) B1

Equation of tangent: $x = 3y + 1$ (convincing) B1

(*b*) $(3y+1)^2 - y^3 - 3(3y+1)y + 1 = 0$ M1

$y^3 - 3y - 2 = 0$ A1

Either:

$(y + 1)$ must be a (repeated) factor of $y^3 - 3y - 2$ M1

$y^3 - 3y - 2 = (y+1)(y+1)(y-2)$ A1

At *Q*, $y = 2$, $x = 7$ A1

Or:

Substitution of 2 for y in $y^3 - 3y - 2$ M1

$y = 2$ is a root of $y^3 - 3y - 2 = 0$ A1

At *Q*, $y = 2$, $x = 7$ A1

3. (*a*) $2(2\cos^2\theta - 1) = 3(1 - \cos^2\theta) - 5\cos^2\theta + \cos\theta + 1$
(correct use of $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin^2\theta = 1 - \cos^2\theta$) M1
An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1
$12\cos^2\theta - \cos\theta - 6 = 0 \Rightarrow (4\cos\theta - 3)(3\cos\theta + 2) = 0$
$\Rightarrow \cos\theta = \frac{3}{4}, \quad \cos\theta = -\frac{2}{3}$, (c.a.o.) A1
$\theta = 41{\cdot}41°, 318{\cdot}59°$ B1
$\theta = 131{\cdot}81°, 228{\cdot}19°$ B1 B1
Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) (i) $R = 13$ B1
Correctly expanding $\sin(\phi - \alpha)$, correctly comparing coefficients and using either $13\cos\alpha = 12$ **or** $13\sin\alpha = 5$ **or** $\tan\alpha = \frac{5}{12}$ to find α (f.t. candidate's value for R) M1
$\alpha = 22{\cdot}62°$ (c.a.o.) A1
(ii) $\sin(\phi - 22{\cdot}62°) = -\frac{2}{13}$
(f.t. candidate's values for R, α, $0° < \alpha < 90°$) B1
$\phi - 22{\cdot}62° = -8{\cdot}85°, 188{\cdot}85°, 351{\cdot}15°$ (at least one value)
(f.t. candidate's values for R, α, $0° < \alpha < 90°$) B1
$\phi = 13{\cdot}77°, 211{\cdot}47°$ (c.a.o.) B1

4. (*a*) $(1 + 2x)^{-2} = 1 - 4x + 12x^2$ $(1 - 4x)$ B1
$(12x^2)$ B1

(*b*) (i) $\left(\frac{1 + 3x}{1 + 2x}\right)^2 = (1 + 3x)^2 \times (1 - 4x + 12x^2)$ M1
$\left(\frac{1 + 3x}{1 + 2x}\right)^2 = 1 + (6x - 4x) + (9x^2 + 12x^2 - 24x^2)$
(at least two terms correct) A1
$\left(\frac{1 + 3x}{1 + 2x}\right)^2 = 1 + 2x - 3x^2$
(c.a.o.) A1
(ii) $|x| < {}^1/_2$ B1

5.	(*a*)	The *x*-coordinate of *Q* is *a*		B1
	(*b*) (i)	Volume = $\pi\int_0^a (a^2 - x^2)\,dx$		M1
		$\int (a^2 - x^2)\,dx = a^2x - \frac{x^3}{3}$		B1
		Volume = $\frac{2\pi a^3}{3}$	(c.a.o.)	A1
	(ii)	This is the volume of a hemisphere of radius *a*		E1

6. candidate's *x*-derivative = $-6t^{-3}$ (o.e.)

candidate's *y*-derivative = $12t^2$ (at least one term correct)

and use of

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\frac{12t^5}{6}$ (o.e.) A1

Equation of tangent at *P*: $y - 4p^3 = -\frac{12p^5}{6}\left(x - \frac{3}{p^2}\right)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

Equation of tangent at *P*: $y = -2p^5x + 10p^3$ (o.e) (c.a.o.) A1

7.	(*a*)	$u = 4x + 1 \Rightarrow du = 4dx$		(o.e.)	B1
		$dv = e^{4x-5}\,dx \Rightarrow v = \frac{1}{4}e^{4x-5}$		(o.e.)	B1
		$\int (4x+1)\,e^{4x-5}\,dx = \frac{1}{4}e^{4x-5} \times (4x+1) - \int \frac{1}{4}e^{4x-5} \times 4dx$		(o.e.)	M1
		$\int (4x+1)\,e^{4x-5}\,dx = \frac{1}{4}e^{4x-5} \times (4x+1) - \frac{1}{4}e^{4x-5} + c$			
		$\int (4x+1)\,e^{4x-5}\,dx = x\,e^{4x-5} + c$			A1
	(*b*)	(i)	$x = 2\sqrt{2} \Rightarrow \theta = \pi/4$		B1
			An attempt to express each of x^2, $\sqrt{(16 - x^2)}$ and dx in terms of θ only		M1
			$\int \frac{x^2}{\sqrt{(16 - x^2)}}\,dx = \int \frac{16\sin^2\theta \times 4\cos\theta\,d\theta}{\sqrt{(16 - 16\sin^2\theta)}}$		A1
			$\int \frac{x^2}{\sqrt{(16 - x^2)}}\,dx = \int 16\sin^2\theta\,d\theta$		A1
		(ii)	Use of $\sin^2\theta = \frac{(1 - \cos 2\theta)}{2}$		B1
			$\int (p + q\cos 2\theta)\,d\theta = p\theta + {}^1/_2\,q\sin 2\theta$	$(p \neq 0,\ q \neq 0)$	B1
			Correct substitution of candidate's upper limit and 0 in candidate's integrated expression of the form $m\theta + n\sin 2\theta$ $(m \neq 0,\ n \neq 0)$		M1
			$\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16 - x^2)}}\,dx = 2\pi - 4$	(c.a.o.)	A1

Note: Answer only with no working earns 0 marks

8.	(*a*)	$\frac{dV}{dt} = kV^{3/2}$		B1
	(*b*)	$\int \frac{dV}{V^{3/2}} = \int k\,dt$	(o.e.)	M1
		$\frac{V^{-1/2}}{-{}^1/_2} = kt + c$		A1
		Substituting $V = 900$, $t = 0$		M1
		$c = -\frac{1}{15}$	(c.a.o.)	A1
		Substituting $V = 1600$, $t = 3$		M1
		$k = \frac{1}{180}$	(f.t. one slip in the evaluation of *c*)	A1
		Substituting $t = 8$		M1
		$V = 8100$	(f.t. one slip in the evaluation of c and *k*)	A1

9. (*a*) $\mathbf{p}.\mathbf{q} = -37$ B1

$|\mathbf{p}| = \sqrt{62}$, $|\mathbf{q}| = \sqrt{53}$ (at least one correct) B1

Correctly substituting candidate's derived values in the formula

$\mathbf{p}.\mathbf{q} = |\mathbf{p}| \times |\mathbf{q}| \times \cos\theta$ M1

$\theta = 130{\cdot}2°$ (c.a.o.) A1

(*b*) (i) **Either:**

Position vector of $E = \frac{3}{4}(4\mathbf{a} - \mathbf{b}) + \frac{1}{4}(-10\mathbf{a} + 5\mathbf{b})$

(allow $\frac{1}{4}(4\mathbf{a} - \mathbf{b}) + \frac{3}{4}(-10\mathbf{a} + 5\mathbf{b})$) M1

Position vector of $E = \frac{1}{4}(12\mathbf{a} - 3\mathbf{b} - 10\mathbf{a} + 5\mathbf{b})$ A1

Position vector of $E = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ A1

Or:

An attempt to find **CD** and then use of position vector of E =

$\mathbf{OC} + \frac{1}{4}\mathbf{CD}$ (allow $\mathbf{OC} + \frac{3}{4}\mathbf{CD}$) M1

Position vector of $E = (4\mathbf{a} - \mathbf{b}) + \frac{1}{4}(-14\mathbf{a} + 6\mathbf{b})$ A1

Position vector of $E = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ A1

Or:

An attempt to find **CD** and then use of position vector of E =

$\mathbf{OD} - \frac{3}{4}\mathbf{CD}$ (allow $\mathbf{OD} - \frac{1}{4}\mathbf{CD}$) M1

Position vector of $E = (-10\mathbf{a} + 5\,\mathbf{b}) - \frac{3}{4}(-14\mathbf{a} + 6\mathbf{b})$ A1

Position vector of $E = \frac{1}{2}(\mathbf{a} + \mathbf{b})$ A1

(ii) *E* is the midpoint of *AB* E1

10. Assume that there is a real and positive value of x such that $25x + \frac{4}{x} < 20$

$25x^2 - 20x + 4 < 0$ B1

$(5x - 2)^2 < 0$ B1

This is impossible since the square of a real number cannot be negative and thus $25x + \frac{4}{x} \geq 20$ B1

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – FP1 (LEGACY)
0977-01

INTRODUCTION

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GCE MATHEMATICS – FP1

SUMMER 2018 FINAL MARK SCHEME

Ques	Solution	Mark	Notes
1(a)	Let $\dfrac{1}{n(n+1)} = \dfrac{A}{n} + \dfrac{B}{n+1} = \dfrac{A(n+1)+Bn}{n(n+1)}$	**M1**	Or by inspection.
	$A = 1$, $B = -1$ so $\dfrac{1}{n(n+1)} = \dfrac{1}{n} - \dfrac{1}{n+1}$	**A1**	
(b)	$S_n = \sum_{r=1}^{n}\left(\dfrac{1}{r} - \dfrac{1}{r+1}\right)$	**M1**	
	$= 1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots + \dfrac{1}{n}$ $- \dfrac{1}{2} - \dfrac{1}{3} - \ldots\ldots\ldots - \dfrac{1}{n+1}$	**A1**	
	$= 1 - \dfrac{1}{n+1}$	**A1**	
	$= \dfrac{n}{n+1}$ $(a = 1, b = 1)$	**A1**	
2(a)	METHOD 1		
	$(2+\mathrm{i})^4 = 2^4 + 4\times2^3\times\mathrm{i} + 6\times2^2\times\mathrm{i}^2 + 4\times2\times\mathrm{i}^3 + \mathrm{i}^4$	**M1**	
	$= -7 + 24\mathrm{i}$	**A1**	
	METHOD 2		
	$(2 + \mathrm{i})^2 = 3 + 4\mathrm{i}$	**B1**	
	$(2 + \mathrm{i})^4 = (3 + 4\mathrm{i})^2 = -7 + 24\mathrm{i}$	**B1**	
(b)	Denoting the quartic by $f(x)$,		
	$f(2+\mathrm{i}) = -7 + 24\mathrm{i} + 2(3+4i) - 32(2+\mathrm{i}) + 65$	**M1**	
	$= -7 + 6 - 64 + 65$		
	$+ \mathrm{i}(24 + 8 - 32) = 0$	**A1**	
	Therefore $2 + \mathrm{i}$ is a root.	**A1**	
(c)	It follows that $2 - \mathrm{i}$ is also a root.	**B1**	
	$x^2 - 4x + 5$ is a factor of the quadratic.	**B1**	
	The quartic factorises as follows.		
	$(x^4 + 2x^2 + 2x + 65) = (x^2 - 4x + 5)(x^2 + 4x + 13)$	**M1A1**	
	The other two roots are		
	$\dfrac{-4 \pm \sqrt{16-52}}{2}$	**M1**	Award no marks for unsupported roots
	ie $-2 \pm 3\mathrm{i}$	**A1**	

