



GCE AS/A level

975/01

**MATHEMATICS C3
PURE MATHEMATICS**

A.M. MONDAY, 1 June 2009

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for

$$\int_1^{1.8} \sqrt{8+x^3} \, dx.$$

Show your working and give your answer correct to four decimal places. [4]

2. (a) Show, by counter-example, that the statement

$$\cos\theta + \cos 3\theta \equiv 2 \cos 2\theta \cos 4\theta$$

is false. [2]

- (b) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$\cot^2\theta - 9 = \operatorname{cosec}\theta - \operatorname{cosec}^2\theta. [6]$$

3. (a) Given that

$$x^3 + y^2 + x \tan 2y = 8,$$

find $\frac{dy}{dx}$ in terms of x and y . [4]

- (b) Given that $x = 3t + t^2$, $y = \frac{1+4t}{3+2t}$, find

(i) $\frac{dy}{dt}$,

(ii) $\frac{dy}{dx}$, simplifying your answer as much as possible. [5]

4. (a) Show that $f(x) = (2x-3)e^{2x} - 4x + 5$ has a stationary value when x satisfies

$$(x-1)e^{2x} - 1 = 0. [6]$$

- (b) Show that the equation

$$(x-1)e^{2x} - 1 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$x_{n+1} = 1 + e^{-2x_n}$$

with $x_0 = 1.1$ may be used to find α . Find and record the values of x_1, x_2, x_3 . Write down the value of x_3 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

5. Differentiate each of the following with respect to x , simplifying your answers where possible.

(a) $\ln(3 + 2x^2)$ (b) $x^2 \tan^{-1} x$ (c) $(5 + 7x^2)^{10}$ [2], [2], [3]

6. Solve the following.

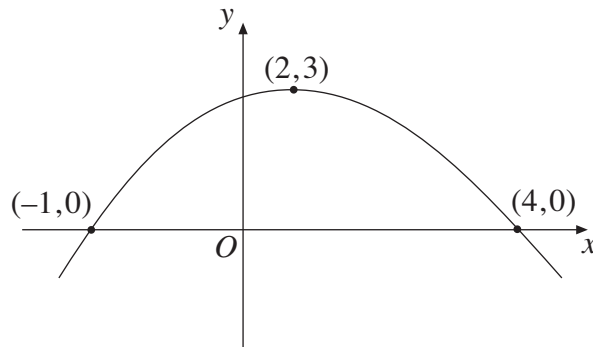
(a) $|9x - 7| \leq 3$ [3]

(b) $\sqrt{5|x| + 1} = 3$ [2]

7. (a) Find (i) $\int \sin 5x \, dx$, (ii) $\int \frac{3}{(2x + 7)^3} \, dx$. [4]

(b) Evaluate $\int_0^3 \frac{2}{5x + 3} \, dx$, giving your answer correct to three decimal places. [4]

8.



The diagram shows a sketch of the graph of $y = f(x)$. The graph has its highest point at $(2, 3)$ and intersects the x -axis at the points $(-1, 0)$ and $(4, 0)$. Sketch the graph of $y = 3f(x - 2)$, indicating the coordinates of three points on the graph. [3]

TURN OVER

9. The function f has domain $(-\infty, \infty)$ and is defined by

$$f(x) = 3e^{2x}.$$

The function g has domain $(0, \infty)$ and is defined by

$$g(x) = \ln 4x.$$

- (a) Write down the domain and range of fg . [2]

- (b) Solve the equation

$$fg(x) = 12. \quad [5]$$

10. The function f has domain $(0, \infty)$ and is defined by

$$f(x) = 1 - \frac{2}{3x^2 + 2}.$$

- (a) Show that $f'(x)$ is always positive. [4]

- (b) Write down the range of f . [1]

- (c) Find an expression for $f^{-1}(x)$. State the domain and range of f^{-1} . [6]