

981/01

MATHEMATICS M2

Mechanics 2

P.M. WEDNESDAY, 22 June 2005

(1½ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9.8 ms^{-2} .

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. One end of a light elastic string, of natural length 0.8 m, is attached to a fixed point O , and the other end is attached to a particle of mass 5 kg. When the particle hangs in equilibrium vertically below O , the length of the string is 1.3 m.

(a) Calculate the modulus of elasticity of the string. [3]

(b) Determine the elastic energy stored in the string. [2]

2. A particle moves in a straight line such that its acceleration $a \text{ ms}^{-2}$ is given by

$$a = 4 - 6t \quad \text{for } t \geq 0.$$

At time $t = 0$, the particle is at the point O and its velocity is 4 ms^{-1} .

(a) Find an expression for the velocity of the particle at time t s. [3]

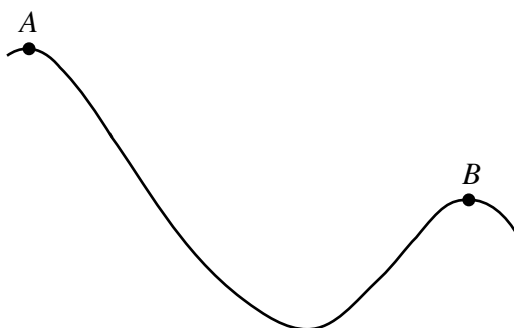
(b) Find an expression for the displacement of the particle from O at time t s. [3]

(c) Determine the time when the particle comes to rest instantaneously and the distance of the particle from O at this time. [3]

(d) Calculate the **speed** of the particle when $t = 3$, and determine whether or not the **speed** of the particle is increasing or decreasing at this time. [3]

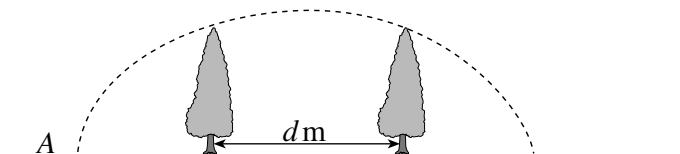
3. A car, of mass 1250 kg, is travelling up a hill inclined at an angle α to the horizontal at a constant speed of 7.5 ms^{-1} . The car's engine is working at a rate of 30 kW and the resistance to motion of the car is 1550 N. Find the value of α , giving your answer in degrees correct to one decimal place. [6]

4. The diagram shows two points A and B on a roller coaster ride in an amusement park.



The heights of A and B above ground level are 30 m and 22 m respectively. The length of the track between A and B is 88 m. The resistance to motion of the carriage may be assumed to have a constant magnitude of 132 N. A carriage, of total mass 240 kg, has speed 2 ms^{-1} at A . Calculate the speed of the carriage at B . [8]

5. A golfer hits a ball from the point A with initial velocity 24.5 ms^{-1} at an angle α above the horizontal, where $\sin \alpha = 0.8$. The ball just clears the tops of two trees. The tops of the trees are both 14.7 m above the level of A and are a horizontal distance $d \text{ m}$ apart.



- (a) (i) Find the time taken for the ball to reach the top of the first tree. [8]
 (ii) Determine the value of d . [8]
- (b) Find the magnitude and direction of the velocity of the ball 0.75 s after it was hit. [6]

6. At time $t \text{ s}$, a particle P has position vector $\mathbf{r} \text{ m}$ with respect to an origin O given by

$$\mathbf{r} = (2t - 5)\mathbf{i} + (t - 3)\mathbf{j} + (7 - 2t)\mathbf{k}.$$

- (a) Show that the distance of the particle from the origin at time $t \text{ s}$ is given by

$$OP^2 = 9t^2 - 54t + 83,$$

and find the value of t when P is closest to O . [5]

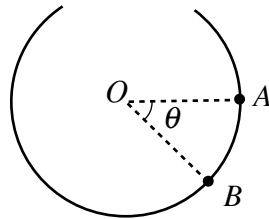
- (b) Find the velocity of P and determine its magnitude. [3]
- (c) Show that, when P is closest to O , the direction of the velocity of P is perpendicular to the line OP . [3]

7. A rider on a motorcycle is travelling at a constant speed of 42 ms^{-1} in a horizontal circle on a track banked at an angle of 25° to the horizontal. There is no tendency to sideslip at this speed. The total mass of the motorcycle and the rider is 600 kg .
 Modelling the motorcycle and the rider as a particle,

- (a) calculate the normal reaction of the track on the motorcycle, [3]
- (b) determine the radius of the circle. [4]

TURN OVER.

8. A marble, of mass 0.3 kg , moves on the inner surface of a spherical bowl of radius 0.4 m . Initially, the marble is held at A , on the inner surface of the bowl, where OA is horizontal, as shown in the diagram.



The marble is then projected with a speed of 2 ms^{-1} vertically downwards. When the marble is at the point B , $\widehat{AOB} = \theta$.

- (a) Find, in terms of θ , an expression for v^2 , where $v \text{ ms}^{-1}$ is the speed of the marble at B . [4]
- (b) Show that the reaction $R \text{ N}$ of the surface of the bowl on the marble is given by

$$R = 3 + 8.82 \sin \theta . \quad [4]$$

- (c) Find the greatest possible value of θ , and briefly describe the subsequent motion of the marble. [4]