YSGOL CREUDDYN

DAWN DYSG DAIONI

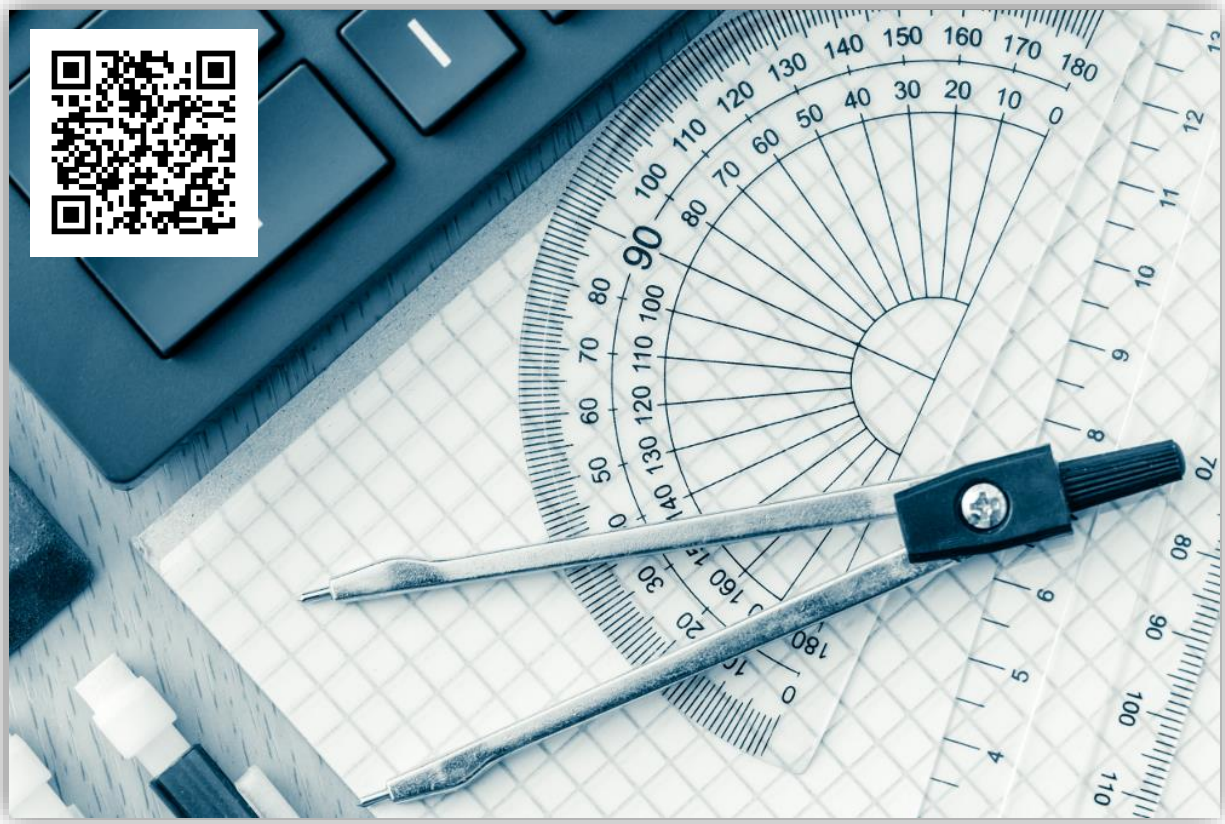
The Mathematics Department

7

Introducing Angles

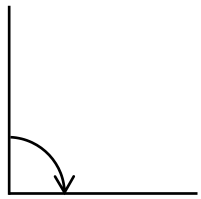
Name:

Contents



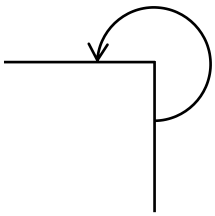
Types of Angles

An angle measures **turn**. Here are some examples of different turns.

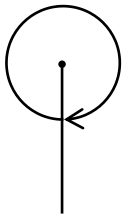


A quarter of a turn clockwise

Half a turn anticlockwise



Three quarters of a turn anticlockwise



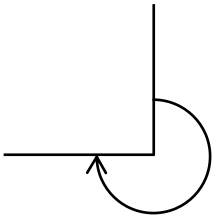
A full turn clockwise

Exercise 1

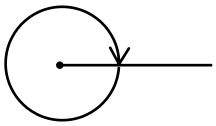
Skill 2

What **type** of turns do you see below?

(a)

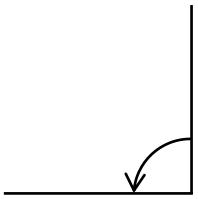


(b)



(c)

(d)

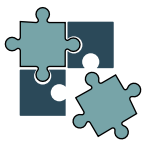


Exercise 2

2

Experiment with turning in class in different ways.

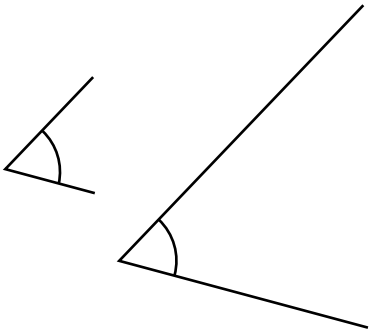
Exercise 3



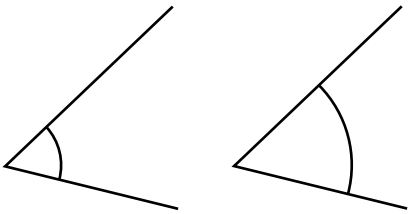
2

Explain which angle is the **largest** in each pair of angles below.

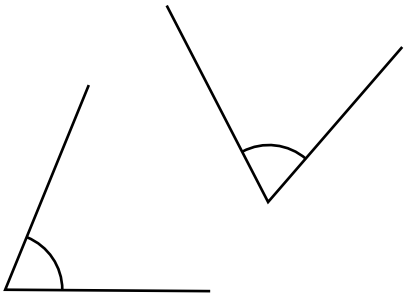
(a)



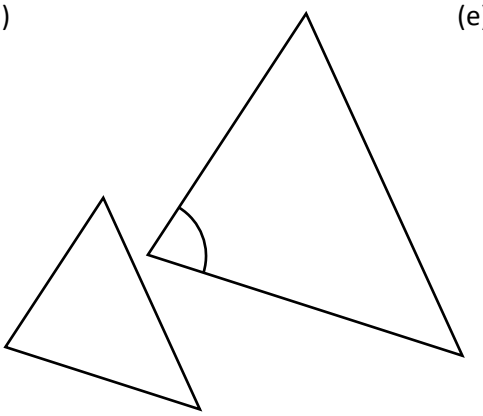
(b)



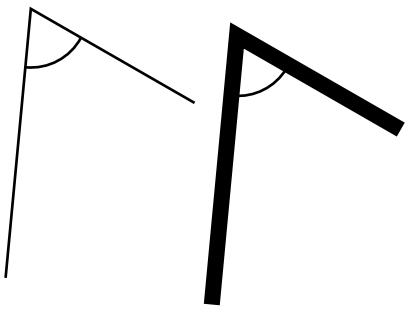
(c)



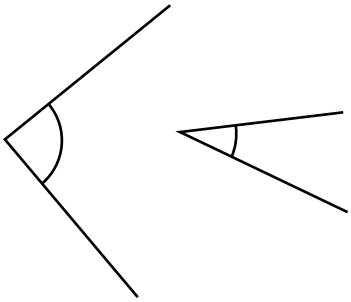
(d)



(e)

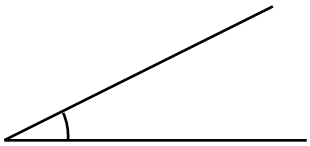


(f)

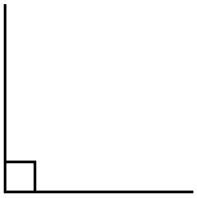


It is possible to measure an angle in degrees. There are **360°** in a full turn, so there are 180° in half a turn; 90° in a quarter of a turn, and so on.

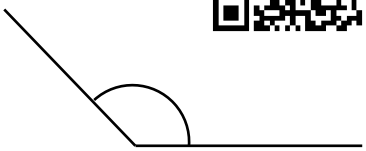
We use the following names for the different types of angles.



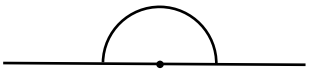
Acute Angle
Any angle between 0° and 90°



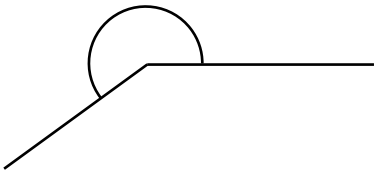
Right Angle
An angle that is exactly 90°



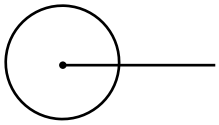
Obtuse Angle
Any angle between 90° and 180°



Straight Line
An angle that is exactly 180°



Reflex Angle
Any angle between 180° and 360°



Full Turn
An angle that is exactly 360°

Challenge!

Why do we use 360° for the size of a full turn, and not some other number?

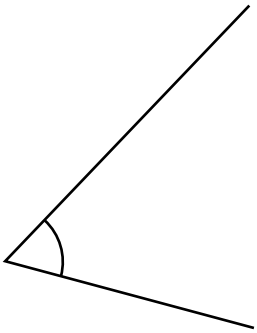
Aiming Higher

Skill 5

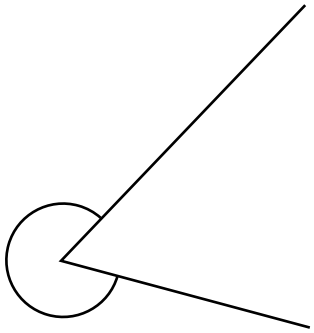
Exercise 4

What **type** of angles are the following angles?

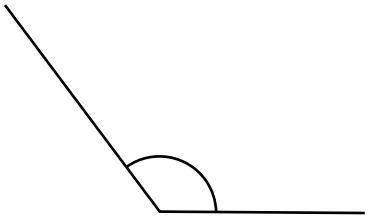
(a)



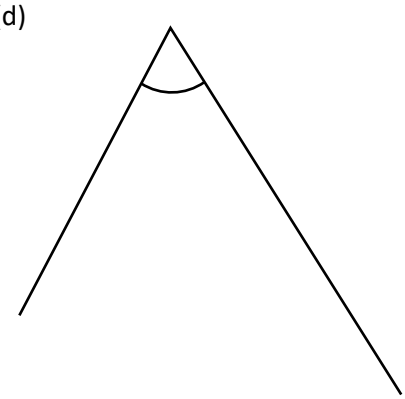
(b)



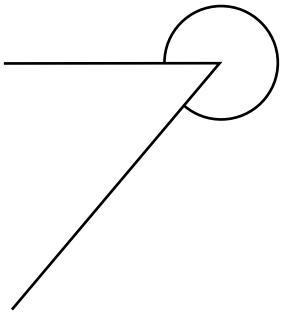
(c)



(d)



(e)



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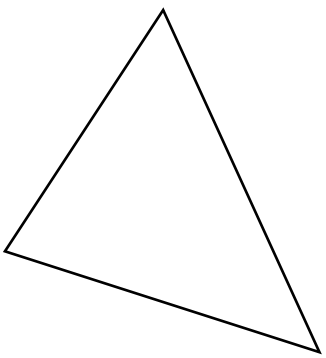


Exercise 5

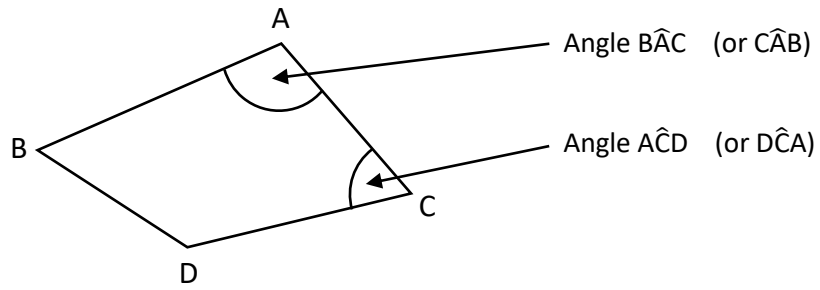
5 Skill

What **type** of angles are the following?