3(a)	$\frac{1+17\text{i}}{1+2\text{i}} = \frac{(1+17\text{i})(1-2\text{i})}{(1+2\text{i})(1-2\text{i})}$	**M1**	Award M0 for unsupported answer
	$= \frac{1+17\text{i}-2\text{i}-34\text{i}^2}{1+2\text{i}-2\text{i}-4\text{i}^2}$	**A1**	
	$= 7 + 3\text{i}$	**A1**	
(b)	Substituting $z = x + \text{i}y$,		FT from (a)
	$2\text{i}(x + \text{i}y) + 3(x - \text{i}y) = 7 + 3\text{i}$	**M1**	
	$-2y + 3x = 7$		
	$2x - 3y = 3$	**A1**	
	$(x,y) = (3,1)$	**A1**	
	$\lvert z\rvert = \sqrt{10}$, $\arg(z) = \tan^{-1}(1/3)$	**A1A1**	
	$z = 3.16(\cos 0.322 + \text{i}\sin 0.322)$	**A1**	Accept 18.4$^\text{o}$ for 0.322
4(a)	Rotation matrix = $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Translation matrix = $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	$\mathbf{T} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	$= \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$		Answer given
(b)	We need to solve		
	$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$	**M1**	
	$b - 1 = 1$ $-a + 2 = -1$	**A1**	A1 for both equations
	$(a,b) = (3,2)$	**A1**	A1 for both coordinates

5	$\alpha+\beta+\gamma=2$ $\beta\gamma+\gamma\alpha+\alpha\beta=4$ $\alpha\beta\gamma=-3$	**B1**	
	Let the roots be *a*,*b*,*c*.		
	$abc=\dfrac{1}{\alpha^2\beta^2\gamma^2}$	**M1**	
	$=\dfrac{1}{9}$	**A1**	
	$bc+ca+ab=\dfrac{1}{\alpha^2\beta\gamma}+\dfrac{1}{\beta^2\gamma\alpha}+\dfrac{1}{\gamma^2\alpha\beta}$	**M1**	
	$=\dfrac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha^2\beta^2\gamma^2}$	**A1**	
	$=\dfrac{4}{9}$	**A1**	
	$a+b+c=\dfrac{1}{\beta\gamma}+\dfrac{1}{\gamma\alpha}+\dfrac{1}{\alpha\beta}$	**M1**	
	$=\dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$	**A1**	
	$=-\dfrac{2}{3}$	**A1**	
	The required equation is $x^3+\dfrac{2}{3}x^2+\dfrac{4}{9}x-\dfrac{1}{9}=0$ (or equivalent)	**B1**	FT from their values

6(a)(i)	Determinant = $\lambda(3\lambda-2)+5-12+2(8-5\lambda)$	**M1**	
	$= 3\lambda^2-12\lambda+9$	**A1**	May be seen later
(ii)	Putting λ = 3, det = 27 – 36 + 9 = 0	**A1**	Accept solving the quadratic equation
	(So M is singular)		
	The other value is λ = 1.	**A1**	
(b)	$\begin{bmatrix}3&1&2\\4&3&1\\5&2&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\\mu\\2\end{bmatrix}$		
	$\begin{bmatrix}3&1&2\\0&5&-5\\0&1&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\3\mu-12\\-9\end{bmatrix}$	**M1** **A1**	
	We can see at this stage that the 2nd row is 5 times the 3rd row so that for consistency	**M1**	
	$3\mu-12=-45$		
	$\mu=-11$ cao	**A1**	
(c)(i)	Cofactor matrix = $\begin{bmatrix}4&-7&-2\\1&-4&1\\-3&6&0\end{bmatrix}$	**M1** **A1**	Award M1 if at least 5 elements are correct.
	Adj(**M**) = $\begin{bmatrix}4&1&-3\\-7&-4&6\\-2&1&0\end{bmatrix}$	**A1**	
(ii)	det(**M**) = -3	**B1**	
	$\mathbf{M}^{-1}=\dfrac{-1}{3}\begin{bmatrix}4&1&-3\\-7&-4&6\\-2&1&0\end{bmatrix}$	**A1**	FT their adg(**M**) from (i)

7	Putting n = 1 gives 1 which is correct so the result is true for n = 1. Let the result be true for $n = k$, ie $$\sum_{r=1}^{k} r^2 = \frac{k(k+1)(2k+1)}{6}$$ Consider (for $n = k + 1$), $$\sum_{r=1}^{k+1} r^2 = \sum_{r=1}^{k} r^2 + (k+1)^2$$ $$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$ $$= \frac{(k+1)(2k^2+k+6k+6)}{6}$$ $$= \frac{(k+1)(k+2)(2k+3)}{6}$$ $$= \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$$ Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	**B1** **M1** **M1** **A1** **A1** **A1** **A1**	 Award the final A1 for a perfect solution including the last line
8(a) **(b)**	Put $z = x + iy$ $\|x + i(y + 2)\| = 2\|(x - 3) + iy\|$ $x^2 + (y+2)^2 = 4(x-3)^2 + 4y^2$ $x^2 + y^2 + 4y + 4 = 4x^2 - 24x + 36 + 4y^2$ $x^2 + y^2 - 8x - \frac{4}{3}y + \frac{32}{3} = 0$ (This is the equation of a circle) $(x-4)^2 + (y-\frac{2}{3})^2 = -\frac{32}{3} + 16 + \frac{4}{9} = \frac{52}{9}$ Radius = $\frac{\sqrt{52}}{3}$ (or equiv) ; Centre = $\left(4, \frac{2}{3}\right)$ cao	 **M1** **A1** **A1** **A1** **M1** **A1A1**	 Accept 3 × LHS Attempt to complete the square in a circle equation Accept use of formulae
9(a) **(b)(i)** **(ii)**	Taking logs, $\ln f(x) = x\ln\sin x$ Differentiating, $\frac{f'(x)}{f(x)} = \ln\sin x + x\cot x$ (So $g(x) = \ln\sin x + x\cot x$) $g(0.1) = -1.3$ $g(1) = 0.47$ $g(1.6) = -0.047$ (Because $f(x) > 0$ in the given domain), the conclusion is that there are **at least two** stationary points on the graph of $f(x)$.	**M1** **A1A1** **B2** **B1**	 Award B1 for 2 correct values Accept anything which rounds to these values FT from (b)(i) if one B1 awarded above and there is a change of sign

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – FP2 (LEGACY)
0978-01

INTRODUCTION

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It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

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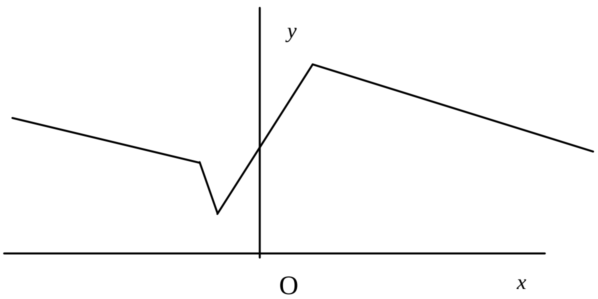
GCE MATHEMATICS – FP2

SUMMER 2018 MARK SCHEME

Ques	Solution	Mark	Notes
1	$\lvert 3 + 4i \rvert = 5$ $\arg(3 + 4i) = 0.9273$ Cube root 1 $= 5^{1/3}(\cos 0.3091 + i\sin 0.3091)$ $= 1.63 + 0.52i$ Cube root 2 $= 5^{1/3}(\cos(0.3091 + 2\pi/3)$ $+ i\sin(0.3091 + 2\pi/3))$ $= -1.26 + 1.15i$ Cube root 3 $= 5^{1/3}(\cos(0.3091 + 4\pi/3)$ $+ i\sin(0.3091 + 4\pi/3))$ $= -0.36 - 1.67i$	**B1** **B1** **M1M1** **A1** **M1** **A1** **A1**	 Accept 53.13 deg M0 no working
2	Consider for $x > 0$, $f(x) = -\sqrt{x}$ $f(-x) = \sqrt{x}$ Therefore f is an odd function.	 **M1** **A1** **A1**	Accept a well presented graphical solution M1 for considering $f(x)$ and $f(-x)$ First A1 for a correct consideration of $f(x)$ and $f(-x)$ Award the second A1 only if the first A1 is awarded
3(a) **(b)**	$3 + 2x - x^2 = 4 - (x^2 - 2x + 1)$ $= 4 - (x - 1)^2$ $\int_0^2 \frac{1}{\sqrt{3 + 2x - x^2}}dx = \int_0^2 \frac{1}{\sqrt{4 - (x-1)^2}}dx$ $= \left[\sin^{-1}\frac{(x-1)}{2}\right]_0^2$ $= \frac{\pi}{6} - \left(-\frac{\pi}{6}\right)$ $= \frac{\pi}{3}$ cao	**M1** **A1** **M1** **A1** **A1**	 FT from (a) if of the form $a - (x - b)^2$

