



**GCE AS/A level**

0976/01



S15-0976-01

**MATHEMATICS – C4**  
**Pure Mathematics**

A.M. FRIDAY, 12 June 2015

1 hour 30 minutes

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that  $f(x) = \frac{2x^2 + 5x + 25}{(x + 3)^2(x - 1)}$ ,

(a) express  $f(x)$  in terms of partial fractions, [4]

(b) evaluate

$$\int_3^{10} f(x) dx,$$

giving your answer correct to two decimal places. [3]

2. The curve  $C$  has equation

$$x^4 + 3x^2y - 2y^2 = 34.$$

(a) Show that  $\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$ . [3]

(b) Find the coordinates of each of the points on  $C$  where the tangent is parallel to the  $y$ -axis. [4]

3. (a) Find all values of  $x$  in the range  $0^\circ \leq x \leq 180^\circ$  satisfying

$$\tan(x + 45^\circ) = 8 \tan x. \quad [5]$$

(b) (i) Express  $\sqrt{13} \sin \theta - 6 \cos \theta$  in the form  $R \sin(\theta - \alpha)$ , where  $R$  and  $\alpha$  are constants with  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ .

(ii) Find all values of  $\theta$  in the range  $0^\circ \leq \theta \leq 360^\circ$  satisfying

$$\sqrt{13} \sin \theta - 6 \cos \theta = -4. \quad [6]$$

4. The line  $L$  has equation  $y = mx$ , where  $m > 0$ . The region  $R$  is bounded by  $L$ , the  $x$ -axis and the line  $x = a$ , where  $a > 0$ .

(a) Using integration, find an expression, in terms of  $a$  and  $m$ , for the volume  $V$  generated when  $R$  is rotated through four right angles about the  $x$ -axis. [3]

(b) The point with coordinates  $(a, b)$  lies on  $L$ .

(i) Rewrite the expression for the volume  $V$  found in part (a) in terms of  $a$  and  $b$ .

(ii) Give a geometrical interpretation of your answer. [3]

5. Expand  $\left(1 + \frac{x}{8}\right)^{-\frac{1}{2}}$  in ascending powers of  $x$  up to and including the term in  $x^2$ .

State the range of values of  $x$  for which your expansion is valid.

Hence, by writing  $x = 1$  in your expansion, find an approximate value for  $\sqrt{2}$  in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are integers whose values are to be found. [5]

6. The parametric equations of the curve  $C$  are  $x = at^2$ ,  $y = 2at$ , where  $a$  is a positive constant. The points  $P$  and  $Q$  lie on  $C$  and have parameters  $p$  and  $q$  respectively.

(a) Simplifying your answer in each case, find

(i) the gradient of the tangent to  $C$  at the point  $P$ ,

(ii) the equation of the tangent to  $C$  at the point  $P$ . [4]

(b) (i) Find an expression, in its simplest form, for the gradient of the line  $PQ$ .

(ii) Explain how you could use the answer of (b)(i) to derive the gradient of the tangent to  $C$  at the point  $P$ . [4]

7. (a) Use the substitution  $u = 12 - x^3$  to evaluate

$$\int_0^2 \frac{x^2}{(12 - x^3)^2} dx. \quad [4]$$

(b) (i) Find  $\int x \cos 2x dx$ .

(ii) Use the result of (b)(i) to find

$$\int x \sin^2 x dx. \quad [7]$$

8. The position vectors of the points  $A$  and  $B$  are given by

$$\begin{aligned} \mathbf{a} &= 5\mathbf{i} - \mathbf{j} - \mathbf{k}, \\ \mathbf{b} &= 4\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}, \end{aligned}$$

respectively.

(a) (i) Write down the vector  $\mathbf{AB}$ .

(ii) Find the vector equation of the line  $AB$ . [3]

(b) The vector equation of the line  $L$  is given by

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Show that the lines  $AB$  and  $L$  intersect and find the position vector of the point of intersection. [6]

**TURN OVER**

9. A bookseller values a rare book at £ $A$  on August 1st 2010. The value, £ $P$ , of the book  $t$  years after this date may be modelled as a continuous variable. The rate of increase of  $P$  may be assumed to be directly proportional to  $P^2$ .

(a) Write down a differential equation satisfied by  $P$ . [1]

(b) Show that

$$\frac{1}{k} \left( \frac{P - A}{PA} \right) = t,$$

where  $k$  is a constant. [4]

(c) The value of the book is £800 on August 1st 2013 and £900 on August 1st 2014. Find the value of  $A$ . [3]

10. Prove by contradiction the following proposition.

If  $a$  and  $b$  are odd integers such that 4 is a factor of  $a - b$ , then 4 is **not** a factor of  $a + b$ .

The first lines of the proof are given below.

*Assume that 4 is a factor of  $a + b$ .*

*Then there exists an integer  $c$  such that  $a + b = 4c$ .* [3]

**END OF PAPER**