(a) 140° (b) 14° (c) 280° (d) 104° (e) 325° (f) 94° (g) 180°
(h) 54° (i) 205° (j) 90° (k) 270° (l) 2° (m) 360° (n) 103°

In order to **name** an angle, we label the vertices in a diagram with capital letters, before using **three letters** to identify a specific angle. (We need to add a **circumflex accent** to the letter in the middle.)

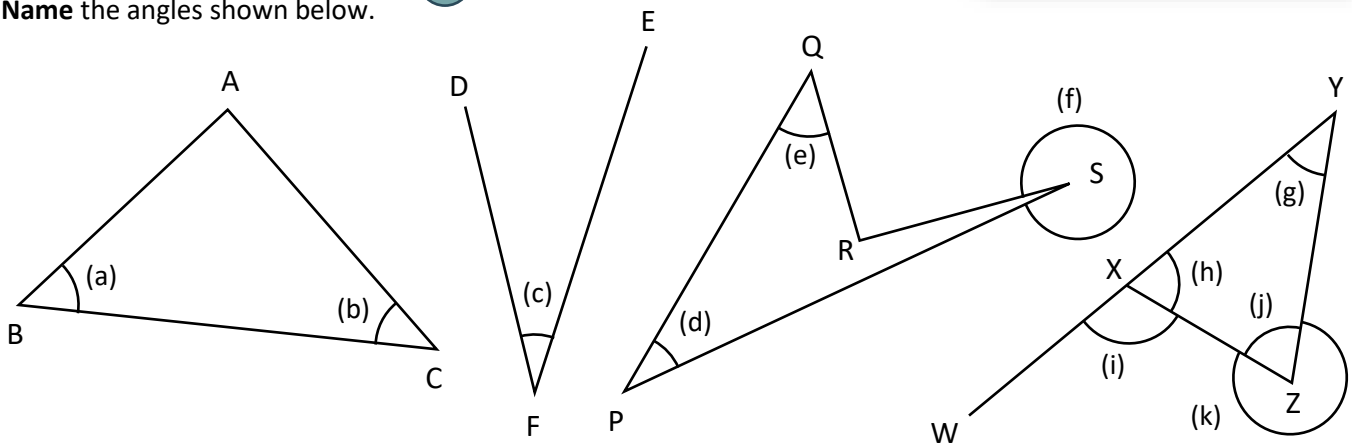
Example



Exercise 6

5

Name the angles shown below.



Exercise 7

5

What **type** of angles are the ones shown in Exercise 6?

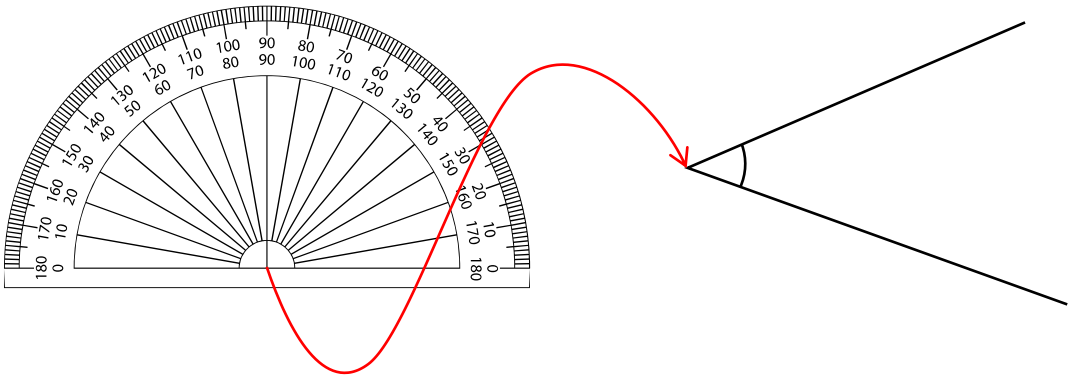
Evaluation

Key Words	Corrections	I am happy with…	I need to revise…

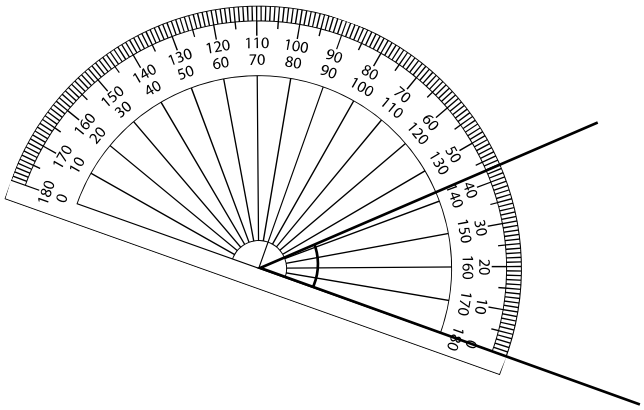
Measuring Angles

We use a **protractor** to measure angles. It is possible to obtain a protractor in the shape of half a circle (measures up to 180°) or in the shape of a full circle (measures up to 360°). To measure an angle:

(i) Put the **centre of the protractor** on the **vertex** (corner) of the angle.



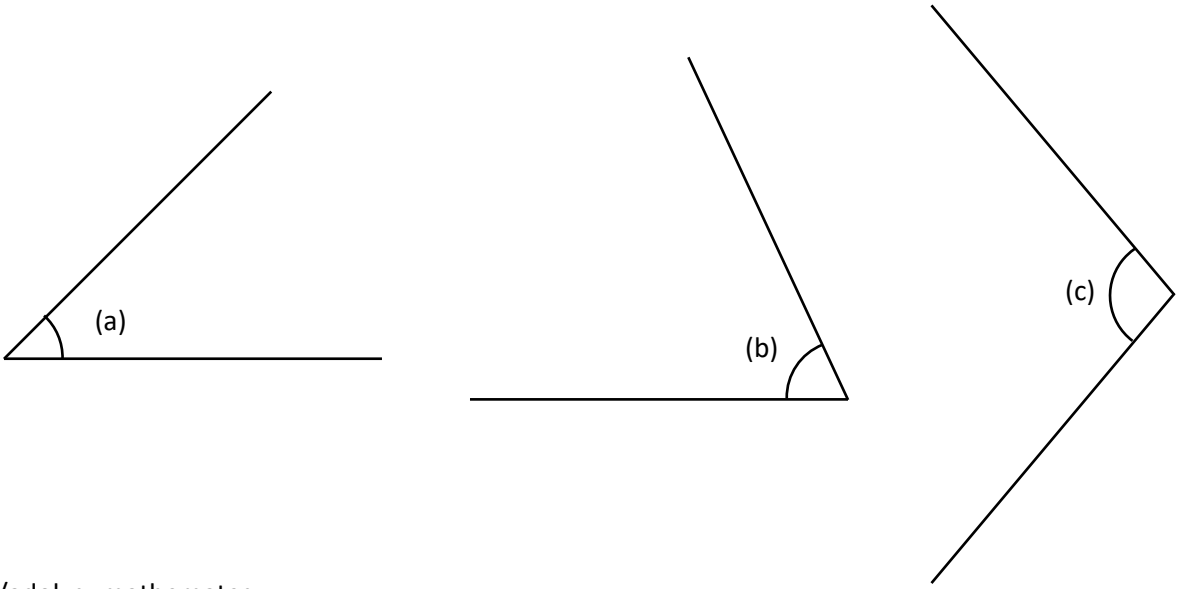
(ii) **Turn** the protractor so that the angle **0°** lies on one of the **lines** of the angle.



(iii) **Count up from 0°** in order to measure the angle. (It is important to do this as there are two sets of numbers on a protractor.) What angle is shown by the protractor above?

Exercise 8 5

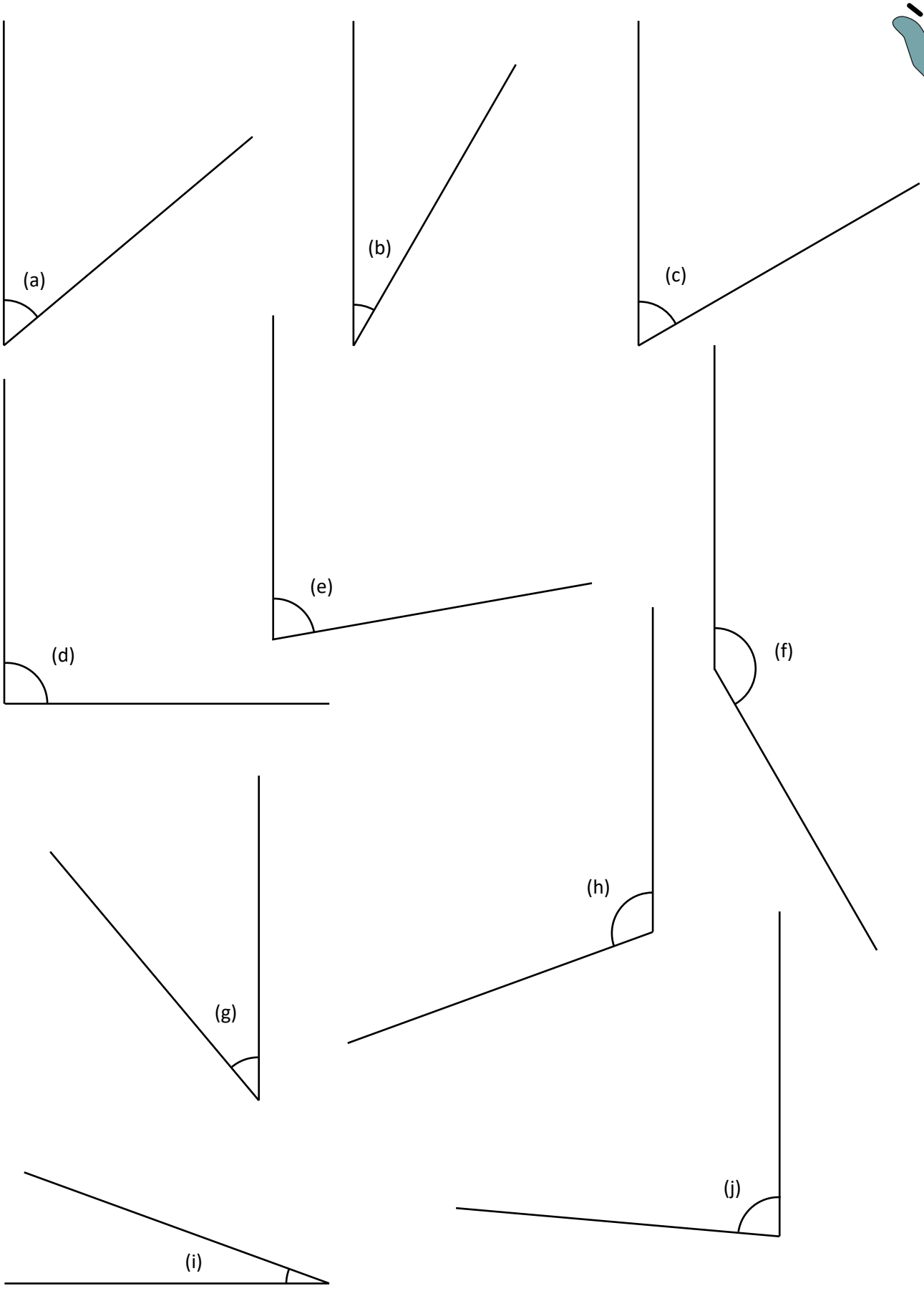
Estimate the size of each of the following angles in degrees. Write your estimate down before **measuring** the angle using a protractor. How accurate were you in estimating the size of the angles?