4(a)	Putting $t = \tan\frac{x}{2}$, $\frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2} = 2$ $1 + 2t + t^2 = 2 - 2t^2$ $3t^2 + 2t - 1 = 0$	**M1A1** **A1**	
(b)	$(3t-1)(t+1) = 0$ $t = -1, 1/3$ $t = -1$ gives $\frac{x}{2} = -\frac{\pi}{4} + n\pi$ $x = -\frac{\pi}{2} + 2n\pi$ This however is not a possible solution (since both sec*x* and tan*x* are not defined for these values.) $t = 1/3$ gives $\frac{x}{2} = 0.3217.. + n\pi$ $(18.43^\circ ... + 180n^\circ)$ $x = 0.644 + 2n\pi$ $(36.9^\circ + 360n^\circ)$	**M1** **A1** **M1** **A1** **B1** **M1** **A1**	Accept $360n^\circ - 90^\circ$ Award all 3 marks if this comment, or equivalent, is made.
5(a)	$\frac{5}{(x+1)(x^2+4)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+4)}$ $= \frac{A(x^2+4) + (Bx+C)(x+1)}{(x+1)(x^2+4)}$ $A = 1$ $B = -1$ $C = 1$	**M1** **A1** **A1** **A1**	
(b)	$\int_1^2 \frac{5}{(x+1)(x^2+4)}\,dx = \int_1^2 \frac{1}{x+1}\,dx + \int_1^2 \frac{1}{(x^2+4)}\,dx - \int_1^2 \frac{x}{(x^2+4)}\,dx$ $= [\ln(x+1)]_1^2 + \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_1^2 - \left[\frac{1}{2}\ln(x^2+4)\right]_1^2$ $= \ln 3 - \ln 2 + \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}\frac{1}{2} - \frac{1}{2}\ln 8 + \frac{1}{2}\ln 5$ $= 0.331$	**M1** **A1A1A1** **A1** **A1**	FT from (a) if M1 awarded in (a) M0 no working Limits can be seen later This line seen or implied 3 sig figs required if FT applied

6(a)	Using deMoivre's Theorem, $z^n - z^{-n} = \cos n\theta + \mathrm{i}\sin n\theta - \cos(-n\theta) - \mathrm{i}\sin(-n\theta)$ $= \cos n\theta + \mathrm{i}\sin n\theta - \cos(n\theta) + \mathrm{i}\sin(n\theta)$ $= 2\mathrm{i}\sin n\theta$ $z^n + z^{-n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ $= 2\cos n\theta$	**M1** **A1** **M1** **A1**	This line must be seen
(b)	$(z - z^{-1})^3(z + z^{-1}) = (z^3 - 3z + 3z^{-1} - z^{-3})(z + z^{-1})$ $= z^4 - 3z^2 + 3 - z^{-2} + z^2 - 3 + 3z^{-2} - z^{-4}$ $= z^4 - z^{-4} - 2z^2 + 2z^{-2}$ $-8\mathrm{i}\sin^3\theta \times 2\cos\theta = 2\mathrm{i}\sin 4\theta - 4\mathrm{i}\sin 2\theta$ $\sin^3\theta\cos\theta = \frac{1}{4}\sin 2\theta - \frac{1}{8}\sin 4\theta$	**M1** **A1** **A1A1** **A1**	
7(a)(i)	Use of $b^2 = a^2(1 - e^2)$ $e = \frac{\sqrt{5}}{3}$	**M1** **A1**	
(ii)	Coordinates of the foci are $(\pm\sqrt{5}, 0)$	**A1**	FT their value of *e* provided <1
(iii)	Equations of the directrices are $x = \pm\frac{9}{\sqrt{5}}$	**A1**	
(b)	Differentiating, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} / \frac{\mathrm{d}x}{\mathrm{d}\theta} = -\frac{2\cos\theta}{3\sin\theta}$ Grad of normal = $\frac{3\sin\theta}{2\cos\theta}$ Equation of normal is $y - 2\sin\theta = \frac{3\sin\theta}{2\cos\theta}(x - 3\cos\theta)$ $3x\sin\theta - 2y\cos\theta = 5\sin\theta\cos\theta$ (or equivalent)	**M1A1** **A1** **M1** **A1**	
(c)	The normal meets the *x*-axis where *y* = 0 Coords of A are $\left(\frac{5}{3}\cos\theta, 0\right)$ Coords of B are $\left(0, -\frac{5}{2}\sin\theta\right)$ Coords of midpoint are $\left(\frac{5}{6}\cos\theta, -\frac{5}{4}\sin\theta\right)$ The equation of the locus is $\frac{x^2}{25/36} + \frac{y^2}{25/16} = 1$ (This is the equation of an ellipse)	 **M1A1** **A1** **A1** **A1**	FT their equation from (b) if both M1s awarded M1 for either *x* or *y* intercept Or those are the parametric coordinates of an ellipse

8(a)	$y = 1$	**B1**	
(b)(i)	$f'(x) = \dfrac{(1+2x)(1-x+x^2)-(2x-1)(1+x+x^2)}{(1-x+x^2)^2}$	**M1**	Accept differentiation of $1+\dfrac{2x}{1-x+x^2}$
	$= \dfrac{1-x+x^2+2x-2x^2+2x^3-2x-2x^2-2x^3+1+x+x^2}{(1-x+x^2)^2}$	**A1**	
	$= \dfrac{2-2x^2}{(1-x+x^2)^2}$	**A1**	
	The stationary points satisfy $f'(x)=0$, therefore	**M1**	
	(1,3) and $\left(-1,\dfrac{1}{3}\right)$	**A1**	
(ii)	When $x > 1$, $f'(x) < 0$ so (1,3) is a maximum.	**M1A1**	Accept correct numerical substitution M1 for either point
	When $x < -1$, $f'(x) < 0$ so $\left(-1,\dfrac{1}{3}\right)$ is a minimum.	**A1**	
(c)		**G2**	G1 correct shape G1 completely correct including asymptotic approach (but asymptote need not be drawn)
(d)	Consider $\dfrac{1+x+x^2}{1-x+x^2} = 2$	**M1**	
	$x^2-3x+1=0$	**A1**	
	$x = \dfrac{3\pm\sqrt{5}}{2}$	**A1**	Accept 0.382, 2.62
	$f^{-1}(S) = \left(\dfrac{3-\sqrt{5}}{2},1\right) \cup \left(1,\dfrac{3+\sqrt{5}}{2}\right)$	**A1A1**	Award A1if the corresponding single interval given including 1 – 1if [ ] used instead of ( )

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – FP3 (LEGACY)
0979-01

INTRODUCTION

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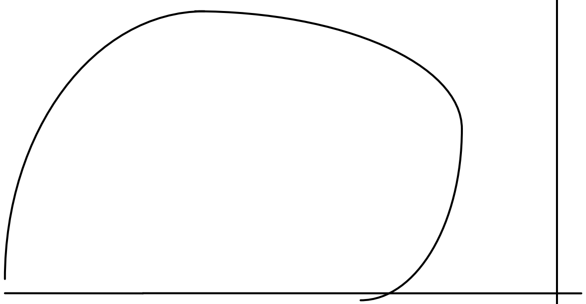
GCE MATHEMATICS – FP3

SUMMER 2018 MARK SCHEME

Ques	Solution	Mark	Notes
1(a)	$f'(x) = 3\cosh^2 x \sinh x - 2\cosh x$	**B1B1**	
(b)	At a stationary point,		
	$3\cosh^2 x \sinh x - 2\cosh x = 0$	**M1**	FT from (a) provided chain rule used
	We can cancel cosh$x$ since it cannot be zero (or equivalent)	**A1**	
	$3\sinh x \cosh x = 2$	**A1**	Candidates who substitute exponential functions:
	$\sinh 2x = \frac{4}{3}$	**A1**	$e^{2x} = 3$ A1
	$x = \frac{1}{2}\sinh^{-1}\left(\frac{4}{3}\right)$=0.549	**A1**	$x = \frac{1}{2}\ln 3 = 0.549$ A1
2(a)	$f(x) = \ln\cos x \quad f(0) = 0$	**B1**	
	$f'(x) = -\tan x \quad f'(0) = 0$	**B1**	
	$f''(x) = -\sec^2 x \quad f''(0) = -1$	**B1**	
	$f'''(x) = -2\sec^2 x \tan x \quad f'''(0) = 0$	**B1**	
	$f''''(x) = -4\sec^2 x \tan^2 x - 2\sec^4 x \quad f''''(0) = -2$	**B1**	
	Use of $\ln\cos x = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + ...$	**M1**	FT values of $f^n(0)$ provided at least two terms
	$= -\frac{x^2}{2} - \frac{x^4}{12} + ...$	**A1**	
(b)(i)			FT from (a) provided at least two terms
	$\ln\sec^2 x = 2\ln\sec x$	**M1**	
	$= -$ 2ln$\cos x$	**A1**	
	$= x^2 + \frac{x^4}{6} + ...$	**A1**	
(ii)	Differentiating,	**M1**	
	$\tan x = x + \frac{x^3}{3} + ...$	**A1**	

3	EITHER		
	$I = -\frac{1}{3}\int_0^{\pi/3} e^{2x}\,\mathrm{d}(\cos 3x)$	**M1**	
	$= -\frac{1}{3}\left[e^{2x}\cos 3x\right]_0^{\pi/3} + \frac{2}{3}\int_0^{\pi/3} e^{2x}\cos 3x\,\mathrm{d}x$	**A1A1**	
	$= \frac{1}{3}\left(e^{2\pi/3}+1\right) + \frac{2}{9}\int_0^{\pi/3} e^{2x}\,\mathrm{d}(\sin 3x)$	**A1A1**	
	$= \frac{1}{3}\left(e^{2\pi/3}+1\right) + \frac{2}{9}\left[e^{2x}\sin 3x\right]_0^{\pi/3} - \frac{4}{9}I$	**A1**	
	$\frac{13}{9}I = \frac{1}{3}\left(e^{2\pi/3}+1\right)$	**m1**	
	$I = \frac{3}{13}\left(e^{2\pi/3}+1\right)$	**A1**	
	OR		
	$I = \frac{1}{2}\int_0^{\pi/3} \sin 3x\,\mathrm{d}\left(e^{2x}\right)$	**(M1)**	
	$= \frac{1}{2}\left[e^{2x}\sin 3x\right]_0^{\pi/3} - \frac{3}{2}\int_0^{\pi/3} e^{2x}\cos 3x\,\mathrm{d}x$	**(A1A1)**	
	$= 0 - \frac{3}{4}\int_0^{\pi/3} \cos 3x\,\mathrm{d}\left(e^{2x}\right)$	**(A1A1)**	
	$= -\frac{3}{4}\left[e^{2x}\cos 3x\right]_0^{\pi/3} - \frac{9}{4}I$	**(A1)**	
	$\frac{13}{4}I = \frac{3}{4}\left(e^{2\pi/3}+1\right)$	**(m1)**	
	$I = \frac{3}{13}\left(e^{2\pi/3}+1\right)$	**(A1)**	

