



GCE AS/A Level – **LEGACY**

0975/01



MATHEMATICS – C3
Pure Mathematics

WEDNESDAY, 5 JUNE 2019 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \cot^2 x \, dx.$$

Show your working and give your answer correct to two decimal places. [4]

- (b) Use your answer to part (a) to deduce an approximate value for the integral

$$\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \operatorname{cosec}^2 x \, dx. \quad [2]$$

2. Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$9 \tan^2 \theta + 5 \sec \theta = 6 \sec^2 \theta + 3. \quad [6]$$

3. (a) Given that

$$2x^4 - x^2 \sin y + y^5 - 4x + 17 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y . [4]

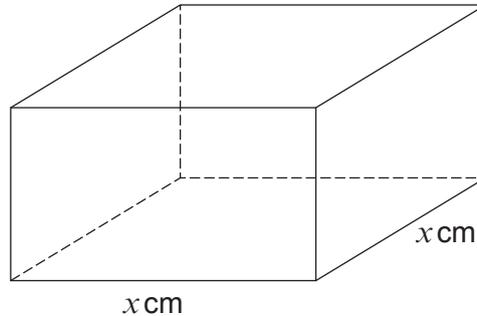
- (b) Given that $x = 4e^{3t} - 3e^{-2t}$, $y = 2e^t - 7e^{3t}$,

(i) find an expression for $\frac{dy}{dx}$ in terms of t ,

(ii) give a mathematical reason to explain why there is no value for t such that

$$\frac{dy}{dx} = -2. \quad [6]$$

4. Eleri owns a jewellery box which is in the form of a **closed** cuboid with a square base. The volume of the box is 9600 cm^3 and the total surface area of the six faces of the box is 2800 cm^2 . The length of each of the edges of the base is denoted by $x \text{ cm}$.



(a) Show that $x = \frac{1400x - 19200}{x^2}$. [3]

- (b) Eleri estimates the value of x as being between 24 and 26 but wants to find a more accurate value for x . She therefore decides to use the recurrence relation

$$x_{n+1} = \frac{1400x_n - 19200}{x_n^2}$$

to try to find x . Use this recurrence relation with $x_0 = 24$ to find and record the values of x_1, x_2, x_3, x_4 . Write down the value of x_4 correct to two decimal places and prove that this is in fact the value of x correct to two decimal places. [5]

5. Differentiate each of the following with respect to x , simplifying your answer wherever possible.

(a) $(7 - 6x)^{\frac{1}{3}}$ (b) $\ln(\sin x)$ [2], [3]

(c) $x^4 \cos 5x$ (d) $\frac{e^{9x}}{(3x+2)^6}$ [3], [4]

6. (a) Find each of the following integrals, simplifying your answer wherever possible.

(i) $\int \frac{7}{8x+1} dx$ (ii) $\int \frac{10}{(3x-2)^5} dx$ (iii) $\int \sec^2 3x dx$ [6]

- (b) Given that $\frac{\pi}{3} < a < 3$ and that

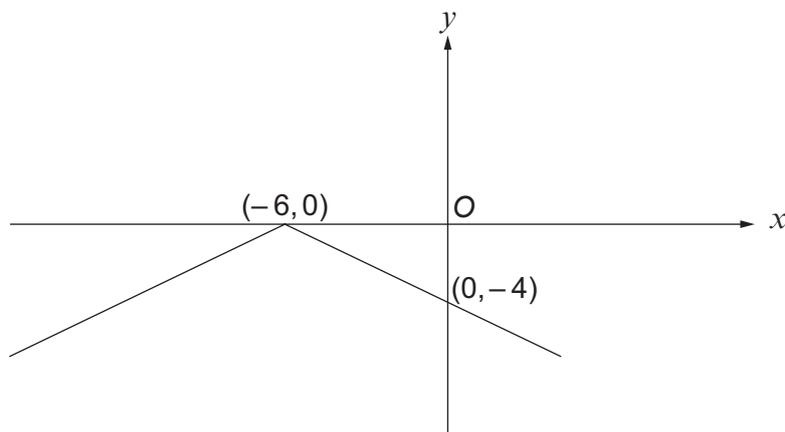
$$\int_{\frac{\pi}{3}}^a \cos \frac{1}{2}x dx = 0.92,$$

find the value of the constant a . Give your answer correct to three decimal places. [5]

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7. (a) Solve the inequality $|3x - 5| > 7$. [3]

(b) The diagram below shows a sketch of the graph of $y = a|x + b|$, where a, b are constants. The graph meets the x -axis at the point $(-6, 0)$ and the y -axis at the point $(0, -4)$.



Find the value of a and the value of b . [2]

(c) Show, by counter-example, that the following statement is false.

'If the positive integers m and n are such that m is a factor of n^2 , then m must be a factor of n .'

[2]

8. The function f has domain $(-\infty, 1]$ and is defined by

$$f(x) = \frac{4x + 3}{7 - 5x}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1} . [2]

(c) (i) Find the exact value of $f^{-1}(0.5)$.

(ii) By carrying out an appropriate calculation involving f , verify that your answer to (c) (i) is correct. [3]

9. The function g has domain $(-\infty, \infty)$ and is defined by

$$g(x) = \sqrt{4x^2 + 5}.$$

Solve the equation

$$gg(x) = 2x + 7. \quad [6]$$

END OF PAPER