The Mathematics Department

11

Developing Probability

Name:

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Relative Frequency

Revision F

Exercise 1 (Revision of previous work on probability)

(a) Use “no chance”; “low chance”; “even chance”; “good chance” or “certain” to describe the probability of the following events.

(i) You land on “tails” when throwing a fair coin.
(ii) The next person you meet writes with their left hand.
(iii) You obtain a number less than 5 when rolling a normal fair die.
(iv) St. David’s Day will be on March 1st next year.
(v) You will obtain 101% in your end of year mathematics examination.

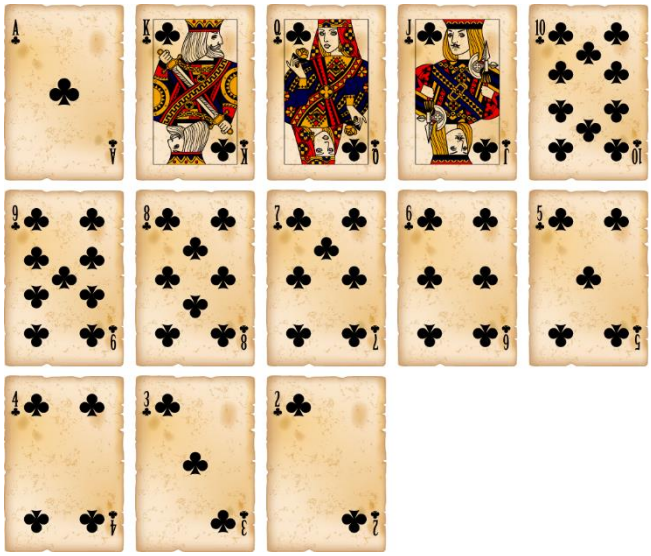
(b) Draw a probability scale. Mark the points i, ii, iii, iv in order to show how probable, in your opinion, the following events are.

(i) A man will be driving the next car you see.
(ii) It will snow during the day tomorrow.
(iii) A story about politics will feature on the news tonight.
(iv) Germany will win the next football world cup.

(c) Answer with a number between 0 and 1: what is the probability that someone will walk to the top of Snowdon tomorrow?

(d) Answer the following questions using fractions.
What is the probability of obtaining…

(i) The number 4 when rolling a normal fair die?
(ii) “Heads” when throwing a fair coin?
(iii) A square number when spinning a spinner showing the numbers 1 to 8?



(e) Rheinallt shuffles the 52 cards in a standard deck of playing cards and chooses one card randomly from the deck. What is the probability that the chosen card is:

(i) a diamond?
(ii) 6?
(iii) a face card?
(iv) a spade showing an even number?
(v) a red card less than 5?

(f) The probability that Meira goes to the shop to buy a loaf of bread tomorrow is 0.4. What is the probability that Meira does not go to the shop to buy a loaf of bread tomorrow?

(g) A red die and a blue die are labelled from 1 to 6.
Gethin rolls both dice and adds the scores obtained.

(i) Use a sample space diagram to list all the possible outcomes.
(ii) What is the probability that the sum of both numbers is 12?
(iii) What is the probability that the sum of both numbers is a one-digit number?

(h) The probability that Ellie goes on a training run on any day is 0.7.
There are 30 days in April. On how many days can you expect Ellie to go running?

Relative Frequency

The **frequency** of an event refers to how many times the event has occurred during a number of trials.

The **relative frequency** of an event compares the frequency to the number of trials.

$$\text{Relative frequency of an event} = \frac{\text{How many times the event has occurred}}{\text{Number of trials}}$$

It is possible to use relative frequency to **estimate the probability of an event**.