4(a)		**G1**	
(b)	Consider $y = r\sin\theta = (1-\cos\theta)\sin\theta$	**M1**	
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \sin^2\theta + \cos\theta(1-\cos\theta)$	**M1A1**	
	We require $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \sin^2\theta + \cos\theta(1-\cos\theta) = 0$	**M1**	
	$1-\cos^2\theta + \cos\theta - \cos^2\theta = 0$		
	$2\cos^2\theta - \cos\theta - 1 = 0$	**A1**	
	$\cos\theta = -\frac{1}{2}$	**A1**	
	Coords are $\left(\frac{3}{2}, \frac{2\pi}{3}\right)$	**A1**	Accept 120^{o}
(c)	Use of Area = $\frac{1}{2}\int r^2\mathrm{d}\theta$	**M1**	
	$= \frac{1}{2}\int_0^{\pi}(1-\cos\theta)^2\mathrm{d}\theta$	**A1**	
	$= \frac{1}{2}\int_0^{\pi}\left(1-2\cos\theta+\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$	**A1**	Only award the last three marks if working shown
	$= \frac{1}{2}\left[\frac{3\theta}{2} - 2\sin\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi}$	**A1**	
	$= \frac{3\pi}{4}$ (2.36)	**A1**	

5(a)	EITHER $y^2 = 4x$ $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{y}$ $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \frac{4}{y^2} = \frac{1}{x}$	 M1 A1 A1	
	OR $y = 2x^{1/2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{x}}$ $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \frac{1}{x}$	 (M1) (A1) (A1)	
	Arc length = $\int_1^4 \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ $= \int_1^4 \sqrt{1+\frac{1}{x}}\,\mathrm{d}x$	 A1	 Convincing
(b)	$x = \sinh^2 u$ $\mathrm{d}x = 2\sinh u \cosh u\,\mathrm{d}u\,; [1,4] \to [\sinh^{-1} 1, \sinh^{-1} 2]$	 B1	 Limits are 0.881 and 1.44
	Arc length = $\int_{\sinh^{-1} 1}^{\sinh^{-1} 2} \sqrt{1+\frac{1}{\sinh^2 u}} \times 2\sinh u \cosh u\,\mathrm{d}u$	M1	M0 no working
	$= \int_{\sinh^{-1} 1}^{\sinh^{-1} 2} \sqrt{\frac{\cosh^2 u}{\sinh^2 u}} \times 2\sinh u \cosh u\,\mathrm{d}u$	A1	
	$= \int_{\sinh^{-1} 1}^{\sinh^{-1} 2} 2\cosh^2 u\,\mathrm{d}u$	A1	
	$= \int_{\sinh^{-1} 1}^{\sinh^{-1} 2} (1+\cosh 2u)\,\mathrm{d}u$	A1	
	$= \left[u + \frac{1}{2}\sinh 2u\right]_{\sinh^{-1} 1}^{\sinh^{-1} 2}$	A1	
	$= \sinh^{-1} 2 - \sinh^{-1} 1 + 2\sqrt{5} - \sqrt{2}$ = 3.62	A1 A1	Award penultimate A1if line not seen but correct answer given

6(a)	A: $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sinh^{-1}\cos\theta\right)=\frac{1}{\sqrt{1+\cos^2\theta}}\times-\sin\theta$ When θ = 0.7, value of derivative = – 0.512 The iteration A is convergent because |– 0.512| < 1 B: $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\cos^{-1}\sinh\theta\right)=-\frac{1}{\sqrt{1-\sinh^2\theta}}\times\cosh\theta$ When θ = 0.7, value of derivative = – 1.93 The iteration B is divergent because |– 1.93| > 1	M1A1 A1 A1 M1A1 A1 A1	
(b)	Successive values are 0.7 0.7049... 0.7024... 0.7037... 0.7030... 0.7034... Value of root = 0.703 correct to three decimal places.	M1A1 A1 A1	At least 4 decimal places required in the table

7(a)	$I_n = \frac{1}{3}\int_1^2 (\ln x)^n \mathrm{d}\left(x^3\right)$	**M1**	
	$= \frac{1}{3}\left[x^3(\ln x)^n\right]_1^2 - \frac{n}{3}\int_1^2\left(x^3(\ln x)^{n-1}\times\frac{1}{x}\right)\mathrm{d}x$	**A1A1**	
	$= \frac{1}{3}\left[8(\ln 2)^n\right] - \frac{n}{3}\int_1^2 x^2(\ln x)^{n-1}\mathrm{d}x$	**A1A1**	
	$= \frac{8}{3}(\ln 2)^n - \frac{n}{3}I_{n-1}$		Convincing
(b)	EITHER Using the above formula,		
	$I_3 = \frac{8}{3}(\ln 2)^3 - I_2$	**M1**	M0 no working
	$= \frac{8}{3}(\ln 2)^3 - \left(\frac{8}{3}(\ln 2)^2 - \frac{2}{3}I_1\right)$	**A1**	
	$= \frac{8}{3}(\ln 2)^3 - \frac{8}{3}(\ln 2)^2 + \frac{2}{3}\left(\frac{8}{3}(\ln 2) - \frac{1}{3}I_0\right)$	**A1**	
	$= \frac{8}{3}(\ln 2)^3 - \frac{8}{3}(\ln 2)^2 + \frac{16}{9}(\ln 2) - \frac{2}{9}\int_1^2 x^2\mathrm{d}x$	**A1**	
	$= \frac{8}{3}(\ln 2)^3 - \frac{8}{3}(\ln 2)^2 + \frac{16}{9}(\ln 2) - \frac{14}{27}$	**B1**	
	= 0.321 cao	**A1**	
	OR		
	$I_0 = \int_1^2 x^2\mathrm{d}x = \left[\frac{x^3}{3}\right]_1^2 = \frac{7}{3}$	**(B1)**	FT their I_0
	$I_1 = \frac{8}{3}\ln 2 - \frac{7}{9}$	**(M1A1)**	
	$I_2 = \frac{8}{3}(\ln 2)^2 - \frac{2}{3}\left(\frac{8}{3}\ln 2 - \frac{7}{9}\right)$	**(A1)**	
	$I_3 = \frac{8}{3}(\ln 2)^3 - \frac{8}{3}(\ln 2)^2 + \frac{16}{9}(\ln 2) - \frac{14}{27}$	**(A1)**	
	= 0.321 cao	**(A1)**	

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – M1 (LEGACY)
0980-01

INTRODUCTION

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GCE MATHEMATICS – M1

SUMMER 2018 MARK SCHEME

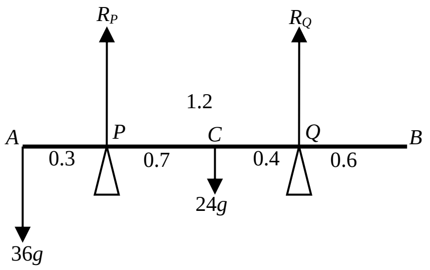
Q	Solution	Mark	Notes
1(a)(i)	N2L on lift, upwards +ve	M1	dim correct, all forces T and 1200g opposing
	$T - 1200g = 1200a$		
	$T = 1200(9.8 +0\ 2)$	A1	any correct form
	$T = 14160$ (N)	A1	cao
1(a)(ii)	$T = 1200g$ (= 11760) (N)	B1	
1(b)	$Mg - R = Ma$	M1	dim correct, all forces No extra
	$M(9.8 - 3) = 442$	A1	any correct form
	$M = 65$	A1	cao

Q	Solution	Mark	Notes
2	Resolve in one direction	M1	obtain comp of resultant All forces, no extra
	$X = 16 - 9\cos75^{\circ} - 21\sin60^{\circ}$ $X = -4.5159$	A1	
	Resolve in perpendicular direction	M1	obtain comp of resultant All forces, no extra.
	$Y = 8 + 21\cos60^{\circ} - 9\sin75^{\circ}$ $Y = 9.8067$	A1	
	$\text{Resultant}^2 = 4.5159^2 + 9.8067^2$	m1	dep on both M's
	Resultant = <u>10.8 (N)</u>	A1	cao
	$\theta = \tan^{-1}\left(\frac{4.5159}{9.8067}\right)$	m1	
	$\theta = \underline{24.7^{\circ}}$	A1	cao

Note

−1 if answers not 1 d.p.

Q	Solution	Mark	Notes
3.			

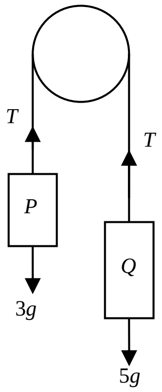


	Moments about *P*	M1	dim correct equation
			All forces, no extra
	$36g\times0.3 + R_Q\times1.1 = 24g\times0.7$	B1	any correct moment
		A1	correct equation
	$R_Q = 53.45$ (N)	A1	cao
	Resolve vertically	M1	dim correct equation
			All forces, no extra
	$R_Q + R_P = 36g + 24g$	A1	
	$R_P = 534.55$ (N)	A1	cao

Notes

	Moments about any point	M1	same conditions as above
	Correct moment	B1	
	Correct equation	A1	
	Attempt at second equation	M1	
	Correct equation	A1	
	Correct answers	A1A1	

Q	Solution	Mark	Notes
4(a)			



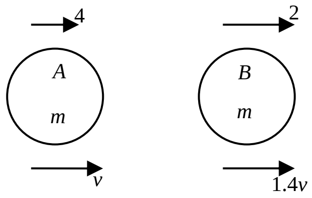
	Apply N2L to *Q*	M1	5*g* and *T* opposing, dim. correct
	$5g - T = 5a$	A1	correct equ, allow –ve *a*
	Apply N2L to *P*	M1	4*g* and *T* opposing, dim. Correct
	$T - 3g = 3a$	A1	correct equ consistent with first equation
	Adding	m1	
	$8a = 2g$		
	a = <u>2.45 (ms^{-2})</u>	A1	cao
	T = <u>36.75 N</u>	A1	cao
4(b)	Light string gives rise to tension constant throughout the string.	B1	
4(c)	Smooth peg means that the tensions in the strings on both sides of the peg are equal.	B1	

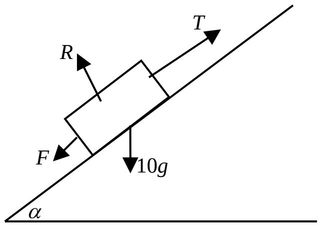
<u>Notes</u> (Newton's method)

	Attempt at Newton's method	M1	forces subtracted, masses added
	$5g - 3g = (5+3)a$	A1	
	$a = 2.45$ (ms^{-2})	A1	cao
N2L applied to either particle		M1	wt and *T* opposing, dim. Correct
	$T - 3g = 3a$	A1	
	$T = 3(9.8 + 2.45)$	m1	substitution
	T = <u>36.75 N</u>	A1	cao

Q	Solution	Mark	Notes
5(a)	I = change in momentum	M1	used
	$I = 0.16(20 - (-12))$		
	$I = 5.12$ (Ns)	A1	
5(b)	$I = Ft$	M1	used
	$5.12 = F \times \frac{1}{8}$		
	$F = 40.96$ (N)	A1	ft answer in (a)

