



GCE AS/A level

0975/01

MATHEMATICS C3

Pure Mathematics

P.M. FRIDAY, 20 January 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^{\frac{\pi}{3}} \cos^2 x \, dx.$$

Show your working and give your answer correct to four decimal places. [4]

- (b) Use your answer to part (a) to deduce an approximate value for the integral

$$\int_0^{\frac{\pi}{3}} \sin^2 x \, dx. \quad [2]$$

2. (a) Show, by counter-example, that the statement

$$\sin(\theta + \phi) \equiv \sin \theta + \sin \phi$$

is false. [2]

- (b) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$\sec^2 \theta + 8 = 4 \tan^2 \theta + 5 \sec \theta. \quad [6]$$

3. (a) A function is defined parametrically by

$$x = 3t^2, \quad y = t^6 - 4t^3.$$

(i) Find $\frac{dy}{dx}$ in terms of t .

(ii) Given that $\frac{dy}{dx} = \frac{7}{2}$, show that $2t^4 - 4t - 7 = 0$. [5]

- (b) Show that the equation

$$2t^4 - 4t - 7 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$t_{n+1} = \left(\frac{4t_n + 7}{2} \right)^{\frac{1}{4}}$$

with $t_0 = 1.6$ can be used to find α . Find and record the values of t_1, t_2, t_3, t_4 . Write down the value of t_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

4. Given that $x^2y^2 + x^4 + 6 = 2y^3 + 2x$, find the value of $\frac{dy}{dx}$ at the point (2, 3). [4]

5. Differentiate each of the following with respect to x , simplifying your answer wherever possible.

(a) $\tan^{-1}4x$ (b) e^{-x^3} [2], [2]

(c) $x^5 \ln x$ (d) $\frac{3-2x^2}{5-4x^2}$ [3], [3]

6. (a) Find each of the following, simplifying your answer wherever possible.

(i) $\int \sin\left(\frac{x}{4}\right) dx$, (ii) $\int e^{\frac{2x}{3}} dx$, (iii) $\int \frac{7}{8x-2} dx$. [6]

(b) Evaluate $\int_1^9 \frac{3}{\sqrt{5x+4}} dx$. [4]

7. Solve the following.

(a) $|4x - 5| \geq 3$, [3]

(b) $(3|x| + 1)^{\frac{1}{3}} = 4$. [2]

8. The function f is defined by $f(x) = e^x$.

(a) Sketch the graph of $y = f(x)$. Write down the coordinates of the point of intersection of the graph with the y -axis. [2]

(b) Using a separate set of axes,

(i) sketch the graph of $y = f(3x) - 4$, indicating the behaviour of your graph for large negative values of x ,

(ii) write down the coordinates of the point of intersection of the graph with the y -axis,

(iii) find the x -coordinate of the point of intersection of the graph with the x -axis. Give your answer correct to three decimal places. [4]

9. The function f has domain $[6, \infty)$ and is defined by

$$f(x) = 3 - \frac{1}{\sqrt{x-2}}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1} . [2]

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10. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = 3x + k,$$

where k is a constant.

(a) Write down, in terms of k , the range of f . [1]

The function g has domain $[-2, \infty)$ and is defined by

$$g(x) = x^2 - 6.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k , for $gf(x)$.

(ii) Given that $gf(2) = 3$, find the value of k . [5]