Exercise 2

Applying

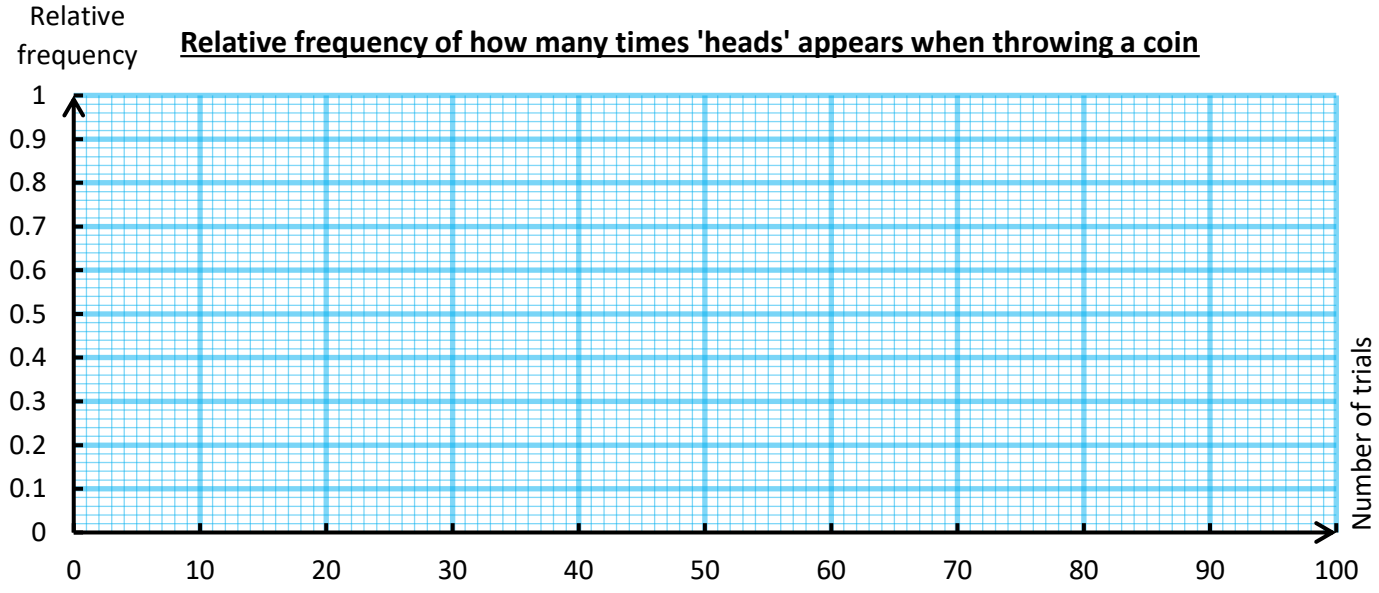
I

You will need a coin for this exercise.

(a) Throw the coin 100 times, recording in the following table, after each set of 10 throws, how many times the coin has landed showing 'heads'.

Total number of trials	Number of heads in these 10 throws	Total number of heads so far	Relative frequency of the number of heads, as a fraction	Relative frequency of the number of heads, as a decimal
10			$\frac{}{10}$	
20			$\frac{}{20}$	
30			$\frac{}{30}$	
40				
50				
60				
70				
80				
90				
100				

(b) Plot, on the following graph paper, a line graph showing what happens to the relative frequency as the number of trials increases.



(c) If a great many more trials were held, how would you expect the graph to change?

Exercise 3 (Buffon's Needle Experiment)

I

For this exercise, you will need a plastic straw of length 3 cm, and lined paper where the lines are separated by 6 cm.