Exercise 9

5

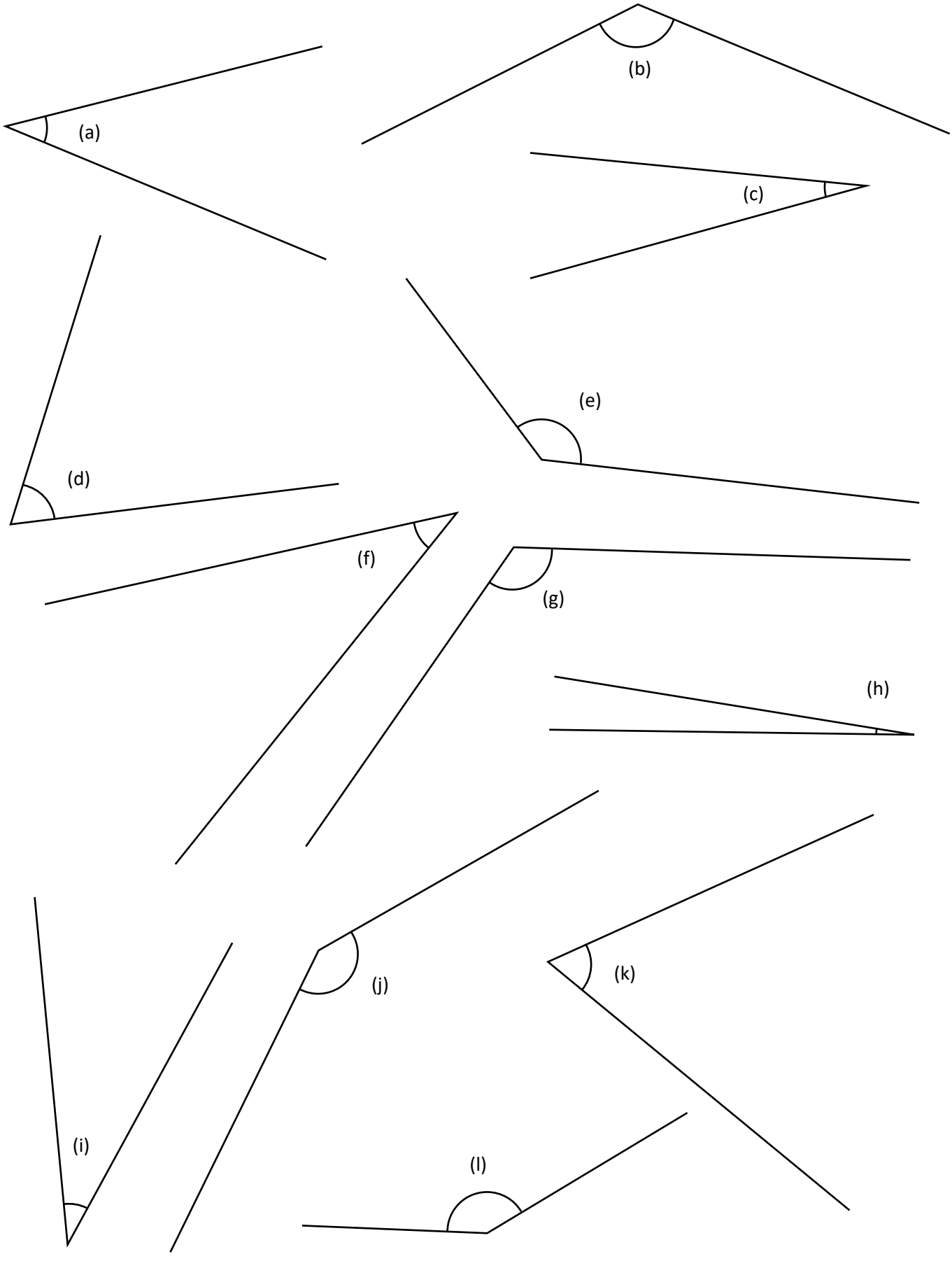
Measure the following angles.



Exercise 10

5

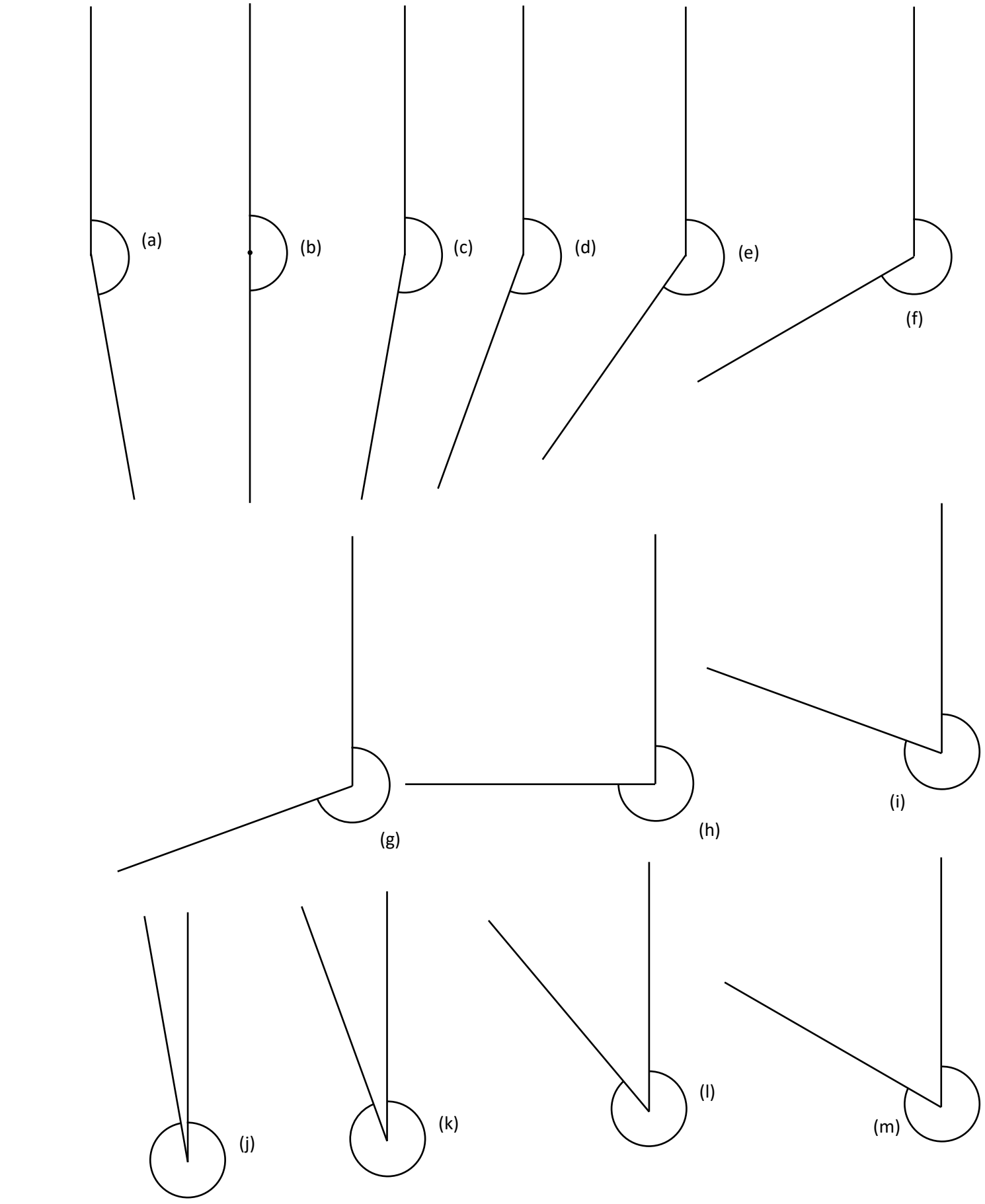
Measure the following angles and state what **type** of angles they are.



Exercise 11

5

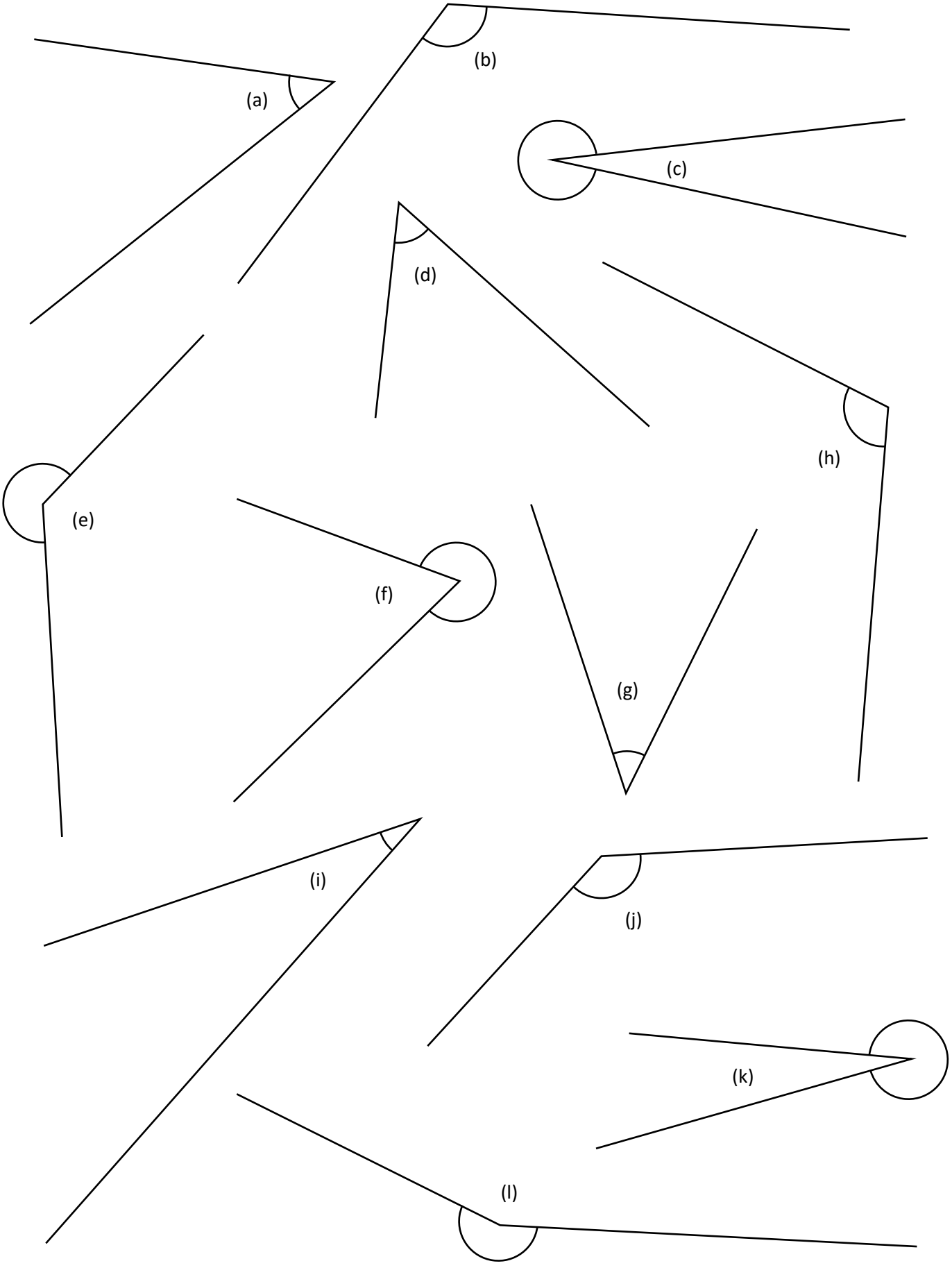
Measure the following angles.



Exercise 12

5

Measure the following angles.



Exercise 13

5 Applying

Measure the following angles.

