WELSH JOINT EDUCATION COMMITTEE CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education Tystysgrif Addysg Gyffredinol

Advanced Level/Advanced Subsidiary Safon Uwch/Uwch Gyfrannol

MATHEMATICS FP3

Further Pure Mathematics

Specimen Paper 2005/2006

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Solve the equation

$$\cosh^2 x = 3 + \sinh x,$$

expressing the roots as natural logarithms. [7]

2. *(a)* By drawing appropriate graphs, show that the equation

$$x^3 = \cot x$$

has one root in the interval $(0, \pi/2)$. [3]

(b) Starting with an initial approximation $x_0 = 1$, use the Newton-Raphson method to calculate successive approximations x_1, x_2 and x_3 to this root. Write down the value of x_3 correct to 6 decimal places and determine whether or not this gives the value of the root correct to 6 decimal places. [8]

3. The arc joining the points (0,0) and (1,1) on the curve $y = x^3$ is rotated through four right-angles about the x-axis.

(a) (i) Show that the area of the curved surface generated is given by

$$2\pi \int_0^1 x^3 \sqrt{1 + 9x^4}\, \mathrm{d}x.$$ [2]

(ii) Use the substitution $u = 1 + 9x^4$ to show this area is equal to

$$\frac{\pi}{27}\left(10\sqrt{10} - 1\right).$$ [6]

4. Given that

$$I = \int_0^{\pi/2} \mathrm{e}^{-2x} \cos x \mathrm{d}x$$

and $$J = \int_0^{\pi/2} \mathrm{e}^{-2x} \sin x \mathrm{d}x,$$

use integration by parts to show that

$$I = \mathrm{e}^{-\pi} + 2J$$

and $$J = 1 - 2I$$

Hence evaluate I and J, giving each answer in the form $a + b\mathrm{e}^{-\pi}$, where a and b are rational numbers. [11]

5. *(a)* Find the Maclaurin series of $\ln(1 + \sin x)$ up to and including the x^3 term. [9]

(b) Use your series to evaluate, approximately, the integral

$$\int_0^{\frac{1}{3}} \ln(1+\sin x)\,dx$$ [4]

6. The curves C_1 and C_2 have polar equations as follows:

$$C_1 : r = 1 - \cos\theta \qquad (-\pi \le \theta \le \pi)$$
$$C_2 : r = \cos 2\theta \qquad \left(-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}\right)$$

(a) Sketch C_1 and C_2 on the same diagram. [2]

(b) Find the area enclosed by C_1. [5]

(c) Find the polar coordinates of the points of intersection of C_1 and C_2. [6]

7. *(a)* Show that

$$\frac{\sin n\theta - \sin(n-1)\theta}{\sin\theta} = \cos(n-1)\theta.$$ [2]

(b) Given that

$$I_n = \int_0^{\pi} \frac{\sin n\theta}{\sin\theta}\,d\theta$$

where n is an integer, show that for $n \ge 2$,

$$I_n = I_{n-2}.$$ [4]

(c) Hence evaluate I_n when n is

(i) an even integer,

(ii) an odd integer. [6]