



**GCE AS/A Level**

0979/01



**MATHEMATICS – FP3**  
**Further Pure Mathematics**

WEDNESDAY, 28 JUNE 2017 – MORNING

1 hour 30 minutes

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Solve the equation

$$2 \sinh \theta + \cosh \theta = 2.$$

Give your answer correct to three significant figures.

[7]

2. By putting  $t = \tan\left(\frac{x}{2}\right)$ , determine the value of the integral

$$\int_0^{\frac{\pi}{2}} \frac{2}{1 + \sin x + 2 \cos x} dx.$$

Give your answer in the form  $\ln N$ , where  $N$  is a positive integer.

[8]

3. The curve  $C$  has equation  $y = x^3$ . The arc joining the points  $(0, 0)$  and  $(1, 1)$  on  $C$  is rotated through an angle  $2\pi$  about the  $x$ -axis. Calculate the curved surface area of the solid generated, giving your answer correct to three significant figures.

[9]

4. The function  $f$  is defined by

$$f(x) = \cos(\ln(1 + x)).$$

- (a) Show that

$$(1 + x)^2 f''(x) + (1 + x) f'(x) + f(x) = 0.$$

[4]

- (b) Hence, or otherwise, show that the Maclaurin series for  $f(x)$  is

$$1 - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \dots$$

[5]

- (c) Deduce the Maclaurin series for  $\sin(\ln(1 + x))$  as far as the term in  $x^2$ .

[4]

5. (a) Show that the equation  $\tan \theta \tanh \theta = 1$  has a root,  $\alpha$ , between 0.9 and 1.1.

[3]

- (b) Consider the sequence defined by

$$\theta_{n+1} = \tan^{-1}\left(\frac{1}{\tanh \theta_n}\right) \quad \text{with } \theta_0 = 1.$$

- (i) Show that

$$\frac{d}{d\theta} \left( \tan^{-1}\left(\frac{1}{\tanh \theta}\right) \right) = -\left(\frac{1 - \tanh^2 \theta}{1 + \tanh^2 \theta}\right).$$

- (ii) Hence show that the sequence defined above is convergent.

[5]

- (c) Using this sequence, with  $\theta_0 = 1$ ,

- (i) write down the value of  $\theta_1$ ,  
 (ii) write down the value of  $\alpha$  correct to three decimal places.

[3]

6. The integral  $I_n$  is given, for  $n \geq 0$ , by

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx.$$

(a) Show that, for  $n \geq 2$ ,

$$I_n = \frac{1}{n-1} - I_{n-2}. \quad [5]$$

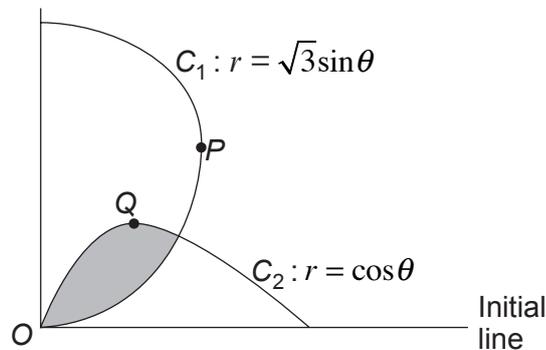
(b) Hence determine the value of the integral

$$\int_0^{\frac{\pi}{4}} (3 + \tan^2 x)^2 dx,$$

leaving your answer in terms of  $\pi$ .

[7]

7.



The diagram shows a sketch of the curve  $C_1$  with polar equation  $r = \sqrt{3} \sin \theta$  and a sketch of the curve  $C_2$  with polar equation  $r = \cos \theta$ , both defined for  $0 \leq \theta \leq \frac{\pi}{2}$ .

(a) The point at which the tangent to  $C_1$  is perpendicular to the initial line is denoted by  $P$  and the point at which the tangent to  $C_2$  is parallel to the initial line is denoted by  $Q$ . Show that the origin  $O$  and the points  $P$  and  $Q$  lie on a straight line. [5]

(b) (i) Show that the polar coordinates of the point of intersection of  $C_1$  and  $C_2$  are  $\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$ .  
(ii) Find the area of the shaded region enclosed by  $C_1$  and  $C_2$ . [10]

**END OF PAPER**