(a) $B\hat{A}N$ (b) $A\hat{B}N$ (c) $A\hat{N}B$

(d) $B\hat{F}H$ (e) $H\hat{G}E$ (f) $N\hat{I}F$

(g) $P\hat{T}W$ (h) Reflex angle $P\hat{T}W$ (i) $Q\hat{M}S$

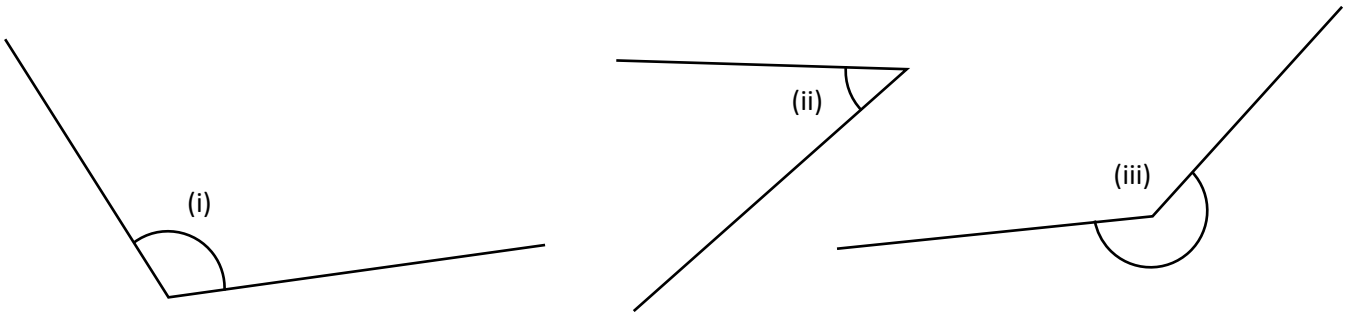
(j) $T\hat{X}W$ (k) $W\hat{U}V$ (l) $V\hat{N}A$

(m) Reflex angle $H\hat{G}J$ (n) $L\hat{K}J$ (o) Reflex angle $Z\hat{Y}X$

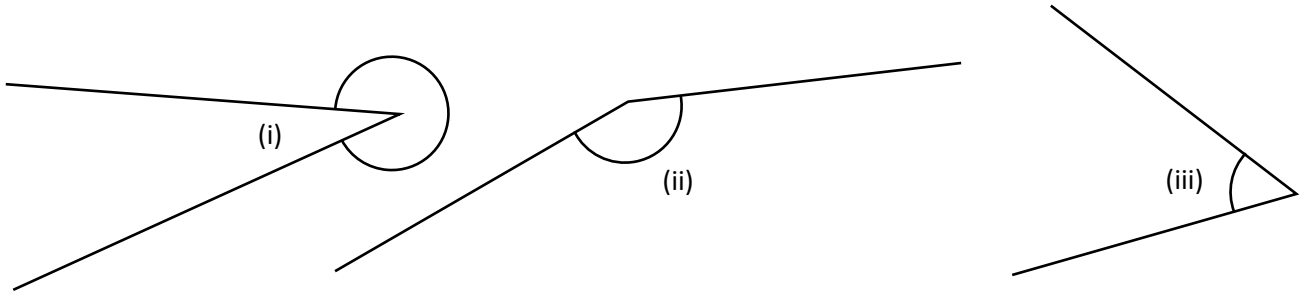
Exercise 14 (Revision)

Skill 5

(a) **Estimate** the size of each of the following angles in degrees.
Write down your estimate before **measuring** the size of the angle using a protractor.



(b) **Measure** the following angles and state what **type** of angles they are.

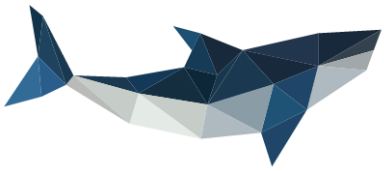


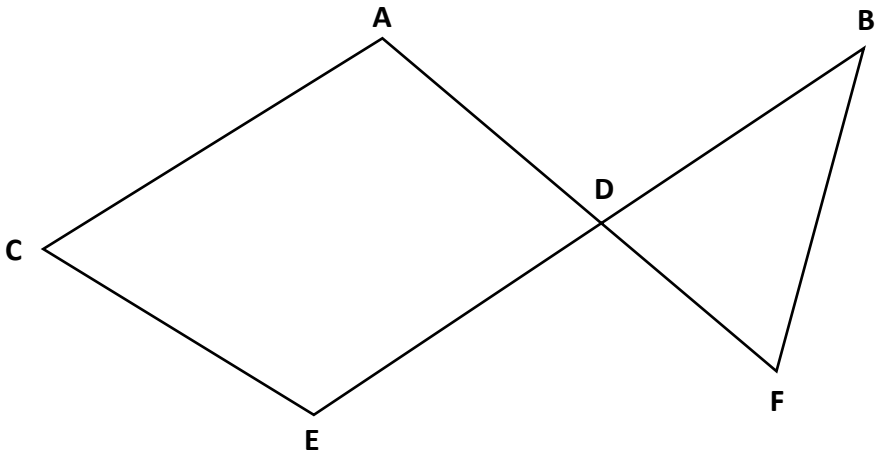
(c) **Measure** the following angles.

(i) $A\hat{D}E$

(ii) $D\hat{B}F$

(iii) Reflex angle $A\hat{C}E$





Evaluation

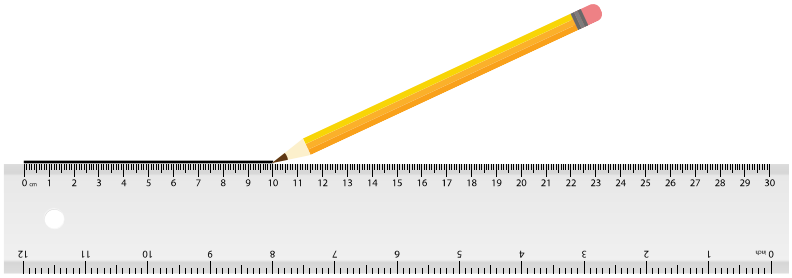
Key Words	Corrections	I am happy with…	I need to revise…

Drawing Angles

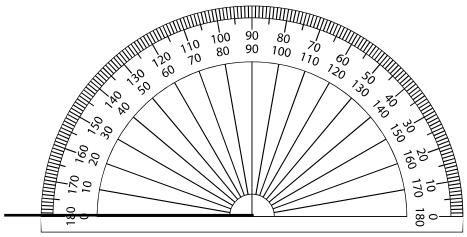
It is possible to draw any angle using a protractor, and it is possible to draw some special angles using a compass.

Drawing Angles Using a Protractor

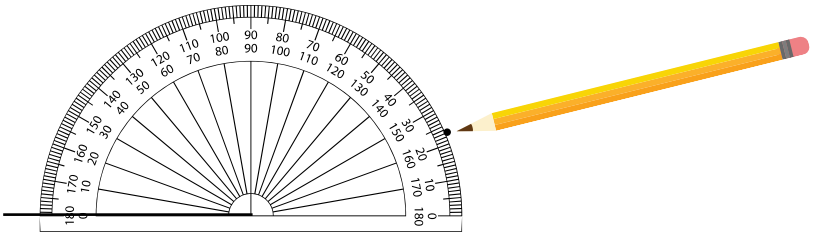
(1) Draw a straight line using a ruler. (Allow plenty of space around the line in order to draw the angle.)



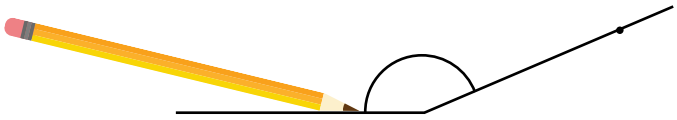
(2) Place the centre of the protractor on one end of the line. Turn the protractor so that the angle 0° lies on the line.



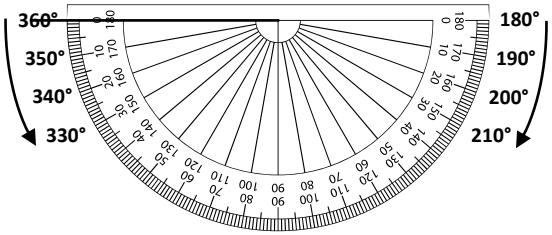
(3) Count up from 0° to the angle you are attempting to draw. Mark the angle (e.g. 157°) using a pencil.



(4) Draw a straight line through the dot, starting from where the centre of your protractor was. Draw the arc of the angle.



Note: If you are drawing a **reflex** angle, hold your protractor as shown below during step (2). Then, during step (3), either count up from 180° on the right side of the protractor, or count down from 360° on the left side of the protractor.



Exercise 15

Skill 5

Draw the following angles using a protractor.

(a) 13° (b) 40° (c) 78° (d) 93° (e) 107° (f) 138° (g) 164°

(h) 193° (i) 216° (j) 254° (k) 265° (l) 284° (m) 303° (n) 341°

Exercise 16

5

(a) Draw an angle of 53° on the left side of the line.

(b) Draw an angle of 147° on the right side of the line.

(c) Draw an angle of 237° on the left side of the line.

(d) Draw a right angle on the right side of the line.

(e) Draw an obtuse angle on the left side of the line.

(f) Draw an angle of 357° on the right side of the line.

Exercise 17 (Revision)

Draw the following angles using a protractor.

(a) 25° (b) 74° (c) 172°
(d) 208° (e) 270° (f) 317°

Key Words	Corrections	I am happy with…	I need to revise…

Calculating Angles

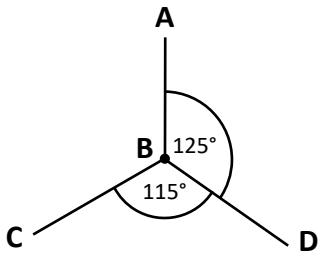
With enough information, it is possible to find the size of some angles without using a protractor, by **calculation**.

Angles Around a Point

From the information on page 3, we know that a complete turn is 360°. It follows that the **angles around any point** must sum to **360°**.

Example

In the diagram on the right, it is possible to find the size of the angle $A\hat{B}C$ without using a protractor, as the sum of $A\hat{B}C$, $C\hat{B}D$ and $A\hat{B}D$ must be 360°.

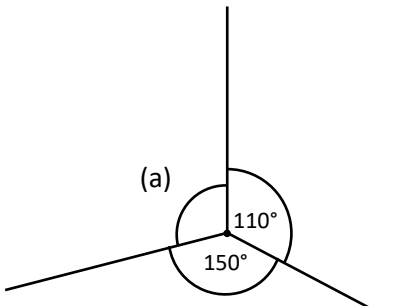


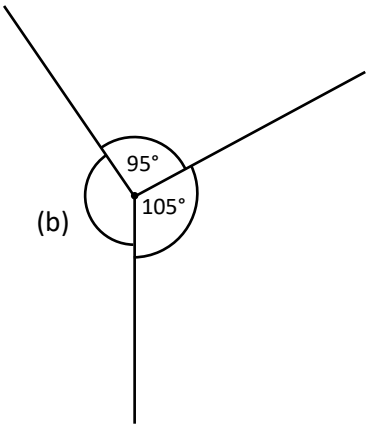
$125° + 115° = 240°$
$A\hat{B}C = 360° - 240°$
$A\hat{B}C = 120°$.

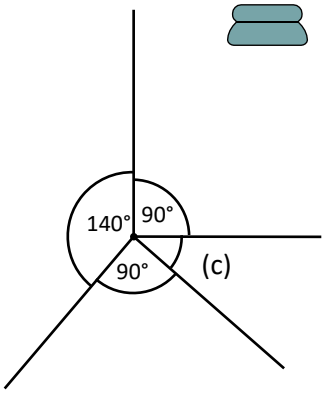
Exercise 18

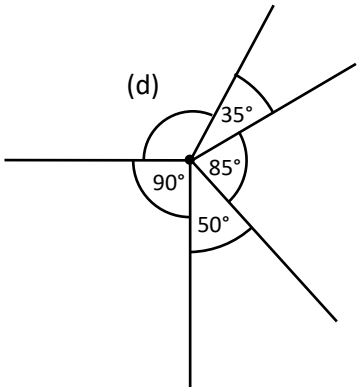
Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

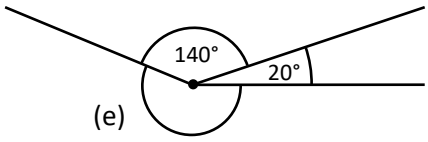
Skill 6

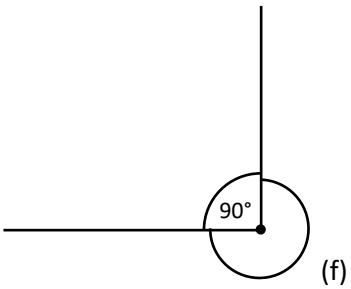


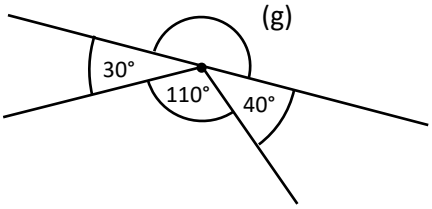


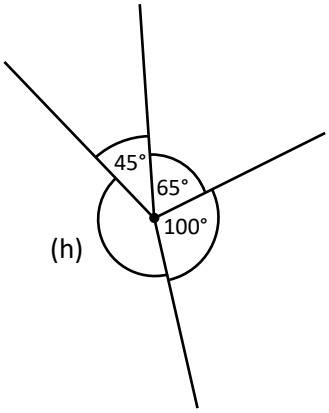


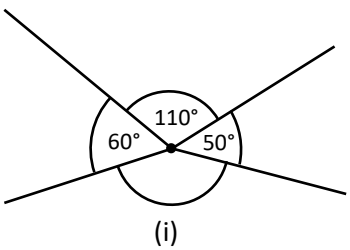










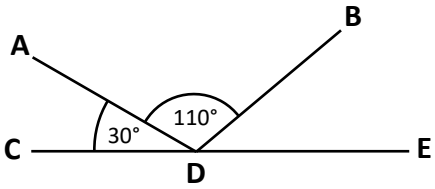


Angles on a Straight Line

The angle for half a turn is 180°, so the angles on any **straight line** must sum to **180°**.

