WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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979/01

MATHEMATICS FP3

Further Pure Mathematics

A.M. MONDAY, 19 June 2006

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Using the exponential definitions of sinh x and cosh x, show that

$$\cosh 2x = 2\sinh^2 x + 1. \qquad [3]$$

(b) Solve the equation

$$\cosh 2x = 3\sinh x$$

giving your answers correct to three significant figures. [6]

2. Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}x}{(1 + 3\cos x)}.$$

3. The function f is defined by

$$f(x) = \ln \sec x.$$

(a) Find the Maclaurin series of $f(x)$ up to and including the term in x^4. [9]

(b) The equation

$$\ln \sec x = 1 - 10x^2$$

has a small positive root α. Use your series to find an approximation to α, giving your answer correct to four decimal places. [4]

4. A curve has parametric equations

$$x = \theta + \sin\theta,\ y = 1 + \cos\theta\ (0 \leqslant \theta \leqslant \pi).$$

(a) Show that

$$\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2 = 4\cos^2\left(\frac{\theta}{2}\right).$$

[5]

(b) Find the total length of the curve. [4]

(c) The curve is rotated through 360° about the x-axis. Find the curved surface area of the solid of revolution generated. [8]

5. The integral I_n is defined for $n \geqslant 0$, by

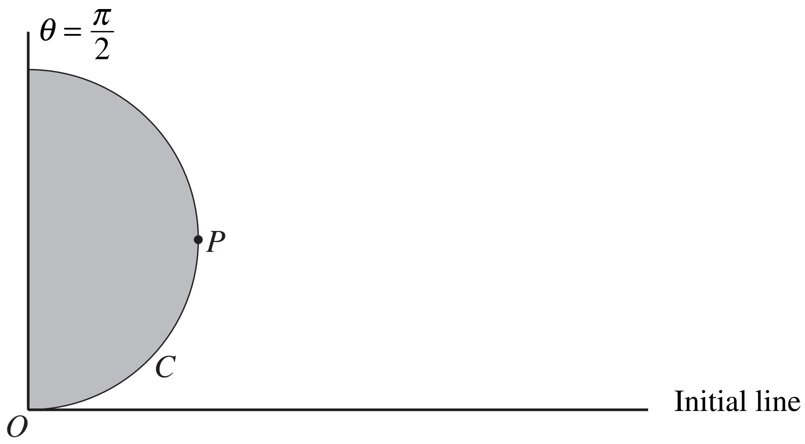
$$I_n = \int_0^{\pi} \theta^n \sin\theta \, d\theta.$$

(a) Show that, for $n \geqslant 2$,

$$I_n = \pi^n - n(n-1)I_{n-2}.$$ [8]

(b) Evaluate I_4, giving your answer in terms of powers of π. [5]

6.



The diagram shows the initial line, the line $\theta = \frac{\pi}{2}$ and the curve C with equation

$$r = \sinh\theta \ (0 \leqslant \theta \leqslant \tfrac{\pi}{2}).$$

(a) Find the area of the shaded region. [5]

(b) The tangent to C at the point P is perpendicular to the initial line.

(i) Show that the θ coordinate of P satisfies the equation

$$\tanh\theta = \cot\theta.$$

(ii) Starting with the initial approximation $\theta_0 = 1$ to the root of this equation, use the Newton-Raphson method **once** to find a better approximation θ_1. Give your answer correct to four significant figures. [10]