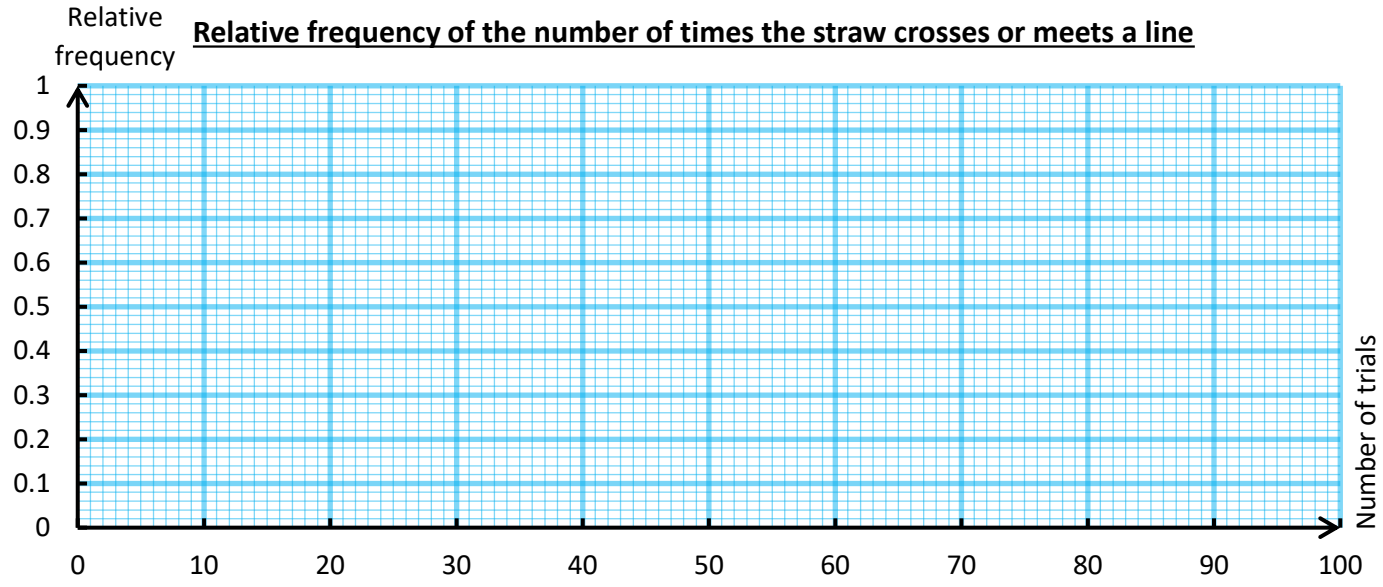
In 1777, a Frenchman called Georges-Louis Leclerc (Comte de Buffon) devised an experiment that can be used to estimate the value of π.

In each trial, a straw is dropped on a piece of paper. The straw should be dropped above the centre of the paper, from arm height. You should record whether or not the straw crosses (or meets) one of the lines.

(a) Perform the experiment 100 times. Record, after each set of 10 trials, how many times the straw crosses (or meets) one of the lines.

Total number of trials	Number of times the straw has crossed a line in these 10 trials	Total number of crossings so far	Relative frequency of the number of crossings, as a fraction	Relative frequency of the number of crossings, as a decimal
10			$\frac{\quad}{10}$	
20			$\frac{\quad}{20}$	
30			$\frac{\quad}{30}$	
40				
50				
60				
70				
80				
90				
100				

(b) Plot, on the graph paper below, a line graph showing what happens to the relative frequency as the number of trials increases.



(c) Calculate the **reciprocal** of your final relative frequency. How close is this value to π?