Q	Solution	Mark	Notes
6(a)	Vel of A when B starts to fall		
	$v^2 = u^2 + 2as$, u=0, a=(±)9.8, s=(±)0.1	M1	oe complete method
	$v^2 = 0 + 2 \times 9.8 \times 0.1$	A1	
	$v = \frac{7}{5}$	A1	cao
6(b)	Vel of A when it reaches the ground		
	$v^2 = u^2 + 2as$, u=0, a=(±)9.8, s=(±)40	M1	
	$v^2 = 0 + 2 \times 9.8 \times 40$		
	$v = 28$	A1	
6(c)	Time of travel of B = time for A to reach ground		
	$v = u + at$, $u=\frac{7}{5}$, v=28, a=9.8	M1	
	$28 = \frac{7}{5} + 9.8t$	A1	ft (a) and (b)
	$t = \frac{19}{7}$	A1	
	Distance travelled by B in that time		
	$s = ut + \frac{1}{2}at^2$, u=0, a=9.8, $t=\frac{19}{7}$	M1	
	$s = 0 + \frac{1}{2} \times 9.8 \times \left(\frac{19}{7}\right)^2$	A1	ft candidates' 19/7
	$s = 36.1$	A1	cao
	Distance between A and B = 40 – 36.1		
	= 3.9 (m)	A1	cao

Q	Solution	Mark	Notes
7(a)			
	Conservation of momentum	M1	dim correct equ.
	$4m + 2m = mv + m \times 1.4v$	A1	
	$2.4v = 6$		
	$v = 2.5$	A1	cao
	Restitution	M1	no more than 1 sign error
	$1.4 \times 2.5 - 2.5 = -e(2 - 4)$	A1	ft v in (a)
	$e = 0.5$	A1	ft v in (a) provided $0<e<1$.
7(b)	Speed of B after collision = v'		
	$v' = 3.5 \times 0.6$	M1	ft v_A
	$v' = 2.1$ (ms^{-1})	A1	ft v_A
7(c)	Distance between A and B at time of collision with the wall = $(3.5 - 2.5)\times 5$ $= 5$	B1	ft v, $1.4v$
	After collision with wall, A and B approach each other with velocity = $2.1 + 2.5 = 4.6$	B1	ft v, v'
	Time to second collision between A and B $= \frac{5}{4 \cdot 6}$		
	= 1.09 (s)(correct to 2 d.p.)	B1	cao

Q	Solution	Mark	Notes
8.			
			
	Resolve perpendicular to plane		
	$R = 10g\cos\alpha$	B1	
	$F = 10g\mu\cos\alpha$		
	With T acting upwards		
	N2L applied to particle	M1	dim correct, all forces
	$T - F - mg\sin\alpha = ma$	A1	
	$98 - F - 10g\sin\alpha = 0$		
	With T acting downwards		
	N2L applied to particle	M1	dim correct, all forces
	$F - T' - 10g\sin\alpha = ma$	A1	
	$F - 49 - 10g\sin\alpha = 0$		
	Adding		
	$98 - 49 = 20\times9.8\times\sin\alpha$		m1
	$\sin\alpha = \frac{1}{4}$	A1	
	$\cos\alpha = \frac{\sqrt{15}}{4}$		
	$\mu = \frac{F}{R}$	M1	
	$\mu = \dfrac{49 + 10\times9\cdot8\times0\cdot25}{10\times9\cdot8\times\frac{\sqrt{15}}{4}}$		
	$\mu = \frac{\sqrt{15}}{5} = \sqrt{\frac{3}{5}} = 0.7746$	A1	cao

Q	Solution			Mark	Notes
9(a).	$\bar{x} = 4$ (cm)			B1	
9(b)	Shape	mass	distance(y)		
	ABCE	40	2.5	B1	2.5
	ECD	36	8	B1	8
	PQR	12	7	B1	7
	ABCDE	64	$\bar{y}$	B1	areas
	Moments about *AB*			M1	dim correct equation
	$64\bar{y} + 12\times7 = 40\times2.5 + 36\times8$			A1	ft table if consistent
	$\bar{y} = 4.75$ (cm)			A1	cao

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – M2 (LEGACY)
0981-01

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

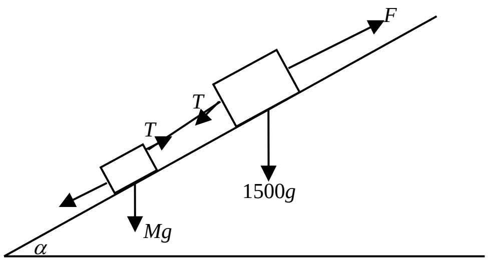
GCE MATHEMATICS – M2

SUMMER 2018 MARK SCHEME

Q	Solution	Mark	Notes
1(a).	$a = \frac{dv}{dt}$	M1	attempt to differentiate
	$a = \frac{1}{20}(16 - 2t)$	A1	any correct form isw
	$a = \frac{1}{10}(8 - t)$		
1(b)	max vel. when $\frac{dv}{dt} = 0$	M1	used
	$t = 8$	A1	cao
	$\frac{d^2v}{dt^2} = -0.1 < 0$, hence maximum	A1	oe, eg –ve quadratic
	max $v = \frac{1}{20}(80 + 16 \times 8 - 8^2)$		
	max $v = 7.2$ (ms^{-1})	A1	cao
1(c)	Distance = $\frac{1}{20}\int_0^{20} 80 + 16t - t^2 \, dt$	M1	attempt to integrate
	Distance = $\frac{1}{20}\left[80t + 8t^2 - \frac{t^3}{3}\right]_0^{20}$	A1	correct integration
	Distance = $\frac{1}{20}\left[80 \times 20 + 8 \times 20^2 - \frac{20^3}{3}\right]$	m1	limits/constant
	Distance = $106\frac{2}{3}$ (m)	A1	cao

Q	Solution	Mark	Notes
2(a)	$\mathbf{v} = \frac{d}{dt}\mathbf{r}$	M1	vector required.
	$\mathbf{v} = (6t)\mathbf{i} + (\cos 4t - 4t\sin 4t)\mathbf{j}$	A1	
	Momentum when $t = 0$ is $8\mathbf{j}$	A1	substitute t=0 and ×8
2(b)	$KE = \frac{1}{2}m\lvert v\rvert^2$	M1	used
	when $t = \pi$, $\mathbf{v} = 6\pi\mathbf{i} + \mathbf{j}$		
	$v^2 = 36\pi^2 + 1$	M1	method for $\lvert v\rvert^2$
	$KE = \frac{1}{2} \times 8 \times (36\pi^2 + 1)$		
	$KE = 4(36\pi^2 + 1)$		
	KE = 1425(.223 J)	A1	cao
2(c)	$\mathbf{a} = \frac{d}{dt}\mathbf{v}$	M1	ft c's $\mathbf{v}$
	$\mathbf{a} = (6)\mathbf{i} + (-4\sin 4t - 4\sin 4t - 16t\cos 4t)\mathbf{j}$	A1	cao
	Force $\mathbf{F} = 8\mathbf{a}$		
	Force $\mathbf{F} = 8(6\mathbf{i} - 16\pi\mathbf{j})$	m1	x8 and substitute π,
	Force $\mathbf{F} = 16(3\mathbf{i} - 8\pi\mathbf{j})$		
	Vector perpendicular force = $8\pi\mathbf{i} + 3\mathbf{j}$	B1	ft c's $\mathbf{F}$
2(d)	Rate of work = $\mathbf{F.v}$	M1	
	Rate of work = $16(3\mathbf{i} - 8\pi\mathbf{j}).(6\pi\mathbf{i} + \mathbf{j})$		
	Rate of work = $16(18\pi - 8\pi)$	M1	correct dot product
	Rate of work = 160π = 502.65 (W)	A1	cao

Q	Solution	Mark	Notes
3			

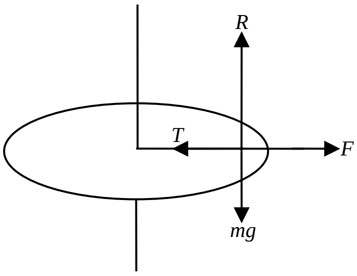


	$F = \dfrac{50\times1000}{25} = (2000)$	B1	use of $P = Fv$, si
	Apply N2L to car & trailer	M1	dim correct eqn, all forces
	$F - R - (1500+M)g\sin\alpha = (1500+M)a$	A1	
	$2000 - 180 - (1500+M)\times9.8\times\dfrac{1}{21}$		
	$= (1500+M)\times0.4$	A1	
	$(0.4 + \dfrac{7}{15})M = 2000-180-700-600$		
	$M = 600$	A1	cao
	Apply N2L to trailer	M1	dim correct eqn.
	$T - 60 - Mg\sin\alpha = Ma$		A1
	$T - 60 - 600g\sin\alpha = 600\times0.4$		
	$T - 60 - 280 = 240$		
	$T = 580$ (N)	A1	ft M if T>0

Alternative Solution

	Apply N2L to car	(M1)	dim correct eqn
	$F - T - 120 - 1500g\sin\alpha = 1500\times0.4$	(A1)	
	$2000 - T - 120 - 700 = 600$		
	$T = 580$ (N)	(A1)	

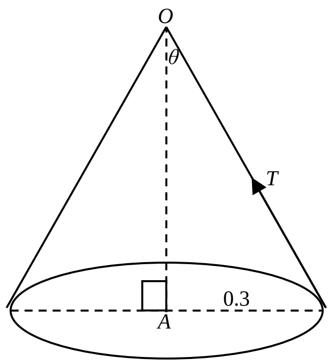
Q	Solution	Mark	Notes
4			



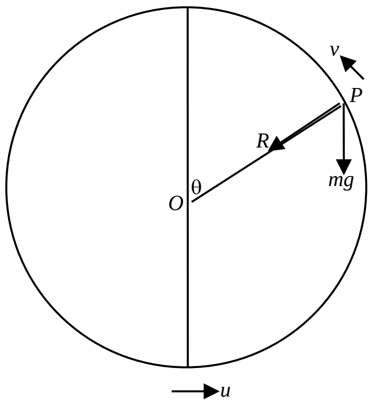
Using Hooke's Law	M1	used one error only
$T = \frac{\lambda x}{l} = \frac{3mg \times 0\cdot05}{0\cdot2}$	A1	
$T = \frac{3}{4}mg$		
$R = mg$	B1	
Max $F = \mu R = 0.4mg$	B1	
Min ω occurs when object about to move inwards, friction acts outwards.	M1	
$T - F = m \times 0.25\ \omega^2$	A1	
$\frac{3}{4}mg - 0.4mg = m \times 0.25\ \omega^2$		
$\omega = 3.704$	A1	cao
Max ω occurs when object about to move outwards, friction acts inwards.	M1	
$T + F = m \times 0.25\ \omega^2$	A1	
$\frac{3}{4}mg + 0.4mg = m \times 0.25\ \omega^2$		
$\omega = 6.714$	A1	cao