Example

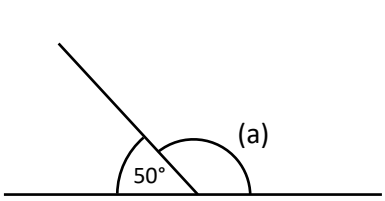
In the diagram on the right, it is possible to find the size of the angle $B\hat{D}E$ without using a protractor, as the sum of $A\hat{D}C$, $A\hat{D}B$ and $B\hat{D}E$ must be 180°.

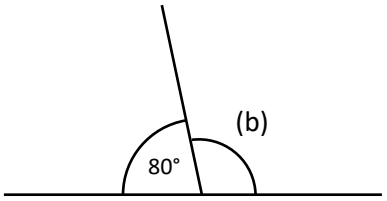


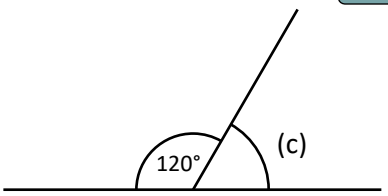
$30° + 110° = 140°$
$B\hat{D}E = 180° - 140°$
$B\hat{D}E = 40°$.

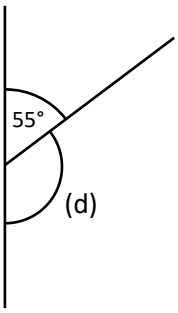
Exercise 19

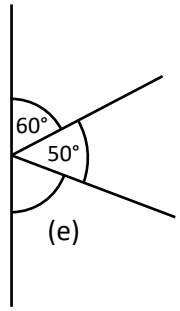
Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

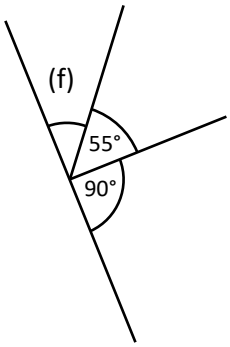


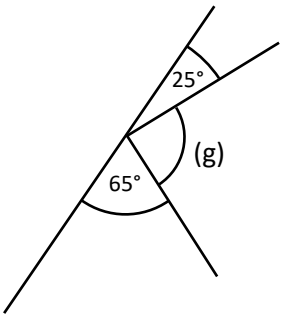


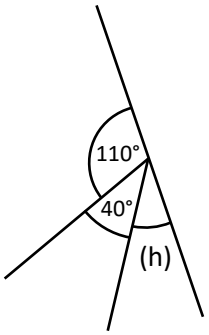


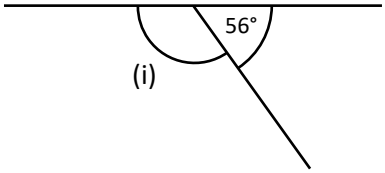


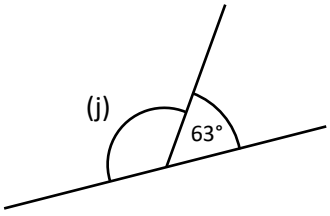


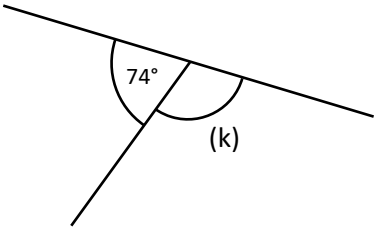


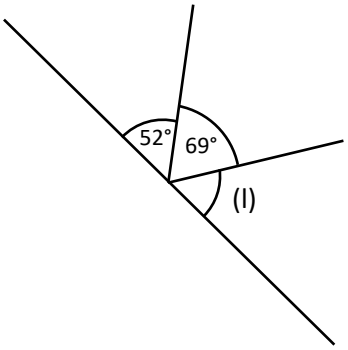


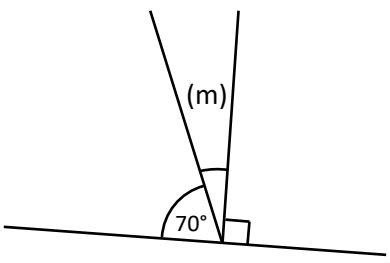


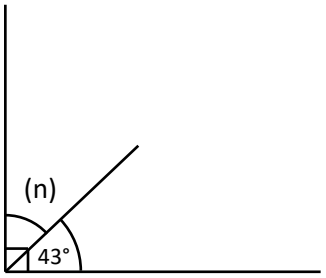






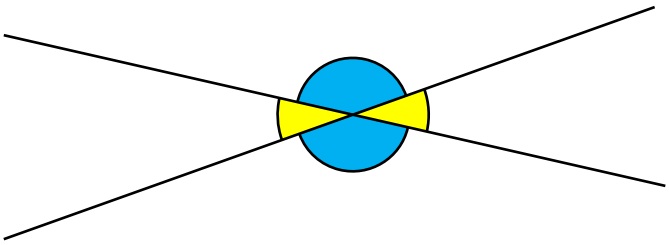






Vertically Opposite Angles

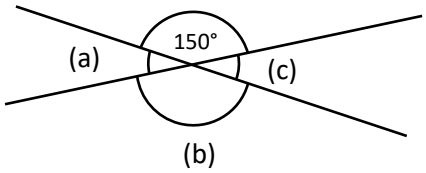
Every time that two lines cross, four angles are formed around the vertex. The angles that are opposite each other (not next to each other) are **equal**. We call these angles **vertically opposite angles**.

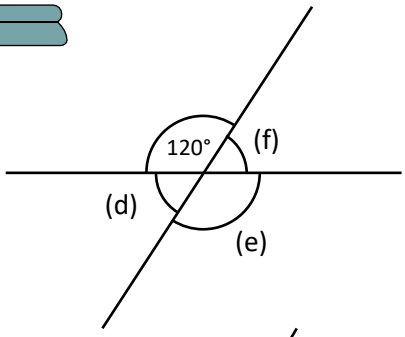


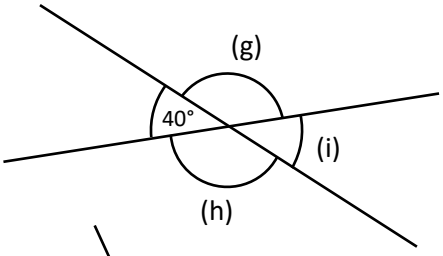
Exercise 20

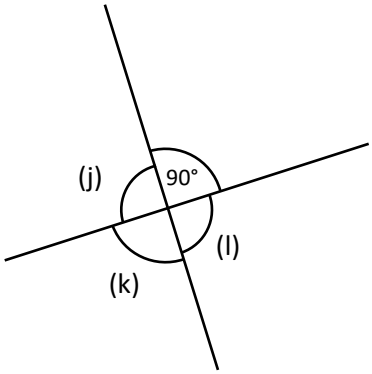
Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

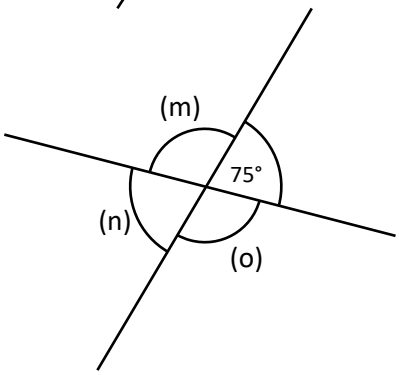
6 Skill

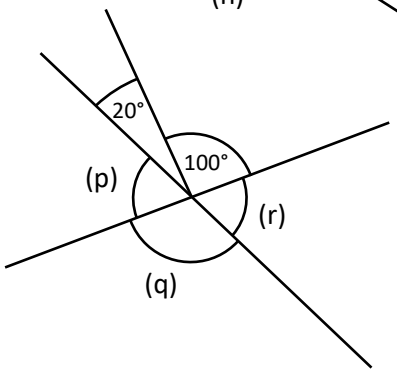


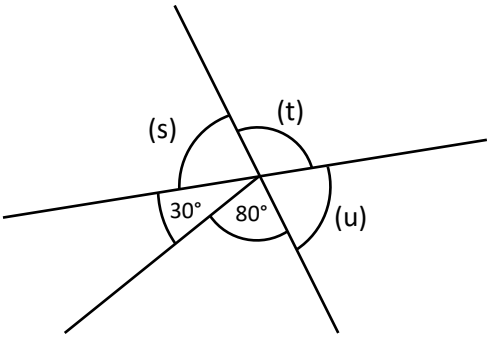


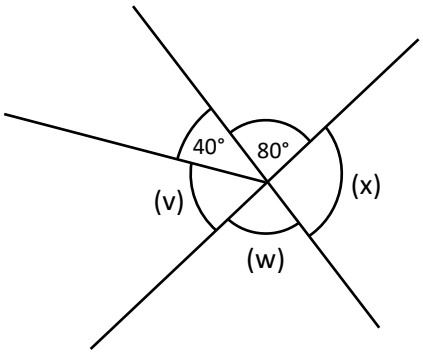


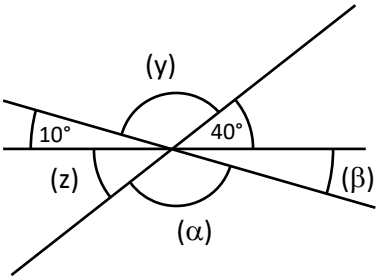






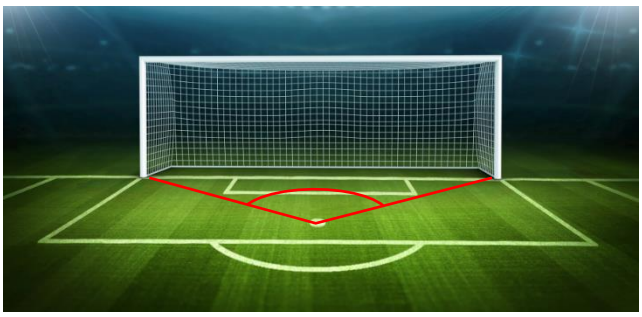






Challenge!

The width of a football goal is 8 yards. The shortest distance from the goal to the penalty spot is 12 yards. Construct a scale drawing in order to find the angle shown in the picture, which is the angle to score a penalty.

