GCE AS/A level

0979/01

S15-0979-01

MATHEMATICS – FP3
Further Pure Mathematics

A.M. WEDNESDAY, 24 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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1. *(a)* Express $5\cosh\theta + 3\sinh\theta$ in the form $r\cosh(\theta + \alpha)$, $r > 0$, where the values of r and α are to be found. [4]

(b) Hence solve the equation

$$5\cosh\theta + 3\sinh\theta = 10.$$ [4]

2. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} e^{2x}\cos x \, dx,$$

giving your answer in the form $\dfrac{ae^{\pi} + b}{5}$, where a and b are integers to be found. [7]

3. The function f is defined by

$$f(x) = 3x^4 - 4x^3 - 3x^2 - 6x + 4.$$

You are given that the graph of f has exactly one stationary point whose x-coordinate is denoted by α.

(a) Show that

(i) α lies between 1·4 and 1·6,

(ii) $\alpha = \left(\dfrac{2\alpha^2 + \alpha + 1}{2}\right)^{\frac{1}{3}}$. [5]

(b) It is suggested that the following sequence could be used to determine the value of α.

$$x_{n+1} = \left(\frac{2x_n^2 + x_n + 1}{2}\right)^{\frac{1}{3}}; \quad x_0 = 1{\cdot}5$$

(i) By considering an appropriate derivative, show that this sequence is convergent.

(ii) Use this sequence to find the value of α correct to three decimal places. [8]

4. The function f is defined by

$$f(x) = \ln(1 + \cosh x).$$

(a) Show that

$$f''(x) = \frac{1}{1 + \cosh x}.$$ [3]

(b) Determine the Maclaurin series for $f(x)$ as far as the term in x^4. [6]

5. The curve C has parametric equations

$$x = t + \sin t, \quad y = 1 - \cos t \quad (0 \leqslant t \leqslant \pi)$$

(a) Show that

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 4\cos^2 \frac{1}{2}t.$$

[3]

(b) (i) Find the arc length of C.

(ii) Find the curved surface area of the solid generated when C is rotated through an angle 2π about the x-axis. [8]

6. *(a)* Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\left(4 - x^2\right)^{\frac{3}{2}}\right) = -3x\left(4 - x^2\right)^{\frac{1}{2}}.$$

[1]

The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_0^2 x^n \sqrt{4 - x^2}\, \mathrm{d}x.$$

(b) Show that, for $n \geqslant 2$,

$$I_n = \left(\frac{4(n-1)}{n+2}\right) I_{n-2}.$$

[5]

(c) (i) Show that

$$I_0 = \pi.$$

(ii) Evaluate I_4, giving your answer in the form $p\pi$ where p is a positive integer. [8]

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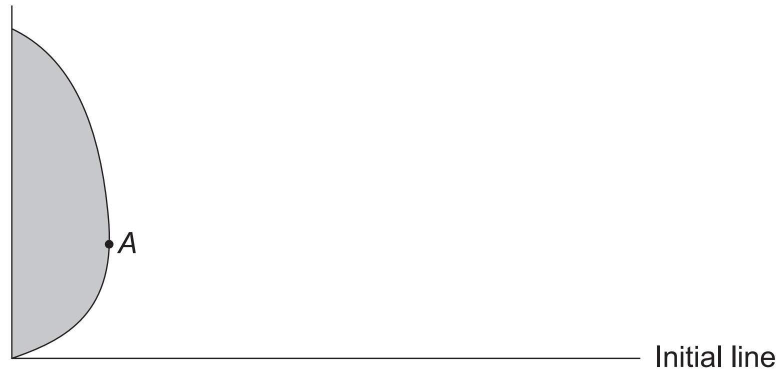
7.



The above diagram shows the curve C with polar equation

$$r = \tan\left(\frac{\theta}{2}\right), \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Show that the θ-coordinate of the point A at which the tangent to C is perpendicular to the initial line satisfies the equation

$$2\tan\theta\tan\left(\frac{\theta}{2}\right) = 1 + \tan^2\left(\frac{\theta}{2}\right).$$

Hence find the polar coordinates of A. [9]

(b) Find the area of the shaded region enclosed between C and the line $\theta = \frac{\pi}{2}$. [4]

END OF PAPER