Q	Solution	Mark	Notes
5	PE lost = mgh	M1	used
	PE lost = $3 \times 9.8 \times 0.9$	A1	
	PE lost = 26.46 (J)		
	KE gained = $\frac{1}{2}mv^2$		
	KE gained = $\frac{3}{2}v^2$	B1	
	EE gained = $\frac{1}{2}\lambda\frac{x^2}{l}$	M1	used
	EE gained = $\frac{1}{2} \times 60 \times \frac{0\cdot3^2}{1\cdot2}$	A1	cao
	EE gained = 2.25 (J)		
	Conservation of energy	M1	dim correct equation PE, KE and EE all present
	$\frac{3}{2}v^2 + 2.25 = 26.46$	A1	
	$v = 4.02$ (ms^{-1})	A1	cao

Q	Solution	Mark	Notes
6(a)	time to wall $t = \dfrac{6}{V\cos\theta}$	B1	
	$s=ut+\frac{1}{2}at^2$, s=(±)3, a=(±)g, u=$V\sin\theta$, t=t	M1	
	$3 = V\sin\theta\times\dfrac{6}{V\cos\theta} - \dfrac{1}{2}\times g\times\dfrac{6^2}{V^2\cos^2\theta}$		
	$3 = 6\tan\theta - \dfrac{18g}{V^2\cos^2\theta}$	A1	convincing
6(b)	time to building $t = \dfrac{24}{V\cos\theta}$		
	$s=ut+\frac{1}{2}at^2$, s=(±)10, a=(±)9.8,u=$V\sin\theta$	M1	
	$10 = V\sin\theta\times\dfrac{24}{V\cos\theta} - \dfrac{1}{2}\times 9.8\times\dfrac{24^2}{V^2\cos^2\theta}$	A1	
	$10 = 24\tan\theta - \dfrac{288g}{V^2cos^2\theta}$		
	× first equation by 16		
	$48 = 96\tan\theta - \dfrac{288g}{V^2cos^2\theta}$		
	subtract		
	$38 = 72\tan\theta$	m1	oe one variable eliminated
	$\tan\theta = \dfrac{19}{36}$		
	$\theta = 27.824°$	A1	cao
	$3 = 6\times\dfrac{19}{36} - \dfrac{18\times 9.8}{V^2cos^2(27.824°)}$	m1	substitute for θ
	$V = 36.786$	A1	cao

Q	Solution	Mark	Notes
7(a)(i)			
	N2L towards centre	M1	
	$T\sin\theta = mr\omega^2$	A1	
	$T\sin\theta = 3\times0.3\times4^2$		
	$T\sin\theta = 14.4$		
	Resolve vertically	M1	
	$T\cos\theta = mg$	A1	
	$T\cos\theta = 3\times g = 29.4$		
	Dividing		
	$\tan\theta = \frac{14\cdot4}{29\cdot4}$	m1	
	$\angle AOP = \theta = \tan^{-1}(\frac{14\cdot4}{29\cdot4})$		
	$\angle AOP = 26.1°$	A1	cao
7(a)(ii)	$T = \frac{14\cdot4}{\sin(26\cdot1°)}$		
	$T = 32.74$ (N)	A1	ft θ
7(b)	$l = \frac{0\cdot3}{\sin(26\cdot1°)}$		
	$l = 0.682$ (m)	B1	ft θ

Q	Solution	Mark	Notes
8.			



	Conservation of energy	M1	
	$\frac{1}{2}mu^2 = mga(1+\cos\theta) + \frac{1}{2}mv^2$	A1	PE allow $\cos\theta = 2/3$
		A1	KE
	$v^2 = u^2 - 2ag(1+\cos\theta)$	A1	allow $\cos\theta = 2/3$, any form si
	N2L towards centre	M1	
	$R + mg\cos\theta = ma$	A1	allow $\cos\theta = 2/3$, allow $R=0$
	$R + mg\cos\theta = \frac{mv^2}{a}$		
	$R + mg\cos\theta = \frac{m(u^2 - 2ag - 2ag\cos\theta)}{a}$	m1	allow $\cos\theta = 2/3$
	particle leaves hoop when $R=0$, $\cos\theta = \frac{2}{3}$	m1	depends on previous M only
	$u^2 - 2ag(1 + \frac{2}{3}) = \frac{2}{3}ag$		
	$u^2 = 4ag$		
	$u = 2\sqrt{ag}$	A1	convincing

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – M3 (LEGACY)
0982-01

INTRODUCTION

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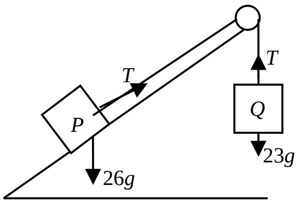
GCE MATHEMATICS – M3

SUMMER 2018 MARK SCHEME

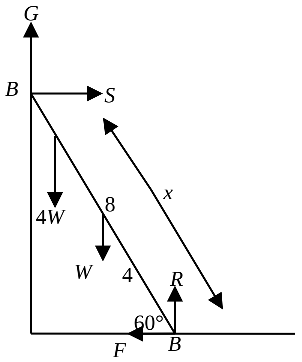
Q	Solution	Mark	Notes
1(a)	N2L to particle, upwards positive		
	$0.3a = -0.3g - 0.12v$	M1	dim. correct, 3 terms
	$0.3\dfrac{dv}{dt} = -0.3\times9.8 - 0.12v$		
	$\dfrac{dv}{dt} = -0.4(24.5 + v)$	A1	AG convincing
1(b)	$\int\dfrac{1}{24\cdot5+v}dv = -0.4\int dt$	M1	separate variables
	$\ln\lvert24\cdot5+v\rvert = -0.4t$ (+ C)	A1	$\ln\lvert24\cdot5+v\rvert$
		A1	all correct
	When $t = 0$, $v = 40$ (or limits)	m1	use of initial conditions
	C = ln(64.5)		
	$t = 2.5\ln\left\lvert\dfrac{64\cdot5}{24\cdot5+v}\right\rvert$	A1	cao
1(c)	Greatest height when $v = 0$.	M1	used
	$t = 2.5\ln\left\lvert\dfrac{64\cdot5}{24\cdot5}\right\rvert$		
	$t = 2.42$ (correct to 3 sig. figs.)	A1	cao
1(d)	$0.3\dfrac{dv}{dt} = 0.3\times9.8 - 0.12v$	B1	
	$\dfrac{dv}{dt} = 0.4(24.5 - v)$		

Q	Solution	Mark	Notes
2(a)	Period = $\frac{2\pi}{\omega}$ = 5×4	B1	
	$\omega = \frac{\pi}{10}$	B1	
	Max speed = $a\omega = \pi$	M1	oe
	A = 10 (m)	A1	convincing
2(b)	max magnitude of the accel = $10\left(\frac{\pi}{10}\right)^2$	B1	ft ω
	max magnitude of the accel = $\frac{\pi^2}{10}$ (=0.987 ms^{-2})		
2(c)	$v^2 = \omega^2(a^2 - x^2)$, a=10, x=6	M1	
	$v^2 = \omega^2(10^2 - 6^2)$		
	$v^2 = \left(\frac{\pi}{10}\right)^2(10^2 - 6^2)$	A1	ft a, ω
	$v = \frac{4\pi}{5}$ (= 2.513 ms^{-1})	A1	cao
2(d)	$x = (\pm)a\cos(\omega t)$	M1	
	$x = (\pm)10\cos\left(\frac{\pi}{10} \times 4\right)$	m1	ft ω
	x = (±)3.09	A1	
	Distance of P from A = 10−3.09 = 6.91 (m)	A1	cao
2(e)	At X, x = −4	M1	
	$-4 = -10\cos\left(\frac{\pi}{10}t\right)$		
	t_X = 3.6901	A1	
	At Y, x = 4		
	$4 = -10\cos\left(\frac{\pi}{10}t\right)$		
	t_Y = 6.3099		
	Required time = $t_Y - t_X$	m1	any correct method
	$t_Y - t_X$ = 2.62 (s)	A1	cao

Q	Solution	Mark	Notes
3(a)			
	Apply N2L to Q	M1	dim correct $23g$ and T opposing
	$23g - T = 23a$	A1	
	Apply N2L to P	M1	dim correct $23g\sin 30$ and T opposing
	$T - 26g\sin 30 = 26a$	A1	
	Adding $23g - 13g = 49a$	m1	
	$a = 2$ (ms^{-2})	A1	cao
3(b)	$v^2=u^2+2as$, $u=0$, $a=(\pm)2$(c), $s=(\pm)\frac{49}{16}$	M1	oe complete method
	$v^2 = 0 + 2 \times 2 \times \frac{49}{16}$	A1	
	$v = \frac{7}{2}$	A1	cao
3(c)	Impulse = change in momentum	M1	
	$J = 26(\frac{7}{2} - v)$	A1	ft $\frac{7}{2}$ (c)
	$J = 23v$	B1	
	$26(\frac{7}{2} - v) = 23v$	m1	
	$49v = 13\times 7$		
	$v = \frac{13}{7} = 1.86$ (ms^{-1})	A1	cao
	$J = 23 \times \frac{13}{7}$		
	$J = \frac{299}{7} = 42.71$ (Ns)	A1	cao

Q	Solution	Mark	Notes
4	Auxiliary equation		
	$m^2 + 2m - 15 = 0$	M1	
	$(m - 3)(m + 5) = 0$		
	$m = -5, 3$	A1	
	CF is $x = Ae^{-5t} + Be^{3t}$	B1	ft values of m
	For PI, try $x = at + b$	M1	
	$\frac{dx}{dt} = a$		
	$\frac{d^2x}{dt^2} = 0$		
	$2a - 15(at + b) = 30t - 19$	A1	
	Comparing coefficients	m1	
	$-15a = 30$		
	$a = -2$		
	$-4 - 15b = -19$		
	$b = 1$	A1	cao both values
	General solution is		
	$x = Ae^{-5t} + Be^{3t} - 2t + 1$		
	When $t = 0$, $x = 10$		
	$10 = A + B + 1$		
	$A + B = 9$		
	$\frac{dx}{dt} = -5Ae^{-5t} + 3Be^{3t} - 2$	B1	ft CF+PI
	When $t = 0$, $\frac{dx}{dt} = -31$,	m1	initial conditions used
	$-31 = -5A + 3B - 2$		
	$-5A + 3B = -29$		
	$3A + 3B = 27$		
	Subtract		
	$8A = 56$	m1	
	$A = 7$	A1	cao
	$B = 2$	A1	cao
	$x = 7e^{-5t} + 2e^{3t} - 2t + 1$		
	When $t = 1$		
	$x = 7e^{-5} + 2e^{3} - 2 + 1$		
	$x = 39.22$	A1	cao