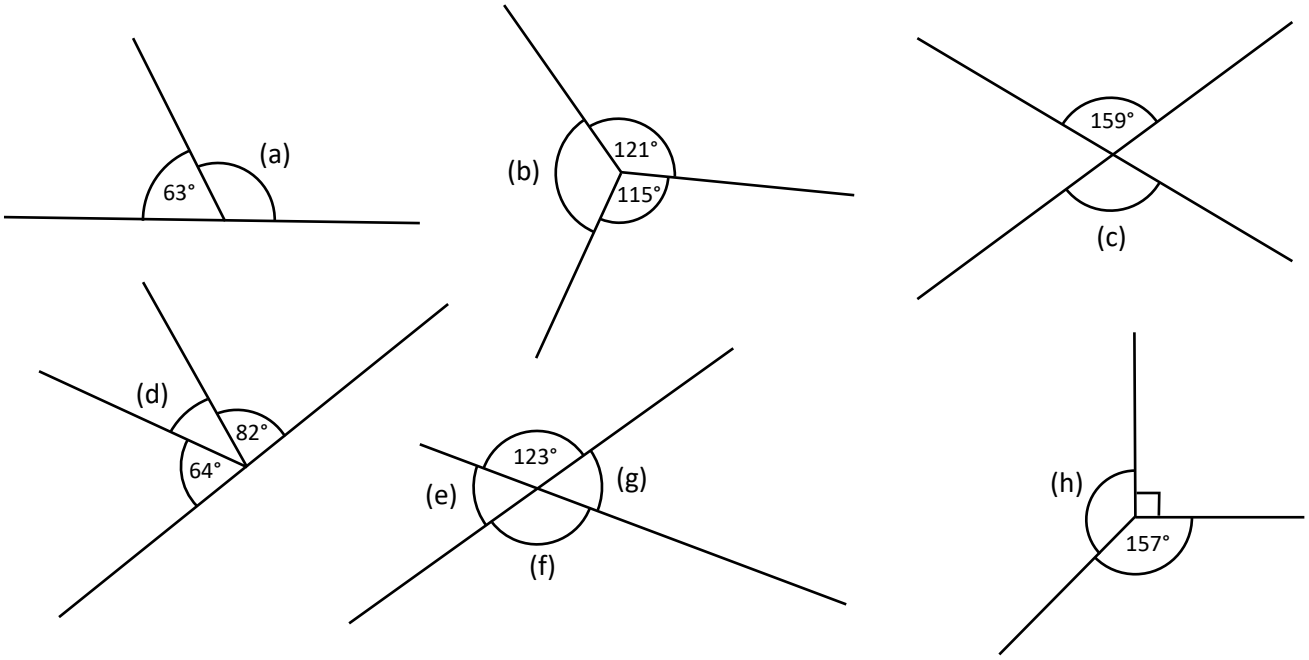


Aiming Higher Applying

Exercise 21

6 Skill

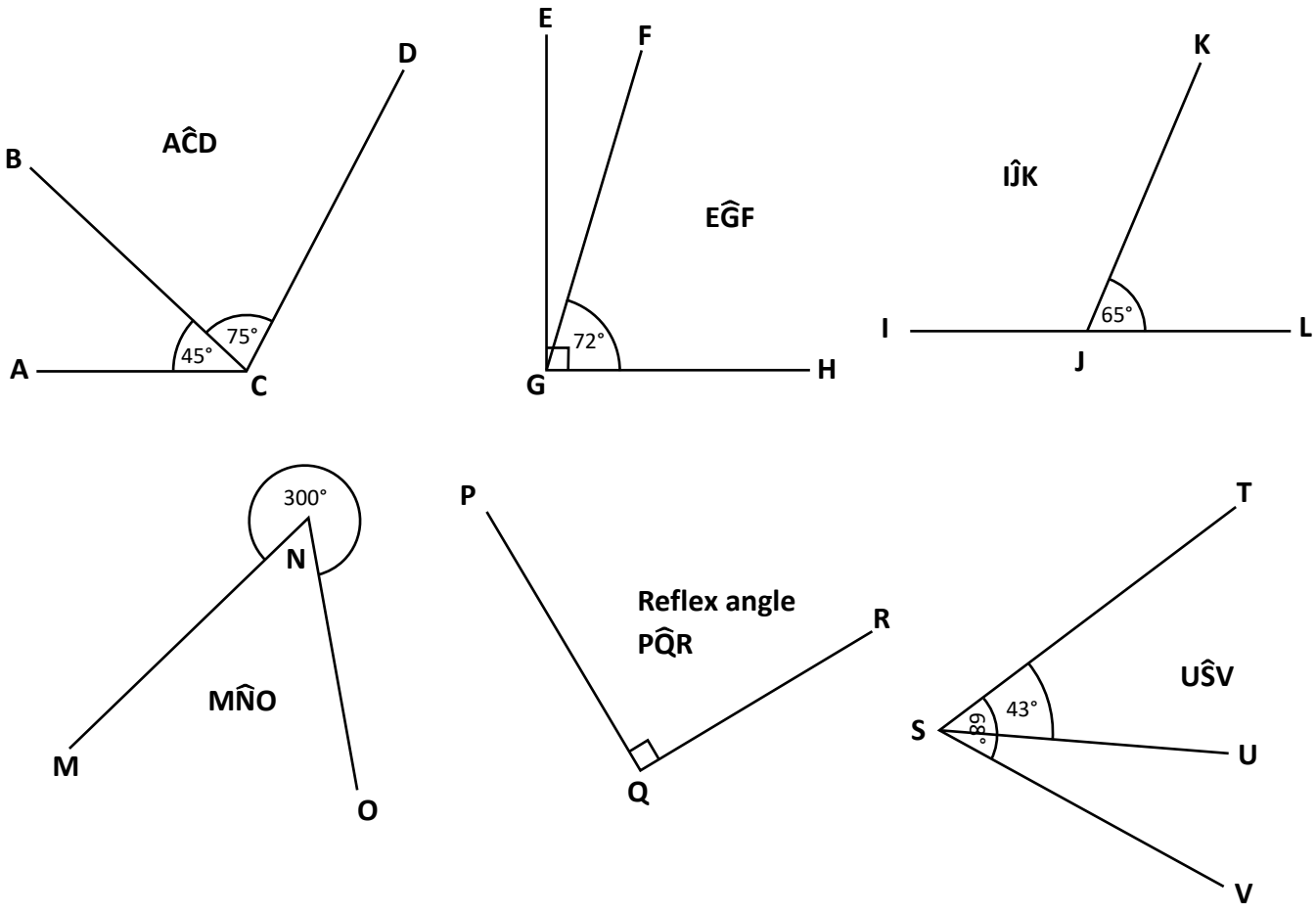
Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



Exercise 22

6

Calculate the **size** of each of the named angles. (The diagrams are not drawn to scale.)

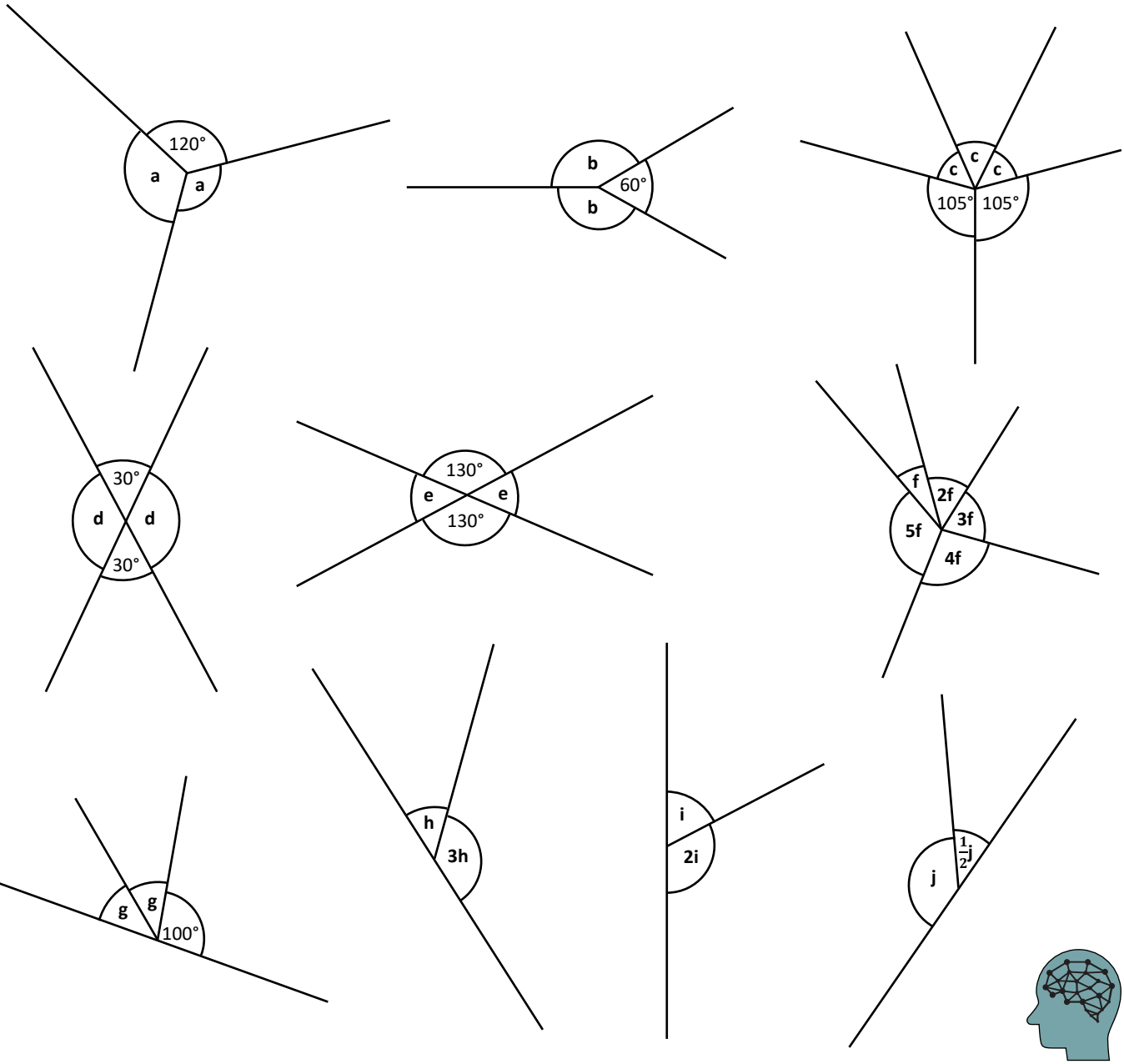


Challenge!

Can you find the following angles? (The diagrams are not drawn to scale.)

Aiming Higher

6 Extension



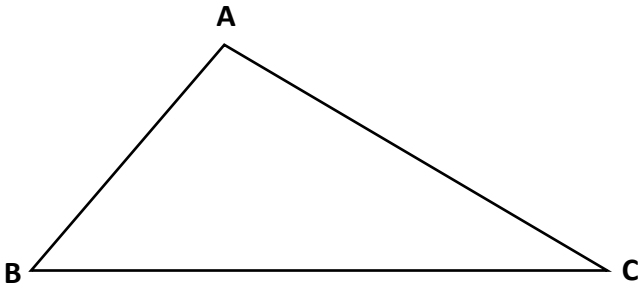
Evaluation

Key Words	Corrections	I am happy with…	I need to revise…

Angles in a Triangle

The angles inside a shape are called **interior angles**.

Use your protractor to measure the three interior angles of the triangle shown on the right.



$A\hat{B}C =$ _____ °
$B\hat{A}C =$ _____ °
$A\hat{C}B =$ _____ °

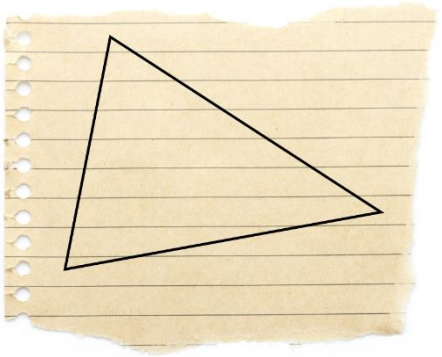
What is the total of your answers?

Total = _____°

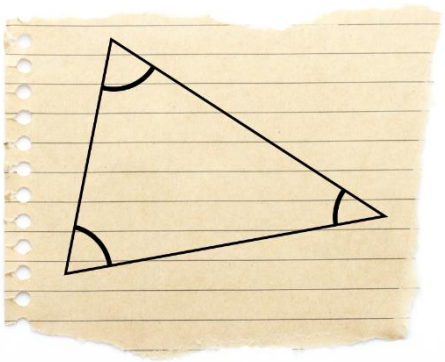
Proving the Sum of the Interior Angles of a Triangle

Follow the steps below to show that the sum of the interior angles of a triangle is always the same.