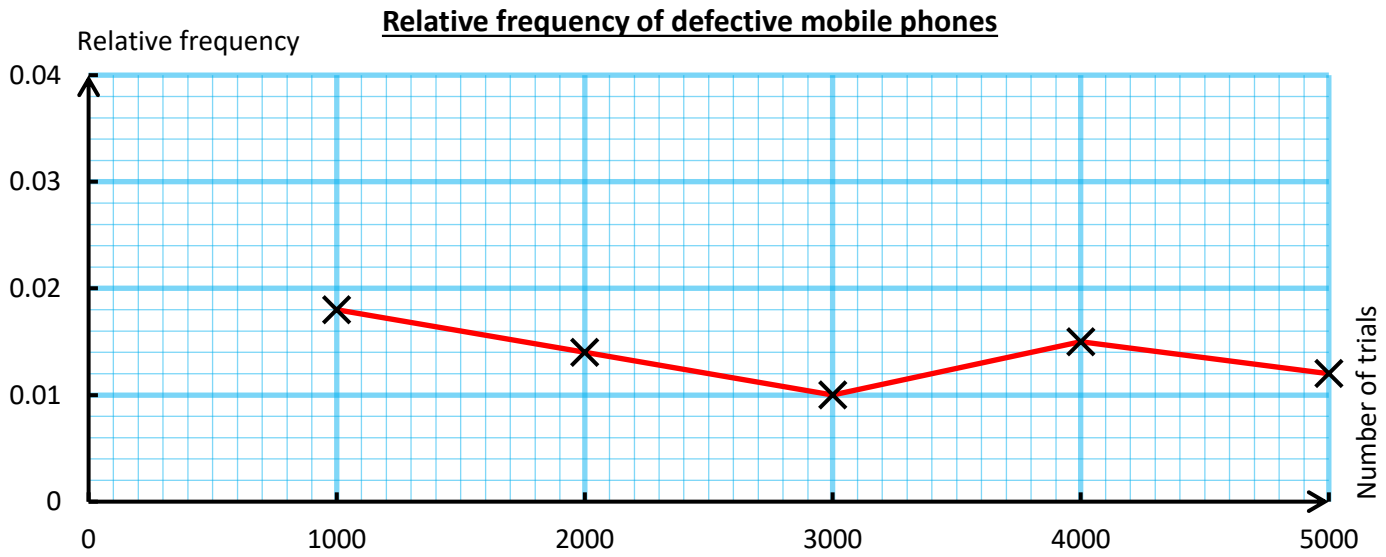
In any experiment,

The more trials are held, the better the relative frequency is as an estimate of the probability.

Exercise 4 I

A factory produces mobile phones. The manager holds a survey to investigate the probability that the factory produces a defective mobile phone.

The relative frequency of defective mobile phones is recorded after testing a total of 1,000, 2,000, 3,000, 4,000 and 5,000 mobile phones. The results are shown in the following graph.



(a) How many of the first 2,000 mobile phones tested were defective?

(b) Write the best estimate for the probability that a randomly selected mobile phone is defective.
You must give a reason for your answer.

Exercise 5 F

(a) Fred throws a die 200 times and records how many times each score occurs.

Score	1	2	3	4	5	6
Frequency	29	34	35	32	34	36

(i) Calculate the relative frequency of each score.
(ii) In your opinion, is Fred's die fair? Give a reason for your answer.

(b) Rhys recorded the results of his favourite football team.

Win	Draw	Lose
32	11	7

(i) Calculate the relative frequency of each of the three possible results.
(ii) Are your answers to part (i) a good estimate for the probability of the result of the next game?
Explain your answer.

(c) A petrol station owner noticed that 287 customers out of a total of 340 customers spent more than £30 filling up their cars. Use these figures to estimate the probability that the next customer spends

(i) more than £30, (ii) £30 or less.

(d) In a survey, 600 people were asked about their favourite crisp flavour.
The following table shows the results.

Flavour	Frequency
Ready Salted	166
Salt & Vinegar	130
Cheese & Onion	228
Other	76

(i) Calculate the relative frequency of each flavour. Give your answers correct to 2 decimal places.
(ii) Explain why it is reasonable to use these figures to estimate the probability of the favourite crisp flavour of the next person to be asked.

(e) A card was picked from a standard pack of cards, and its suit was noted. The card was put back and the pack was shuffled. This was repeated 250 times. The results are shown in the following table.

Suit	Frequency
Spades	52
Hearts	67
Diamonds	61
Clubs	70

Find the relative frequency of
(i) Spades (ii) Hearts (iii) Diamonds (iv) Clubs.
(v) What is the sum of all the relative frequencies?

(f) The following table shows the number of pictures on each page of a newspaper.

Number of pictures	Tally Marks	Frequency
0	𝍸 𝍸	10
1	𝍸 ||	7
2	𝍸 ||||	9
3	𝍸 |	6
4	|||| |	6
5	𝍸	4
6	𝍸 |	6

(i) How many pages does the newspaper have?
(ii) Find the relative frequency of one picture appearing on a page of the newspaper.

Evaluation

Key Words	Further Questions	What went well?	To reach my target grade I will…
			Grade ❑ Target ❑

Combined Events

Combined events involve two or more events that occur together.
For example,

- Throwing a penny and rolling a die at the same time;
- Rolling a die and then rolling it again;
- Spinning a spinner that shows the numbers 1-8, and then randomly choosing a card from a standard deck of playing cards.