Q	Solution	Mark	Notes
5(a)	N2L applied to vehicle	M1	dim correct, 3 terms
	$108v - 12v^3 = 72a$	A1	all correct
	$9v - v^3 = 6v\frac{dv}{dx}$	A1	use of $a = v\frac{dv}{dx}$,convincing
5(b)	$9 - v^2 = 6\frac{dv}{dx}$		
	$\int dx = 6\int \frac{1}{3^2 - v^2} dv$	M1	correct sep variables
	$x = \frac{6}{2\times 3}\ln\left\|\frac{3+v}{3-v}\right\| + (C)$	A1	for $\ln\left\|\frac{3+v}{3-v}\right\|$,
		A1	all correct
	When $x = 0$, $v = 1$	m1	used
	C = -ln2	A1	
	$x = \ln\left\|\frac{3+v}{3-v}\right\| - \ln 2$		
	$e^x = \frac{3+v}{2(3-v)}$	m1	inversion
			any correct form
	$v = 3\left(\frac{2e^x - 1}{2e^x + 1}\right)$	A1	cao any correct expression, no ln
	When $x \to \infty$		
	$v \to 3$	B1	ft similar expression

Q	Solution	Mark	Notes
6			
	$F = 0.4R$	B1	
	$G = 0.4S$	B1	
	Resolve horizontally	M1	dim. correct eqn.
	$S = F$	A1	
	$S = 0.4R$		
	$G = 0.16R$		
	Resolve vertically	M1	dim. correct eqn.
	$G + R = 4W + W$	A1	
	$0.4 \times 0.4\, R + R = 5W$		
	$W = 0.232R$		
	Moments about B	M1	dim. correct eqn. no missing/extra forces
	$W\times4\cos60 + 4W\times x\cos60$		
	$= S\times8\sin60+G\times8\cos60$	A1	2 correct terms, allow θ
		A1	correct equation, allow θ
	$0.232R\times2 + 2\times0.232Rx$		
	$= 0.4R\times4\sqrt{3} + 0.16R\times4$		
	$0.464 + 0.464x = 1.6\times\sqrt{3} + 0.64$		
	$x = 6.35$ (m)	A1	cao in (6.3, 6.4)
	The man can climb 6.35 m up the ladder before it slips.		

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – S1 (LEGACY)
0983-01

INTRODUCTION

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GCE MATHEMATICS – S1

SUMMER 2018 MARK SCHEME

Ques	Solution	Mark	Notes
1(a)	$E(X^2) = \text{Var}(X) + [E(X)]^2$ $= 153$	**M1** **A1**	
(b)	$E(Y) = 4E(X) - 3$ $= 45$ SD(Y) = 4SD(X) (or Var(Y) = 16Var(X)) $= 12$	**M1** **A1** **M1** **A1**	M1A0 for variance M0 for Var(Y) = 4Var(X)
2(a)	We are given that $p_a \times p_b = 0.4$ $p_a + p_b =$ P($A \cup B$) + P($A \cap B$) $= 1.3$ $p_a + \frac{0.4}{p_a} = 1.3$ $p_a^2 - 1.3p_a + 0.4 = 0$ $(p_a - 0.5)(p_a - 0.8) = 0$ $p_a = 0.8$, $p_b = 0.5$	**B1** **M1** **A1** **M1** **m1** **A1A1**	Or by inspection Lose A1 if wrong way round
(b)	$P(A \mid A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$ $= \frac{P(A)}{P(A \cup B)}$ $= \frac{8}{9}$	**M1** **A1** **A1**	FT from (a)
3	P(Beti selects red first time) = $\frac{1}{6}$ P(Beti selects red second time) = $\frac{5}{6} \times \frac{4}{5} \times \frac{1}{4} = \frac{1}{6}$ P(Beti selects red third time) $= \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{6}$ P(Beti selects red) = $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$ (So equal probabilities for Beti and Gwyn)	**B1** **M1A1** **M1A1** **A1**	Special case – award an extra B1if after this first line you see P(Gwyn selects red 1st time) $= \frac{5}{6} \times \frac{1}{5} = \frac{1}{6}$ and no further relevant probabilities evaluated. Accept a solution which gives the probabilities of Gwyn winning each time.

4	E(X) = 10p si SD(X) = $\sqrt{10p(1-p)}$ si We require $\sqrt{10p(1-p)} > 10p$ $10p - 10p^2 > 100p^2$ ($10p(11p - 1) < 0$) $110p < 10$ $(0 <)\ p < \frac{1}{11}$	**B1** **B1** **M1** **A1** **A1**	
5(a) **(b)**	P(>20) = $\frac{1}{6} \times 0.6 + \frac{5}{6} \times 0.24$ = 0.3 P(cycled| > 20) = $\frac{0.2}{0.3}$ = $\frac{2}{3}$ cao	**M1A1** **A1** **B1B1** **B1**	 FT denominator from (a)
6(a)(i) **(ii)** **(b)**	Number X arriving between 9 am and 9.15 am is Poi(3.75) si $P(X = 4) = e^{-3.75} \times \frac{3.75^4}{4!}$ = 0.194 Number Y arriving between 10 am and 10:20 am is Poi(5) si P(Y > 6) = 0.2378 Evidence of using the table in the appropriate vicinity. Mean = 8 t = 32	**B1** **M1** **A1** **B1** **M1A1** **M1** **A1** **A1**	Award M0 if tables used with mean rounded to 3.8 Award M0 if no working shown. M1A0 if reading adjacent row or column

7(a)	$\alpha+\beta+0.5=1$ $\alpha+\beta=0.5$ $E(X)=0.3+2\alpha+3\beta+0.8=2.2$ $2\alpha+3\beta=1.1$ $\alpha=0.4, \beta=0.1$	**M1** **A1** **M1** **A1** **A1**	Special case – award B1 if correct answer given with no working Only award if both M1s given
(b)	The possible values are 1,1,1 ; 2,2,2 ; 3,3,3 ; 4,4,4 si Required prob $= 0.3^3 + 0.4^3 + 0.1^3 + 0.2^3$ $= 0.1$	**B1** **M1** **A1**	FT from (a) Accept α,β here
8(a)(i)	X is binomially distributed with parameters 20, 0.6.	**B1** **B1**	
(ii)	P(X = 15) = $\binom{20}{15}\times 0.6^{15}\times 0.4^{5}$ = 0.0746	**M1** **A1**	Award M0 if no working
(iii)	Let N denote the number not germinating so that N is B(20, 0.4). si We require $P(X \geq 15) = P(N \leq 5)$ = 0.1256	**M1** **A1** **m1** **A1**	
(b)	Y is B(200,0.05) which is approx Poi(10) si	**B1**	
(i)	P(Y = 8) = $e^{-10}\times\frac{10^8}{8!}$ = 0.113	**M1** **A1**	Award M0 if no working seen Accept 0.3328 – 0.2202 or 0.7798 – 0.6672
(ii)	P(Y > 12) = 0.2084	**M1A1**	M1A0 if reading adjacent row or column

9(a)(i)	$P(2 < X < 2.5) = F(2.5) - F(2)$	**M1**	
	$= 0.275$	**A1**	
(ii)	Use of $F(q) = 0.75$	**M1**	
	$q^2 + q - 9.5 = 0$	**A1**	
	$q = \dfrac{-1 \pm \sqrt{1+38}}{2}$	**m1**	
	$= 2.62$	**A1**	
(b)(i)	$f(x) = F'(x)$	**M1**	
	$= \dfrac{1}{10}(2x+1)$	**A1**	
(ii)	Use of $E(X) = \int xf(x)\mathrm{d}x$	**M1**	FT from (b)(i) if M1 awarded
	$= \dfrac{1}{10}\int_1^3 (2x^2 + x)\mathrm{d}x$	**A1**	Limits need not be seen here
	$= \dfrac{1}{10}\left[\dfrac{2x^3}{3} + \dfrac{x^2}{2}\right]_1^3$	**A1**	
	$= 2.13 \quad (32/15)$	**A1**	

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – S2 (LEGACY)
0984-01

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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GCE MATHEMATICS – S2

SUMMER 2018 MARK SCHEME

Ques		Mark	Notes
1(a)	The possibilities are X=1,Y=2 or X=2,Y=1 si	**B1**	
	P(X = 1) = 4 × 0.25 × 0.75^3 (= 0.4218...)	**B1**	Accept use of tables
	P(X = 2) = 6 × 0.25^2 × 0.75^2 (= 0.2109...)	**B1**	
	P(Y = 1) = $e^{-3} \times 3 (= 0.1493...)$	**B1**	
	P(Y = 2) = $e^{-3} \times \frac{3^2}{2} (= 0.2240...)$	**B1**	
	Prob = 0.4218 × 0.2240 + 0.2109 × 0.1493	**M1**	
	= 0.126 cao	**A1**	
(b)	$E(X)$ = 1, Var(X) = 0.75, Var(Y) = 3 si	**B1**	
	$E(U) = E(X)E(Y)$= 1 × 3 = 3	**B1**	
	$E(X^2) = \text{Var}(X) + (E(X))^2$	**M1**	Award this M1 for whichever of X,Y comes first
	= 0.75 + 1 = 1.75	**A1**	
	$E(Y^2) = \text{Var}(Y) + (E(Y))^2$		
	= 3 + 9 = 12	**A1**	
	Var(U) = $E(X^2Y^2) - (E(XY))^2$	**M1**	
	= $E(X^2)E(Y^2) - (E(XY))^2$	**m1**	
	= 1.75 × 12 – 9 = 12	**A1**	
2(a)	Let X = weight of a randomly chosen hen and let Y = weight of a randomly chosen cockerel.		
	Let $U = Y - 2X$.		
	$E(U)$ = 4.2 – 5.2 = –1.0	**B1**	
	Var(U) = $0.25^2 + 4 \times 0.15^2$	**M1**	
	= 0.1525	**A1**	
	We require $P(U > 0)$		
	$z = \frac{1}{\sqrt{0.1525}}$	**M1A1**	
	= 2.56 (Accept – 2.56)	**A1**	
	Required probability = 0.00523	**A1**	
(b)	Let $V = Y_1 + Y_2 + X_1 + X_2 + X_3 + X_4 + X_5$		
	$E(V)$ = 2 × 4.2 + 5 × 2.6 = 21.4	**B1**	
	Var(V) = 2 × 0.25^2 + 5 × 0.15^2 = 0.2375	**M1A1**	
	We require P(V > 21)		
	$z = \frac{-0.4}{\sqrt{0.2375}}$	**M1**	
	= –0.82 (Accept +0.82)	**A1**	
	Required probability = 0.7939	**A1**	

