



GCE AS/A level

979/01

MATHEMATICS FP3
Further Pure Mathematics

P.M. FRIDAY, 24 June 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find the positive root of the equation

$$3 \tanh^2 \theta = 5 \operatorname{sech} \theta + 1,$$

giving your answer in the form $\ln(a + \sqrt{b})$, where a, b are positive integers. [8]

2. Use the substitution $t = \tan \frac{x}{2}$ to show that

$$\int_0^{\frac{\pi}{2}} \frac{1}{2 + \sin x} dx = \frac{\pi}{3\sqrt{3}}. \quad [8]$$

3. Show that the length of the arc joining the points $(2a, 2a)$ and $(4a, 2\sqrt{3}a)$ on the curve with equation $y^2 = 4a(x - a)$ is given by the integral

$$\int_{2a}^{4a} \sqrt{\frac{x}{x-a}} dx.$$

Hence evaluate this length using the substitution $x = a \cosh^2 u$. Give your answer in the form ka where k should be evaluated correct to three significant figures. [14]

4. The function f is defined by

$$f(x) = e^x \cos x.$$

- (a) Show that

$$f''(x) = -2e^x \sin x. \quad [2]$$

- (b) Determine the Maclaurin series for $f(x)$ as far as the term in x^4 . [5]

- (c) By differentiating your series, determine the Maclaurin series for $e^x \sin x$ as far as the term in x^3 . [4]

5. Consider the equation $x \sin x - 0.5 = 0$.

(a) Show that this equation has a root α between 0.6 and 0.8. [2]

(b) (i) Show that the Newton-Raphson iteration to find the value of α can be written in the form

$$x_{n+1} = \frac{x_n^2 \cos x_n + 0.5}{x_n \cos x_n + \sin x_n}.$$

(ii) Starting with $x_0 = 0.7$, find the value of α correct to five decimal places. [6]

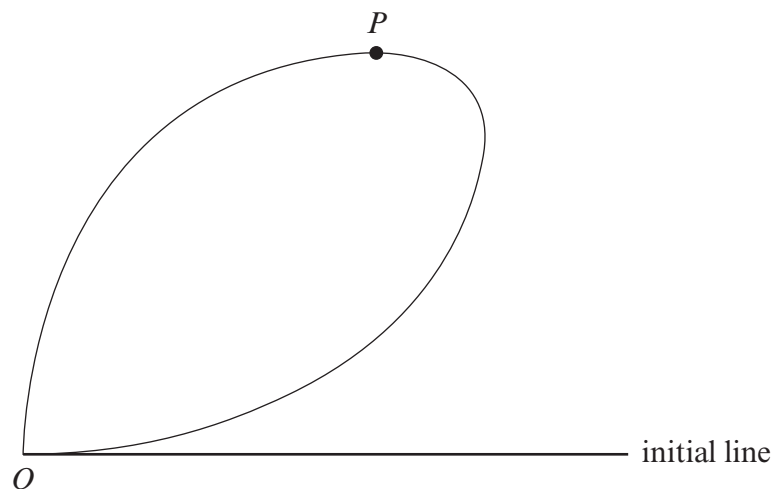
(c) A rearrangement of the equation leads to the iterative sequence

$$x_{n+1} = f(x_n) \text{ where } f(x) = \sin^{-1}\left(\frac{0.5}{x}\right).$$

(i) Obtain an expression for $f'(x)$.

(ii) **Hence** determine whether or not the sequence can be used to find the value of α . [4]

6.



The above diagram shows a sketch of the curve C with polar equation

$$r = \sin 2\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

The point P , marked on the diagram, is the point at which the tangent to C is parallel to the initial line.

(a) Determine the area of the region enclosed by C . [5]

(b) Find the polar coordinates of the point P . [7]

TURN OVER

7. The integral I_n is defined, for $n \geq 0$, by

$$I_n = \int_0^a \tanh^n x \, dx,$$

where $a = \tanh^{-1} 0.5$.

(a) Show that, for $n \geq 2$,

$$I_n = I_{n-2} - \frac{0.5^{n-1}}{n-1}. \quad [5]$$

(b) Giving your answers correct to three significant figures, evaluate

(i) I_0 ,

(ii) I_4 . [5]