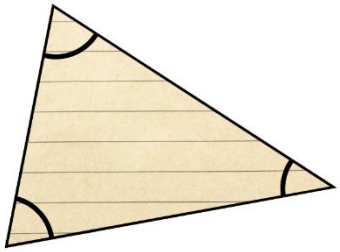
1. Draw any triangle on a piece of scrap paper. The triangle does **not** need to look like the triangle below.



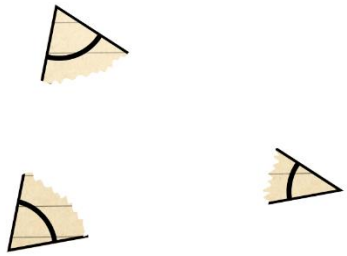
2. Mark the interior angles of your triangle with thick arcs.



3. Cut your triangle out using a pair of scissors.



4. Tear off the corners of your triangle.



5. Arrange the corners to form a straight line.

6. Glue your corners (in a straight line) in the box below.

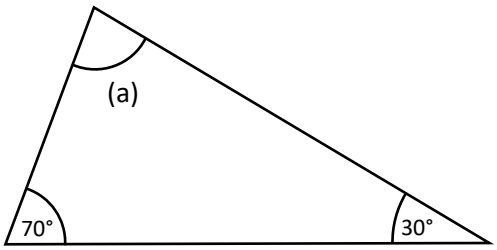
7. Complete the following sentence:

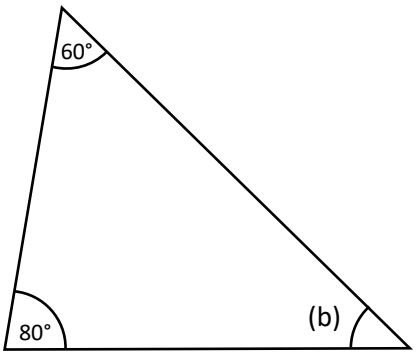
The sum of the interior angles of a triangle is always _____ °

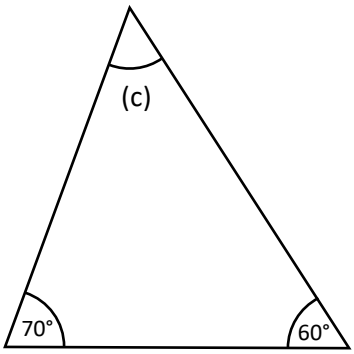
Exercise 23

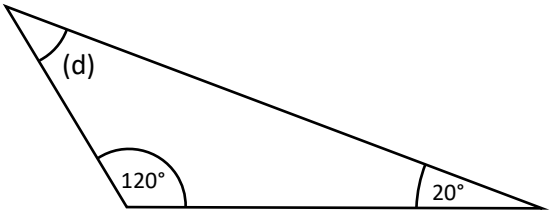
Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)

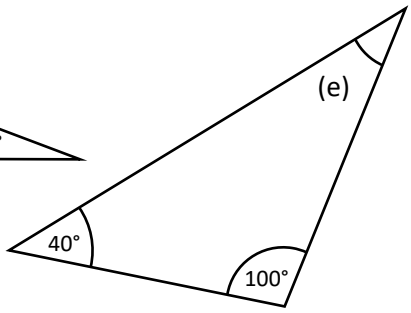
6 Skill

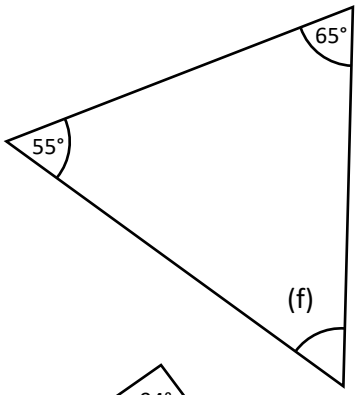


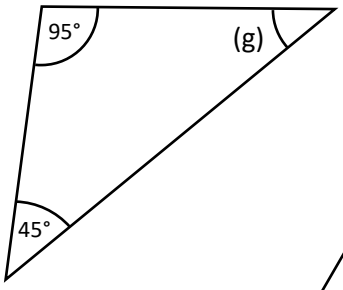


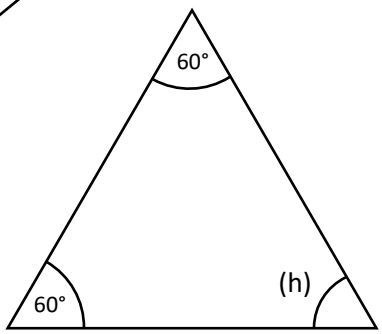


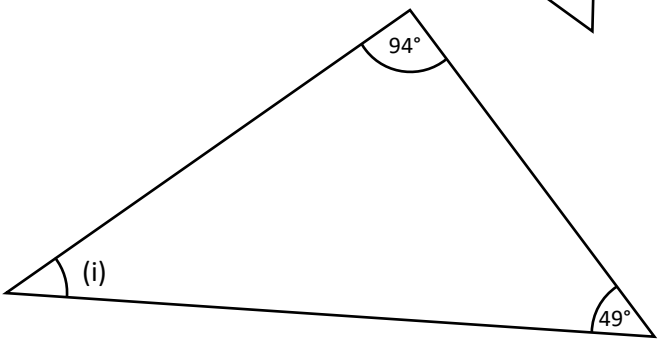


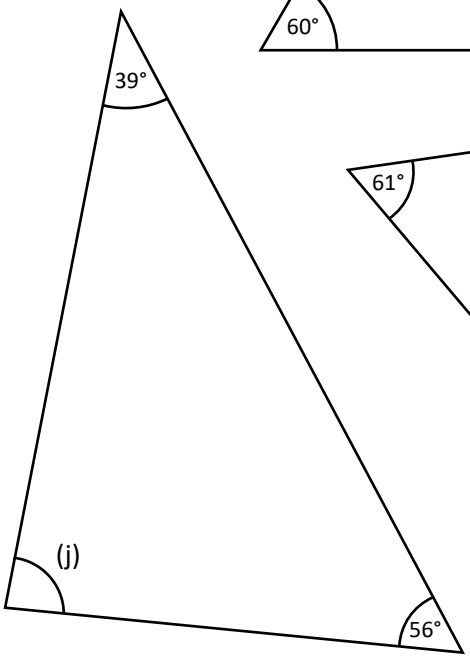


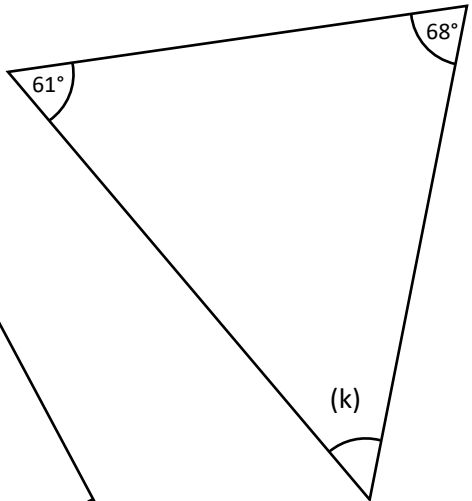


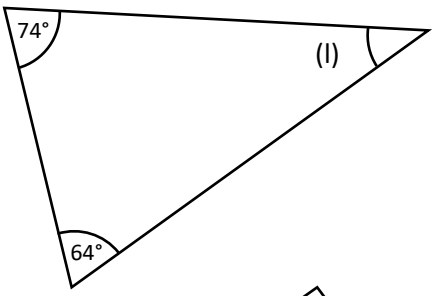


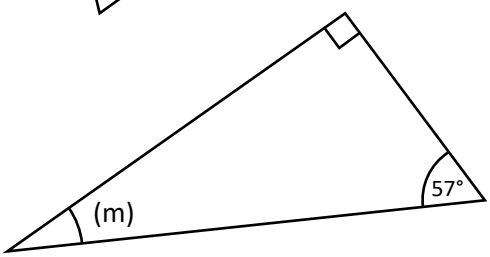






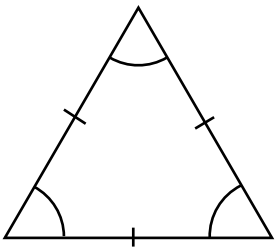




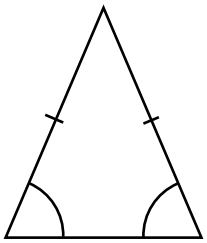


Types of Triangle

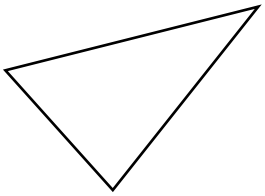
An **equilateral** triangle has three equal sides and three equal angles.



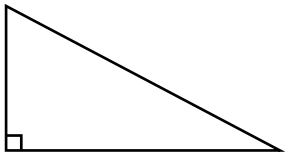
An **isosceles** triangle has two equal sides and two equal angles.



A **scalene** triangle does not have any equal sides or any equal angles.



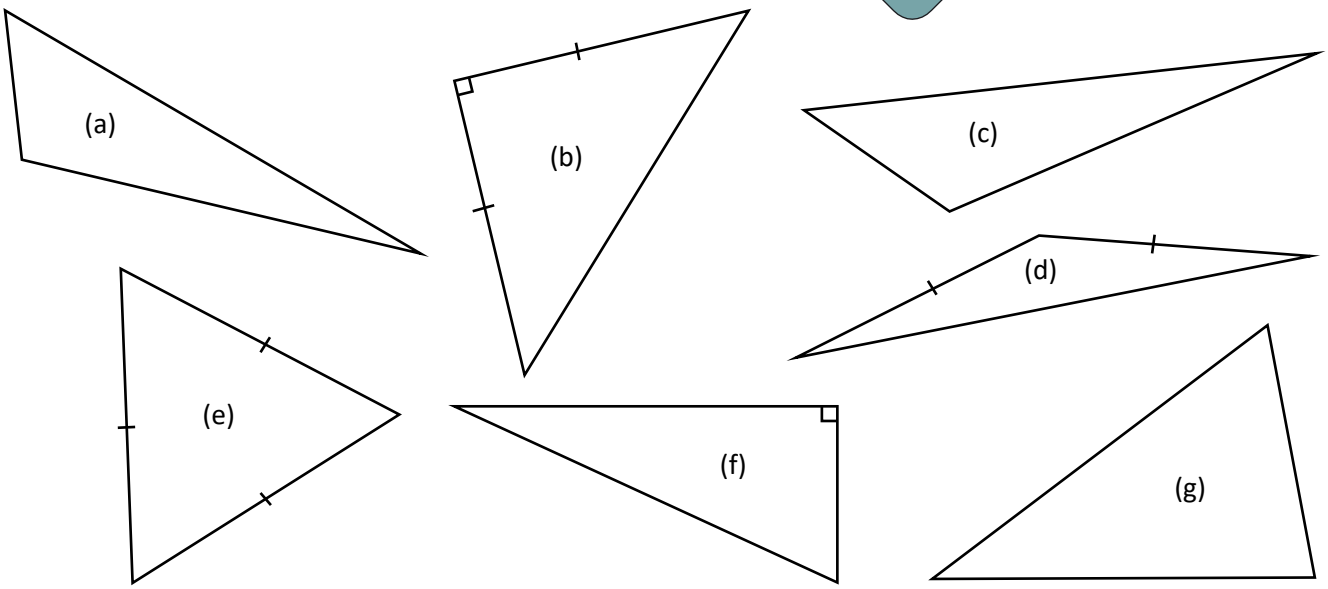
A **right-angled** triangle has one angle that is a right angle, or 90°.



