GCE AS/A level

0974/01

S15-0974-01

MATHEMATICS – C2
Pure Mathematics

A.M. WEDNESDAY, 20 May 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_1^3 \frac{x}{10-\sqrt{x}}\,dx.$$

Show your working and give your answer correct to four decimal places. [4]

2. *(a)* Show that there is no angle θ which satisfies the equation

$$4\cos^2\theta - 2\sin^2\theta - \sin\theta + 8 = 0,$$

giving a mathematical reason to explain how you came to your conclusion. [4]

(b) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\sin(2x - 75°) = -0{\cdot}515.$$ [3]

(c) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 180°$ satisfying

$$4\tan\phi + 7\sin\phi = 0.$$ [4]

3. The triangle *ABC* is such that *AB* = 19 cm, *AC* = 12 cm and $A\hat{B}C$ = 25°.

(a) Find the possible values of $A\hat{C}B$. Give your answers correct to the nearest degree. [2]

(b) Given that $B\hat{A}C$ is an **acute** angle, find

(i) the size of $B\hat{A}C$, giving your answer correct to the nearest degree,

(ii) the area of triangle *ABC*, giving your answer correct to two decimal places. [4]

4. *(a)* The first term of an arithmetic series is 4 and the common difference is 6.

(i) Show that the nth term of the arithmetic series is $6n - 2$.

(ii) The sum of the first n terms of this series is given by

$$S_n = 4 + 10 + \ldots + (6n - 8) + (6n - 2).$$

Without using the formula for the sum of the first n terms of an arithmetic series, **prove** that

$$S_n = n(3n + 1).$$ [4]

(b) The tenth term of another arithmetic series is four times the fifth term.
The sum of the first fifteen terms of the series is 210.

(i) Find the first term and common difference of this arithmetic series.

(ii) Given that the kth term of the series is 200, find the value of k.

[6]

5. *(a)* The eighth and ninth terms of a geometric series are 576 and 2304 respectively. Find the fifth term of the geometric series. [3]

(b) Another geometric series has first term a and common ratio r. The third term of this geometric series is 24. The sum of the second, third and fourth terms of the series is -56.

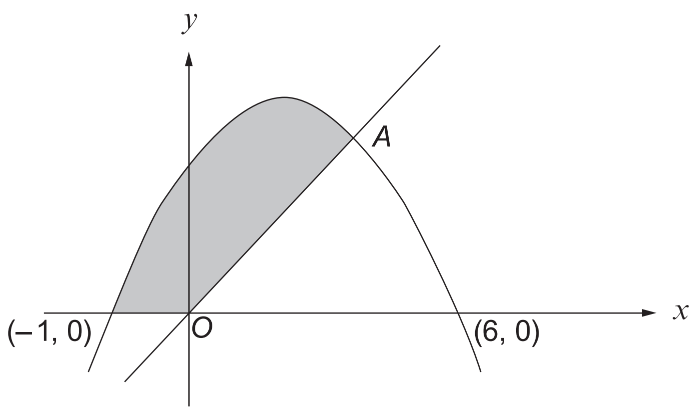
(i) Show that r satisfies the equation

$$3r^2 + 10r + 3 = 0.$$

(ii) Given that $|r| < 1$, find the value of r and the sum to infinity of the series. [8]

6. *(a)* Find $\int \left(\frac{3}{\sqrt{x}} - 6x^{\frac{4}{3}} \right) \mathrm{d}x.$ [2]

(b)



The diagram shows a sketch of the curve $y = 6 + 5x - x^2$ and the line $y = 4x$. The curve and the line intersect at the point *A* in the first quadrant and the curve intersects the x-axis at the points (–1, 0) and (6, 0).

(i) Showing your working, find the x-coordinate of *A*.

(ii) Find the area of the shaded region. [9]

7. *(a)* Given that $x > 0$, $y > 0$, show that

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y.$$ [3]

(b) Find all values of x satisfying the equation

$$\log_a(6x^2 + 9x + 2) - \log_a x = 4\log_a 2.$$ [5]

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8. The circle C has centre A and radius r. The points $P(-2, -3)$ and $Q(8, 1)$ are at opposite ends of a diameter of C.

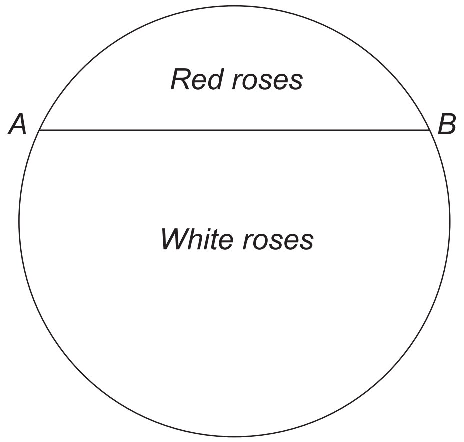
(a) (i) Write down the coordinates of A.

(ii) Show that $r = \sqrt{29}$. [3]

(b) Given that the point $R(5, 4)$ lies on the circle C, find $P\hat{Q}R$. Give your answer in degrees, correct to one decimal place. [3]

(c) The point S lies on the circle C. The tangent to the circle at S passes through the point $T(11, 0)$. Find the length of ST. [3]

9. Gwyn wants to turn part of his garden into a circular flower bed. In order to do this, he digs out a shallow circular hole of radius r m and then divides it into two segments by means of a thin plank AB, as shown in the diagram. He plants red roses in the minor segment and white roses in the major segment.



Let the centre of the flower bed be denoted by O. Show that when $A\hat{O}B$ equals 2·6 radians, the area of the flower bed containing white roses is approximately twice the area containing red roses.

[5]

END OF PAPER