Independent Events (Intermediate Tier)

Two events are **independent** if the result of the first event does *not* affect the probability of the second event. For example, when throwing the same die twice, the fact that the die has landed on 6 the first time does not affect the probability of obtaining a 6 the second time.

For independent events A and B,

$$P(A \cap B) = P(A) \times P(B)$$

The **multiplication** rule for independent events

This means the probability of A **and** B occurring is the product of A's probability and B's probability.

For example, the probability of obtaining heads when throwing a coin and 4 when rolling a die is $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.

Dependent Events (Higher Tier)

Two events are **dependent** if the result of the first event *does* affect the probability of the second event. For example, when choosing two cards from a standard deck of playing cards, **without replacement**, the first card chosen affects the probability of the second card chosen. If the first card is a king, then the probability of the second card being a king is $\frac{3}{51}$, not $\frac{4}{52}$ as for the first card.

For dependent events A and B,

$$P(A \cap B) = P(A) \times P(B|A)$$

The **multiplication** rule for dependent events

This means the probability of A **and** B occurring is the product of A's probability with the probability of B occurring, given that A has already occurred. (The probability $P(B|A)$ is a **conditional** probability.)

For example, the probability of choosing two kings from a standard deck of playing cards is $\frac{4}{52} \times \frac{3}{51} = \frac{12}{2652}$, or $\frac{1}{221}$.

Mutually Exclusive Events (Intermediate Tier)

Two events are **mutually exclusive** if they cannot happen at the same time. For example, when rolling a normal fair die, the events "landing on an odd number" and "landing on 6" are mutually exclusive, because 6 is not an odd number. On the other hand, the events "landing on an odd number" and "landing on a prime number" are not mutually exclusive, because the numbers 3 and 5 are both odd and prime.

For mutually exclusive events A and B,

$$P(A \cup B) = P(A) + P(B)$$

The **addition** rule for mutually exclusive events

This means the probability of A **or** B occurring is the sum of A's probability and B's probability.

For example, the probability of obtaining an odd number or a 6 when rolling a normal fair die is $\frac{3}{6} + \frac{1}{6} = \frac{4}{6}$, or $\frac{2}{3}$.

Exercise 6

Skill

I

For the following pairs of events, decide whether the events are independent or dependent.

(a) Obtaining 'heads' when throwing a coin and obtaining '6' when rolling a normal fair die.

(b) Choosing two queens when choosing two cards from a standard deck of playing cards, without replacement.

(c) Choosing two queens when choosing a card from a standard deck of playing cards, returning the card to the deck, and then choosing another card.

(d) Choosing two red balls when one ball is chosen from a bag containing 4 red balls and 5 blue balls, and the other is chosen from a bag containing 5 red balls and 4 blue balls.

(e) Choosing two red balls from a bag containing 4 red balls and 5 blue balls, without replacement.

Exercise 7

I

For the following pairs of events, decide whether the events are mutually exclusive or not.

(a) Obtaining 'heads' when throwing a coin and obtaining '6' when rolling a normal fair die.

(b) When rolling a normal fair die, obtaining

(i) a number less than 3 and a number greater than 3.
(ii) an even number and a number greater than 4.
(iii) an odd number and a square number.
(iv) an even number and a cube number.
(v) an even number and a prime number.

(c) Obtaining 'heads' when throwing a coin and 'heads' when throwing another coin.

(d) On a spinner showing the numbers 1-10, landing on a multiple of 3 and landing on a multiple of 4.

(e) On a spinner showing the numbers 1-12, landing on a multiple of 3 and landing on a multiple of 4.

Example

Christine has a normal fair die. She rolls the die twice.
Calculate the probability that the die lands on 3 each time.