Exercise 24

6

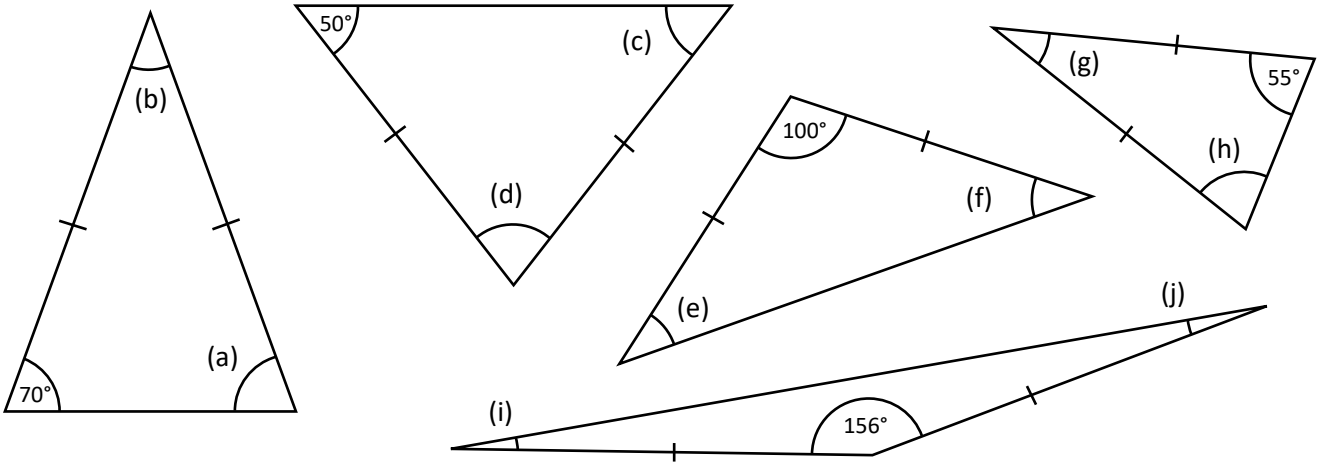
What **type** of triangles are shown below?



Exercise 25

6

Calculate the **size** of the marked angles.
(The diagrams are not drawn to scale.)



Exercise 26

6

(a) Dewi draws a triangle with angles 50°, 80° and 50°.
What type of triangle is this?

(b) Explain whether or not it is possible to draw a triangle with two obtuse angles.

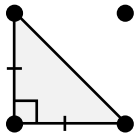
(c) Llinos draws an isosceles triangle that is also a right-angled triangle.
What are the size of the angles in Llinos' triangle?

(d) Explain whether or not it is possible to draw an equilateral triangle that is also a right-angled triangle.

(e) Tomos draws a triangle and measures its angles as 95°, 95° and 170°. Erin measures the angles of the same triangle and gets 85°, 85° and 10°. Explain who has measured the angles correctly. What mistake has the other person made?

Investigation

Draw triangles on the dotty paper below (each corner of a triangle must be placed on a dot).
Identify the type of triangle underneath the dots. How many different triangles are possible?

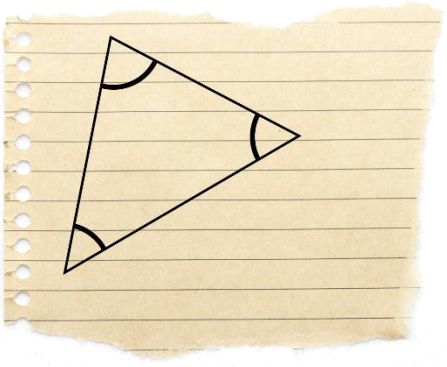
 Isosceles; Right-Angled			

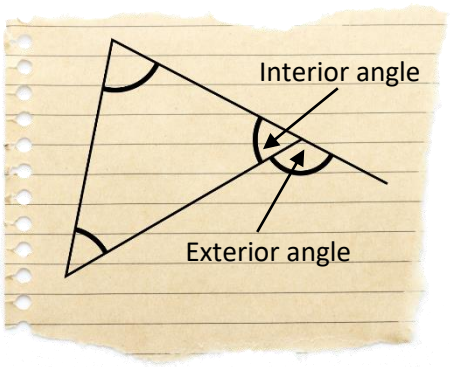
The Exterior Angle of a Triangle

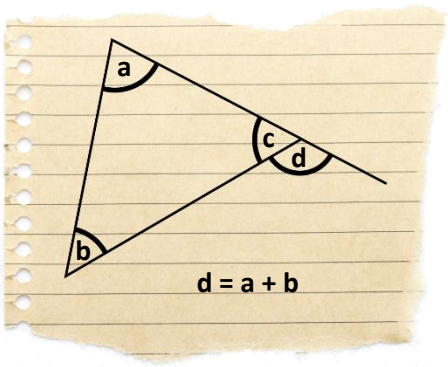
Consider any triangle.

By extending one of the sides of the triangle, we create an **exterior angle**.

The size of the exterior angle is the sum of the **two opposite interior angles**.



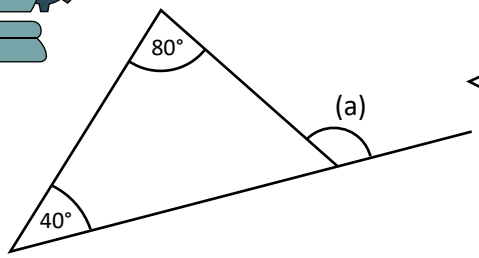


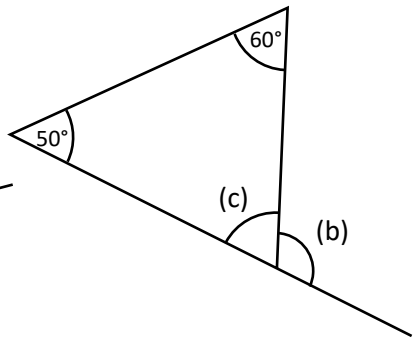


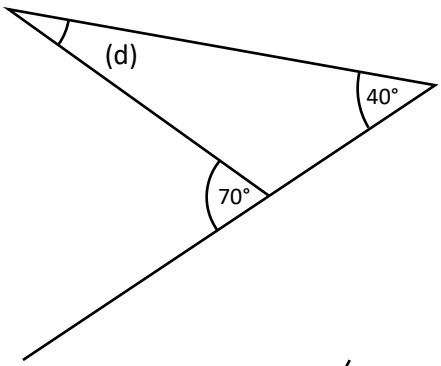
Exercise 27

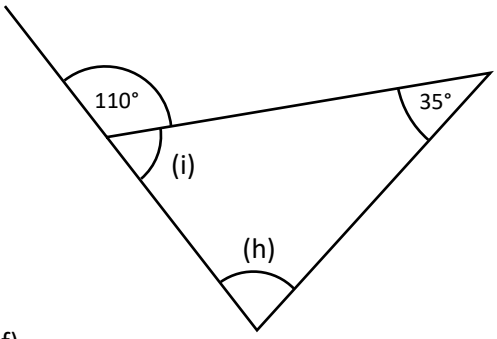
6 Skill

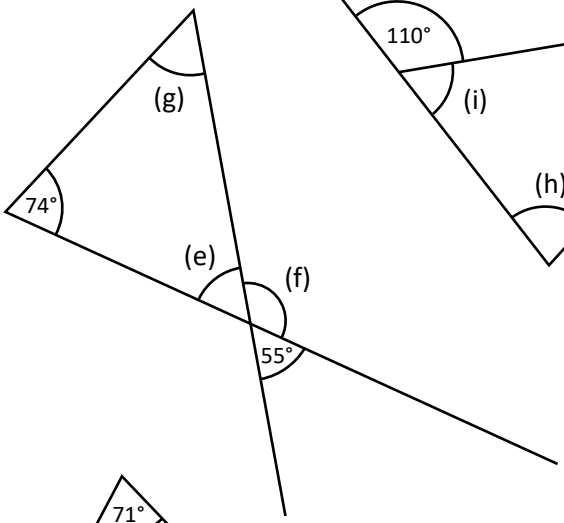
Calculate the **size** of the marked angles. (The diagrams are not drawn to scale.)

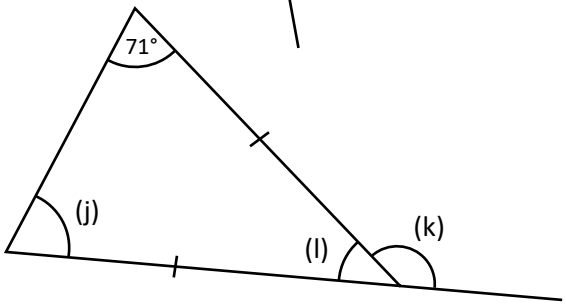












LOGO

6

1. Go to the LOGO website (https://www.mathemateg.com/logo/index.htm).
2. To draw a rectangle, type the instructions on the right.
3. Clear the screen by typing `CLEAN` and then take the turtle back to its original position by typing `HOME`.
4. Try to draw the following triangles.
   (a) An equilateral triangle. (b) An isosceles triangle.
   (c) A right-angled triangle. (d) A scalene triangle.
   Write down the instructions you have used in your book.
5. Try to draw the following quadrilaterals.
   (a) Square. (b) Parallelogram. (c) Trapezium.
   (d) Rhombus. (e) Kite. (f) Arrowhead.
   Write down the instructions you have used in your book.

```
FORWARD 100
RIGHT 90
FORWARD 50
RIGHT 90
FORWARD 100
RIGHT 90
FORWARD 50
```

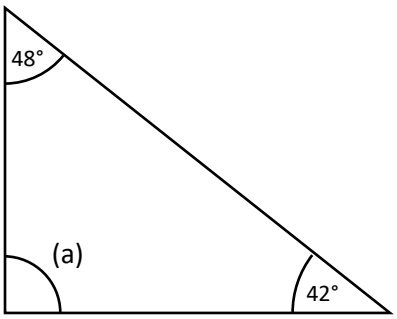
Useful Instructions

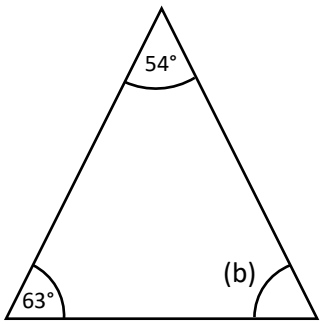
`FORWARD 50` Move the turtle forward 50 steps	`BACK 50` Move the turtle back 50 steps.	`LEFT 72` Turn anticlockwise through 72 degrees.	`RIGHT 56` Turn clockwise through 56 degrees.
`CLEAN` Clear all the lines.	`HOME` Move the turtle back to the centre of the screen.	`PENUP` Raise the pen.	`PENDOWN` Lower the pen.

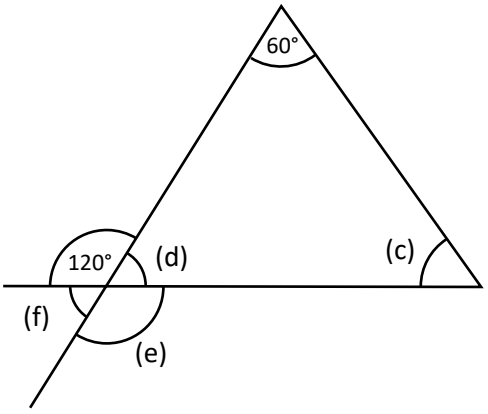
Exercise 26 (Revision)

6

Find the **size** of the marked angles. What **type** of triangles are shown?
(The diagrams are not drawn to scale.)







Key Words	Corrections	I am happy with…	I need to revise…

Introducing Angles

Reflection Sheet

Name: .....

Percentage in the test: .....

	I know this. 	I need to revise this. 	Question in the test:	Correct in the test?
I know the names for the different **types** of angle, e.g. acute angle.			1, 5, 10	
I know how to **name** an angle, e.g. $A\hat{B}C$.			4, 5, 11	
I know how to **estimate** an angle.				
I know how to **measure** an angle with a protractor.			3, 5	
I know how to draw an angle using a **protractor and ruler**.			2	
I know what the **angles around a point** total to.			6, 12	
I know what the **angles on a straight line** total to.			6, 12	
I know how to use **vertically opposite angles** to find the size of some angles.			6, 12	
I know the total of the angles in any **triangle**.			6, 9, 11, 12	
I know what the different types of triangle are, e.g. **equilateral** triangle.			7, 8, 11	
I know how to use **external angles** to find the size of some angles.			6	