GCE AS/A level

979/01

MATHEMATICS FP3

Further Pure Mathematics

A.M. THURSDAY, 24 June 2010

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined for $x \geqslant 0$ by

$$f(x) = \sinh 2x - 14\sinh x + 8x.$$

(a) Show that

$$f'(x) = 2(2\cosh^2 x - 7\cosh x + 3).$$ [2]

(b) Show that there is one stationary point on the graph of f. Find its x-coordinate, giving your answer correct to two decimal places. [5]

(c) Obtain an expression for $f''(x)$ and **hence** classify the stationary point as either a maximum or a minimum. [3]

2. Use the substitution $x = \sinh u$ to evaluate the integral

$$\int_0^3 \frac{x^2}{\sqrt{x^2+1}}\,\mathrm{d}x.$$

Give your answer correct to two decimal places. [7]

3. *(a)* Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^x) = x^x(1+\ln x).$$

[2]

(b) The equation $x^x - 2 = 0$ has one positive root α.

(i) Starting with an initial approximation of 1·5, use the Newton-Raphson formula once to find a better approximation to α. Give your answer correct to two decimal places.

(ii) Prove that the answer to *(b)*(i) is the value of α correct to two decimal places. [5]

(c) (i) The equation given in *(b)* can be rearranged in the form

$$x = \mathrm{e}^{\frac{\ln 2}{x}}.$$

By evaluating an appropriate derivative, show that the iterative process based on this rearrangement is convergent.

(ii) Starting with an initial approximation equal to your answer to *(b)*(i), use this iterative process to find the value of α correct to four decimal places. [5]

4. Find the length of the arc joining the points $(0,0)$ and $(1,1)$ on the curve having equation

$$y^2 = x^3.$$ [7]

5. Consider the function

$$f(x) = \ln(1 + \sinh x).$$

(a) (i) Find the first three non-zero terms of the Maclaurin series for $f(x)$.

(ii) Explain how your result enables you to conclude that f is neither an odd function nor an even function. [10]

(b) The equation

$$\ln(1 + \sinh x) = 10x^2$$

has a small positive root. Use your result in *(a)*(i) to find its approximate value, giving your answer correct to two significant figures. [3]

6. The curve C has polar equation

$$r = \cos\theta + 2\sin\theta \quad (0 \leqslant \theta \leqslant \frac{\pi}{2})$$

(a) Find the polar coordinates of the point on C at which the tangent is perpendicular to the initial line. [7]

(b) Determine the area of the region enclosed between C, the initial line and the line $\theta = \frac{\pi}{2}$. [6]

7. The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_0^{\frac{\pi}{2}} \cos^n x \, dx.$$

(a) Show that, for $n \geqslant 2$,

$$I_n = \left(\frac{n-1}{n}\right) I_{n-2} \; .$$

[5]

(b) Hence evaluate

(i) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^4 x \, dx,$

(ii) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^5 x \sin^2 x \, dx.$ [8]