Answer: The result of the first roll does not affect the second roll, so the events are independent.
We can therefore use the multiplication rule for independent events.

$$\begin{aligned} P(3 \text{ the first time}, 3 \text{ the second time}) &= P(3 \text{ the first time}) \times P(3 \text{ the second time}) \\ &= \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{36} \end{aligned}$$

Exercise 8

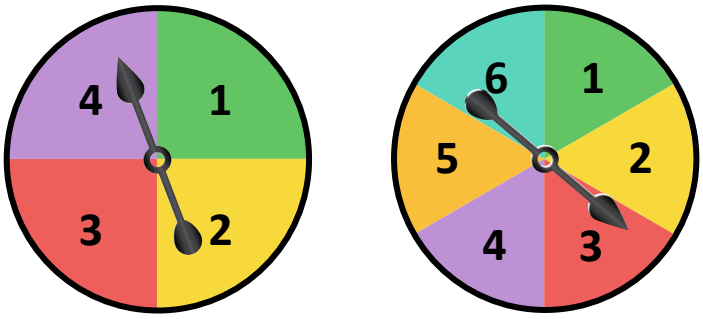
I

(a) David has a normal fair die. He rolls it twice.
Calculate the probability that he rolls 5 the first time and 1 the second time.

(b) Fiona has a fair coin. She throws it 3 times.
Calculate the probability that she obtains 3 'tails'.

(c) Rachel has a normal fair die and a fair coin. She rolls the die and throws the coin.
Calculate the probability that the die lands on 4 and the coin lands on 'tails'.

(d) A game is played where the two spinners below are spun at the same time.



(i) What is the probability that the spinner on the left stops at 4 and the spinner on the right stops at 3?
(ii) What is the probability that the spinner on the left stops at an even number and the spinner on the right stops at an odd number?
(iii) What is the probability that both spinners stop at the same number?

(e) A bag contains 12 counters. 3 are red, 4 are blue and the rest are green. Another bag contains 15 counters. 7 are red, 2 are blue and rest are green. What is the probability of choosing

(i) A red counter from the first bag and a blue counter from the second bag?
(ii) A blue counter from the first bag and a red counter from the second bag?
(iii) Two green counters?

H

Exercise 9

(a) A bag contains 8 counters. 3 are red and 5 are blue. 2 counters are chosen randonly from the bag, without replacement. What is the probability of choosing

(i) Two red counters?
(ii) A red counter first then a blue counter?
(iii) A blue counter first then a red counter?
(iv) Two blue counters?

(b) Tom shuffles a standard pack of playing cards before choosing, at random, two cards from the pack without replacement. What is the probability that Tom chooses

(i) The king of hearts first and then the queen of diamonds?
(ii) Two hearts?
(iii) Two cards showing 7?
(iv) A red card first and then a black card?

(c) A class in a school has 15 girls and 12 boys. Two names are chosen at random from the register in order to represent the class in a survey. What is the probability of choosing two girls?

(d) The probability that James watches television tonight is 0.6. If James watches television tonight, the probability that he reads a book tonight is 0.2. If James does not watch television tonight, the probability that he reads a book tonight is 0.7. What is the probability that James, tonight,

(i) Watches television and reads a book?
(ii) Does not watch television and reads a book?
(iii) Watches television and does not read a book?
(iv) Does not watch television and does not read a book?

(e) An office has 20 workers. 7 of the workers wear spectacles. Two workers are chosen at random. What is the probability that the workers chosen do **not** wear spectacles?

Exercise 10

I

(a) Mariel rolls a normal fair die. What is the probability that Mariel's die lands on

(i) 2 or 3?
(ii) An even number or 5?
(iii) A number less than 3 or a number greater than 4?
(iv) A prime number or a square number?

(b) Heulwen shuffles a standard deck of playing cards before randoomly choosing one card from the pack. What is the probability that Heulwen chooses

(i) Hearts or spades?
(ii) 3 or 5?
(iii) A face card or a card less than 6?
(iv) A black card or a red card?

(c) One number is chosen at random from the grid on the right. What is the probability that the number is:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

(i) 4 or 5?
(ii) A multiple of 5 or a multiple of 7?
(iii) A factor of 8 or a two-digit number?
(iv) A cube number or a prime number?