3(a)	$\bar{x} = \frac{2255.4}{9} = 250.6$	**B1**	
	SE of $\bar{x} = \frac{2}{\sqrt{9}}$ (0.6666….)	**M1A1**	M0 if no working
	90% confidence interval limits are $250.6 \pm 1.645 \times 0.6666...$	**M1A1**	FT their $\bar{x}$
	giving [249.5, 251.7]	**A1**	
(b)	Width of CI = $2 \times z \times \frac{2}{\sqrt{9}} = 2.4$	**M1A1**	FT their SE
	$z = 1.8$	**A1**	
	Tabular value = 0.9641	**A1**	
	Confidence level = 0.928 = 92.8%	**A1**	
4(a)	The number of errors X is Po(10) si	**B1**	
	$P(X = 12) = e^{-10} \times \frac{10^{12}}{12!}$	**M1**	0.3032 – 0.2084 or 0.7916 – 0.6968
	= 0.0948	**A1**	
(b)(i)	Number of errors Y is now Po(15) (under H_o) si	**B1**	
	p-value = $P(Y \le 9)$	**M1**	
	= 0.0699	**A1**	
(ii)	Insufficient evidence to reject H_o because p-value > 0.05	**B1**	FT the p-value
(c)(i)	Number of errors U is now Po(50) ≅ N(50,50) si (p-value = $P(U \le 36)$)	**B1**	
	$z = \frac{36.5 - 50}{\sqrt{50}}$	**M1**	No c/c gives z =–1.98, p = 0.0239 Wrong c/c z =–2.05, p = 0.0202 Withhold final A1 for either of the above cases
	= – 1.91	**A1**	
	p-value = 0.0281	**A1**	
(ii)	Strong evidence to indicate that Alun's typing has improved.	**A1**	FT the p-value

5(a)	Let *x*,*y* denote the measurements on Window 1 and Window 2 respectively. $H_o : \mu_x = \mu_y ; H_1 : \mu_x \neq \mu_y$	**B1**	
(b)(i)	$(\sum x = 9.12; \sum y = 10.85)$ $\bar{x} = 1.52; \bar{y} = 1.55$	**B1B1**	
	Standard error = $\sqrt{\frac{0.02^2}{6} + \frac{0.02^2}{7}}$ (=0.01112...)	**M1A1**	
	$z = \frac{1.55 - 1.52}{0.01112...}$	**M1**	
	= 2.70	**A1**	
	Tabular value = 0.00347	**A1**	Award the final A1 for doubling the tabular value
	p-value = 0.00694	**A1**	
(ii)	Very strong evidence to indicate that the refractive indices of the two windows are not equal.	**B1**	FT the *p*-value Must mention either refractive indices or windows

6(a)	We are given that $\frac{a+b}{2} = 34$ $\frac{(b-a)^2}{12} = 12$ Solving, $a = 28,\ b = 40$	**B1** **B1** **M1** **A1A1**	
(b)	95^{th} percentile $= 28 + 0.95 \times 12$ $= 39.4$	**M1** **A1**	FT their *a*,*b*
(c)(i)	The sample mean (or total) of a large (random) sample from any distribution is approximately normally distributed.	**B1**	
(ii)	EITHER $E(S) = 120 \times 34 = 4080$ $Var(S) = 120 \times 12 = 1440$ $z = \frac{4140-4080}{\sqrt{1440}}$ $= 1.58$ Required probability = 0.0571 OR $P(S > 4140) = P(\overline{X} > 34.5)$ $E(\overline{X}) = 34, \text{Var}(\overline{X}) = \frac{12}{120} (= 0.1)$ $z = \frac{34.5-34}{\sqrt{0.1}}$ $= 1.58$ Required probability = 0.0571	**B1** **B1** **M1A1** **A1** **A1** **(B1)** **(B1)** **(M1A1)** **(A1)** **(A1)**	Do not award the final A1 if a c/c is applied

GCE MARKING SCHEME

SUMMER 2018

MATHEMATICS – S3 (LEGACY)
0985-01

INTRODUCTION

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GCE MATHEMATICS – S3

SUMMER 2018 MARK SCHEME

Ques	Solution	Mark	Notes
1	The following table shows the possibilities. Selection / freq / prob / largest: 111 / 1 / 1/20 / 1 112 / 6 / 6/20 / 2 113 / 3 / 3/20 / 3 122 / 3 / 3/20 / 2 123 / 6 / 6/20 / 3 223 / 1 / 1/20 / 3	**B2** **B2** **B1**	Selections, B1 if 1 error Freq/prob, B1 if 1 error largest
	$E(X) = 1\times\frac{1}{20}+2\times\frac{9}{20}+3\times\frac{10}{20}$ = 2.45	**M1** **A1**	
2(a)	$\bar{x} = \frac{30060}{60}(=501)$	**B1**	
	$s_x^2 = \frac{15060146}{59} - \frac{30060^2}{60\times59}$	**M1**	M0 no working
	= 1.4576... (1.4333.. div by 60)	**A1**	
	Stand error = $\sqrt{\frac{1.4576}{60}}$ = 0.15586.. (0.15455..)	**M1A1**	
	95% confidence limits are 501±1.96×0.15586 giving [500.7, 501.3]	**M1A1** **A1**	M0 no working
(b)	The owner should conclude that the average weight probably exceeds 500 grams (or the nominal weight).	**B1**	FT their confidence interval
3(a)	$H_0 : \mu = 20; H_1 : \mu > 20$	**B1**	
(b)	$\sum x = 206; \sum x^2 = 4248.92$	**B1B1**	
	UE of μ = 20.6	**B1**	No working need be seen
	UE of $\sigma^2 = \frac{4248.92}{9} - \frac{206^2}{90}$	**M1**	M0 division by 10 Answer only no marks
	= 0.591	**A1**	
(c)	$t = \frac{20.6-20}{\sqrt{0.591/10}}$	**M1**	FT from (b)
	= 2.47	**A1**	
	DF = 9	**B1**	DF mark can be implied by T_{crit}
	t_{crit} = 2.821	**B1**	
	Because 2.47 < 2.821, the claim is rejected	**B1**	FT the *t* value

4(a)	$\hat{p} = \frac{42}{80} = 0.525$	**B1**	
(b)	ESE = $\sqrt{\frac{0.525 \times 0.475}{80}}$ = 0.05583.. si	**M1A1**	
(c)	90% confidence limits are $0.525 \pm 1.645 \times 0.05583..$ giving [0.43,0.62] cao	**M1A1** **A1**	M0 no working FT their $\hat{p}$
(d)	Yes the claim is supported because 0.6 lies in the interval.	**B1**	FT from (c)
5(a)	$H_0 : \mu_x = \mu_y ; H_1 : \mu_x \neq \mu_y$	**B1**	
(b)	$\bar{x} = 59.7; \bar{y} = 60.25$ $s_x^2 = \frac{356630}{99} - \frac{5970^2}{99 \times 100} = 2.2323...$ $s_y^2 = \frac{363402}{99} - \frac{6025^2}{99 \times 100} = 3.9974...$ [Accept division by 100 giving 2.21 and 3.9575] SE = $\sqrt{\frac{2.2323}{100} + \frac{3.9974}{100}}$ = 0.24959... (0.24834…) $z = \frac{60.25 - 59.7}{0.24959..}$ = 2.20 (2.21) Tabular value = .0139 (0.01355) Approx *p*-value = 0.0278 (0.0271) Strong evidence for rejecting H_0 (or equivalent)	**B1** **M1A1** **A1** **M1** **A1** **M1** **A1** **A1** **A1** **B1**	Accept – 2.20 FT their *p*-value
(c)	No because the Central Limit Theorem (guarantees approximate normality of the sample means).	**B1**	

6(a)(i)	$S_{xy} = 4478 - 150 \times 164.7/6 = 360.5$	**B1**	
	$S_{xx} = 5500 - 150^2/6 = 1750$	**B1**	
	$b = \dfrac{360.5}{1750} = 0.206$	**M1A1**	M0 no working
	$a = \dfrac{164.7 - 0.206 \times 150}{6} = 22.3$	**M1A1**	
(ii)	Estimated yield = 22.3 + 0.206 × 25 = 27.45 (kg)	**M1** **A1**	Accept the use of $\bar{y}$
(iii)	SE = $\sigma\sqrt{\left(\dfrac{1}{n} + \dfrac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$	**M1**	
	$= \dfrac{0.25}{\sqrt{6}} = 0.102$	**A1**	
(b)(i)	Estimated yield = $\dfrac{22.6 + 32.6}{2} = 27.6$ (kg)	**B1**	
(ii)	SE = $\dfrac{\sigma}{\sqrt{n}}$	**M1**	
	$= \dfrac{0.25}{\sqrt{2}} = 0.177$	**A1**	

7(a)	$E(U) = a(\mu + \mu) + b(2\mu + 2\mu + 2\mu)$	**M1**	
	$= 2a\mu + 6b\mu$	**A1**	
	For an unbiased estimator, $E(U) = \mu$	**A1**	
	leading to $2a + 6b = 1$		
(b)(i)	$\text{Var}(U) = a^2[\text{Var}(X_1) + \text{Var}(X_2)] + b^2[\text{Var}(Y_1) + \text{Var}(Y_2) + \text{Var}(Y_3)]$	**M1**	
	$= 8a^2\sigma^2 + 3b^2\sigma^2$	**A1**	
	$= \sigma^2\left(2 \times (1-6b)^2 + 3b^2\right)$	**A1**	
	$= \sigma^2\left(75b^2 - 24b + 2\right)$		
(ii)	$\dfrac{\text{dVar}(U)}{\text{d}b} = \sigma^2(150b - 24)$		
	$= 0$ for a minimum	**M1**	
	$b = \dfrac{4}{25}; a = \dfrac{1}{50}$	**A1A1**	
	Any valid justification that it is a minimum	**B1**	
(c)(i)	$E(W) = \mu - \mu + 4\mu - 2\mu - 2\mu = 0$	**B1**	
	$\text{Var}(W) = 4\sigma^2 + 4\sigma^2 + 4\sigma^2 + \sigma^2 + \sigma^2$		
	$= 14\sigma^2$	**B1**	
(ii)	$E(V) = \dfrac{E(W^2)}{N}$		
	$= \dfrac{\text{Var}(W)}{N}$	**M1**	FT their Var(W) provided no negative terms in their calculation of Var(W)
	$= \dfrac{14\sigma^2}{N}$	**A1**	
	$= \sigma^2$ when $N = 14$	**A1**	