



**GCE AS/A level**

0981/01

**MATHEMATICS M2**  
**Mechanics**

A.M. TUESDAY, 10 June 2014

1 hour 30 minutes

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The diagram shows a piston, of mass  $0.8 \text{ kg}$ , enclosed in a horizontal tube and attached to a light spring of natural length  $0.2 \text{ m}$  and modulus of elasticity  $625 \text{ N}$ . The other end of the spring is fixed to the end of the tube at point  $B$ .



Initially, the piston is held at rest at a point  $A$  with the spring compressed a distance of  $0.1 \text{ m}$ , so that  $AB$  is the compressed length of the spring.

- (a) Calculate the elastic energy stored in the spring. [2]

The piston is then released. During the subsequent motion, it is subjected to a resistance to motion of constant magnitude  $46 \text{ N}$ .

- (b) Determine the velocity of the piston when the spring reaches its natural length. [5]

2. A particle of mass  $5 \text{ kg}$  moves under the action of a horizontal force given by  $F = 30t^{-2} - 30 \text{ N}$  at time  $t \text{ s}$ , where  $t > 0$ . It also experiences a constant resistance to motion of magnitude  $120 \text{ N}$ .

- (a) Show that the motion of the particle satisfies the differential equation

$$\frac{dv}{dt} = 6t^{-2} - 30,$$

where  $v \text{ ms}^{-1}$  is the velocity of the particle at time  $t \text{ s}$ . [2]

- (b) Calculate the value of  $t$  when the acceleration of the particle is  $24 \text{ ms}^{-2}$ . [2]

- (c) Given that the velocity of the particle is  $18 \text{ ms}^{-1}$  when  $t = \frac{1}{3}$ , find an expression for  $v$  in terms of  $t$ . Hence find the values of  $t$  when  $v = 10$ . [6]

3. A vehicle of mass  $4000 \text{ kg}$  is travelling up a slope inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = \frac{2}{49}$ . The engine of the vehicle is working at a constant rate of  $90 \text{ kW}$ .

- (a) Calculate the resistance to the motion of the vehicle at the instant when its speed is  $4.8 \text{ ms}^{-1}$  and its acceleration is  $1.2 \text{ ms}^{-2}$ . [6]

- (b) Determine the maximum velocity of the vehicle when the resistance to motion has magnitude  $12800 \text{ N}$ . [4]

4. At time  $t = 0$ , an aeroplane  $A$  has position vector  $(3\mathbf{i} + 5\mathbf{j} + 20\mathbf{k})\text{ m}$  and is flying with constant velocity  $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})\text{ ms}^{-1}$ .  
At time  $t = 0$ , another aeroplane  $B$  has position vector  $(-2\mathbf{i} + x\mathbf{j} + 15\mathbf{k})\text{ m}$ , and is flying with constant velocity  $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})\text{ ms}^{-1}$ .
- (a) Find expressions for the position vector of  $A$  and the position vector of  $B$  at time  $t$  s. [3]
- (b) Determine an expression for  $AB^2$ , where  $AB$  is the distance between  $A$  and  $B$  at time  $t$  s. [4]
- (c) Given that the shortest distance between  $A$  and  $B$  occurs at  $t = 5$ , calculate the value of  $x$ . [3]
5. A player kicks a ball from a point  $A$  on horizontal ground so that 2.5 seconds later the ball just clears a bar at a point  $B$ . The point  $B$  is 3 m above the ground. The horizontal distance of  $B$  from  $A$  is 42 m.
- (a) Calculate the horizontal and vertical components of the initial velocity of the ball. [4]
- (b) Find the magnitude of the velocity of the ball and the angle that the direction of the velocity makes with the horizontal as it passes the point  $B$ . [6]
- (c) Determine the horizontal distance from  $B$  to the point where the ball first hits the ground again. [3]
6. A particle of mass 3 kg moves on a horizontal plane. At time  $t = 0$ , the particle has position vector  $-2\mathbf{i} + 3\mathbf{j}\text{ m}$ , where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors along the  $x$ -axis and  $y$ -axis respectively. At time  $t$  s, the particle moves with velocity  $\mathbf{v}\text{ ms}^{-1}$  given by
- $$\mathbf{v} = 4\sin 2t\mathbf{i} + 15\cos 5t\mathbf{j}.$$
- (a) Find the magnitude of the force acting on the particle at time  $t = \frac{3\pi}{2}$  s. [5]
- (b) Determine the position vector of the particle at time  $t$  s. [4]
- (c) Calculate the time and the distance of the particle from the origin when it crosses the  $y$ -axis for the first time. [4]
7. One end of a light rod of length  $l$  metres is freely jointed to a fixed point  $O$  and the other end is attached to a particle of mass  $m$  kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point,  $u\text{ ms}^{-1}$ , is a quarter of its speed at the lowest point of the circle.
- (a) Show that  $u^2 = \frac{4}{15}gl$ . [3]
- (b) When the rod is inclined at an angle  $\theta$  to the **downward** vertical,
- (i) find an expression for the tension in the rod in terms of  $m$ ,  $g$  and  $\theta$ .
- (ii) determine the value of  $\theta$  when the tension in the rod becomes zero. [9]

END OF PAPER