(d) Gareth rolls a fair die with 12 faces.
What is the probability that the die lands on

(i) 1 or 12?
(ii) An odd number or a multiple of 4?
(iii) A square number or 7?
(iv) A multiple of 3 or a factor of 11?

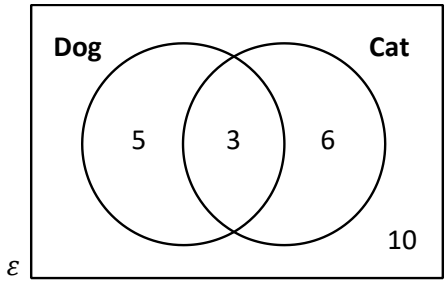
(e) A bag contains 10 counters. 3 are red, 2 are blue and the rest are purple. One counter is chosen at random from the bag. What is the probability that the chosen counter is red or purple?

Venn Diagrams

Example

In a class of 24 learners, 8 own a dog, 9 own a cat and 3 own both a dog and a cat. One learner is chosen at random from the class. What is the probability that the chosen learner does not own either a dog or a cat?

Answer: To begin, let us draw a Venn diagram to illustrate the situation.



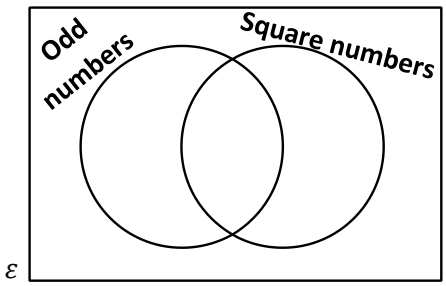
We see from the Venn diagram that 10 learners in the class do not own either a dog or a cat. Therefore, the probability of choosing a learner that does not own either a dog or a cat is $\frac{10}{24}$, or $\frac{5}{12}$.

Exercise 11

F

(a) In a survey someone asked 40 pupis whether they liked football or rugby.
32 pupils liked football.
25 pupils liked rugby.
22 pupils liked both sports.
(i) Draw a Venn diagram to illustrate this information.
(ii) What is the probability that a randomly chosen pupil from this group likes rugby only?

(b) (i) Place the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 into the correct positions in the following Venn diagram.

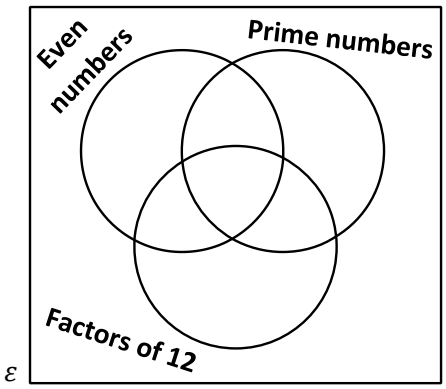


(ii) A number is chosen at random from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
Find the probability that the chosen number
(I) is an odd number; (II) is an odd number and a square number; (III) is neither odd nor a square number.

(c) An ice cream company conducted a taste test in a supermarket. 110 people took part in the survey, where they were asked to taste strawberry, vanilla and chocolate ice creams.

65 people stated that they liked the strawberry ice cream.
80 people stated that they liked the vanilla ice cream.
60 people stated that they liked the chocolate ice cream.
55 people stated that they liked both the strawberry and vanilla flavours.
50 people stated that they liked both the vanilla and chocolate flavours.
45 people stated that they liked both the strawberry and chocolate flavours.
40 people stated that they liked all 3 flavours.
(i) Draw a Venn diagram to illustrate this information.
(ii) What is the probability that a randomly selected person from this group liked
(I) Vanilla only; (II) None of the three flavours; (III) Vanilla or Strawberry?

(d) (i) Place the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 into the correct positions in the following Venn diagram.



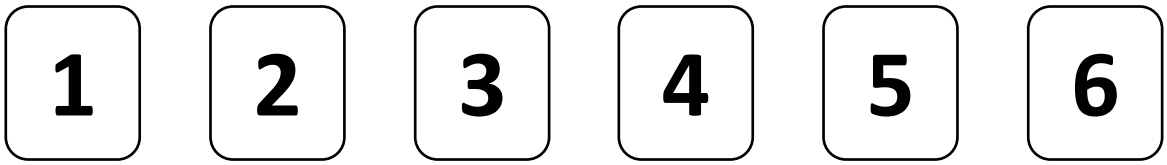
(ii) A number is chosen at random from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
Find the probability that the chosen number is
(I) an even number; (II) a prime number and a factor of 12; (III) a prime number but not an even number.

Sample Space Diagrams and Dependent Events

Higher Tier

Example

Each one of the numbers 1, 2, 3, 4, 5, 6 are written on cards.



Two out of the six cards are chosen at random, **without** replacement.
Find the probability that the **sum** of the numbers shown on the chosen cards is less than 10.

Answer: We can show all the possible combinations in a sample space diagram.

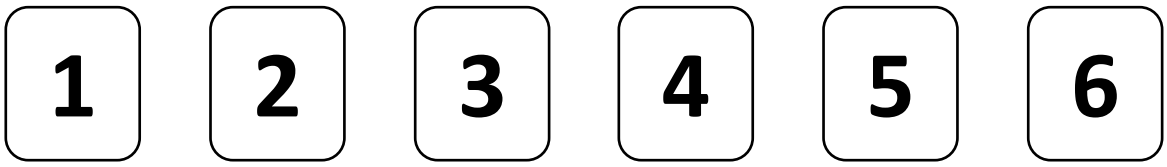
		Number on the second card					
	+	1	2	3	4	5	6
Number on the first card	1	Impossible	3	4	5	6	7
	2	3	Impossible	5	6	7	8
	3	4	5	Impossible	7	8	9
	4	5	6	7	Impossible	9	10
	5	6	7	8	9	Impossible	11
	6	7	8	9	10	11	Impossible

There are 6 × 5 = 30 different combinations, and of these, 26 give a sum that is less than 10. Therefore, the answer to the question is $\frac{26}{30}$, or $\frac{13}{15}$.

Exercise 12

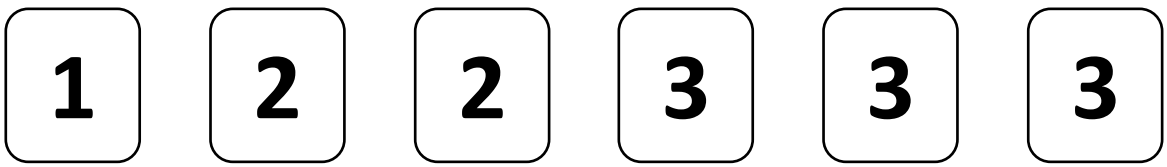
H

(a) Each one of the numbers 1, 2, 3, 4, 5, 6 are written on cards.



Two out of the six cards are chosen at random, **without** replacement.
Find the probability that the **product** of the numbers shown on the chosen cards is less than 10.

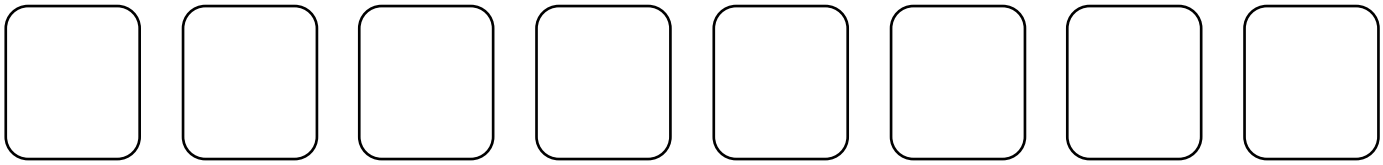
(b) Each one of the numbers 1, 2, 2, 3, 3, 3 are written on cards.



Two out of the six cards are chosen at random, **without** replacement.

(i) Find the probability that the sum of the numbers shown on the chosen cards is less than 6.
(ii) Find the probability that the sum of the numbers shown on the chosen cards is exactly 3.
(iii) Find the probability that the two chosen numbers are the same.
(iv) Find the probability that the two chosen numbers are different.
(v) Find the probability that the card showing 1 is chosen.

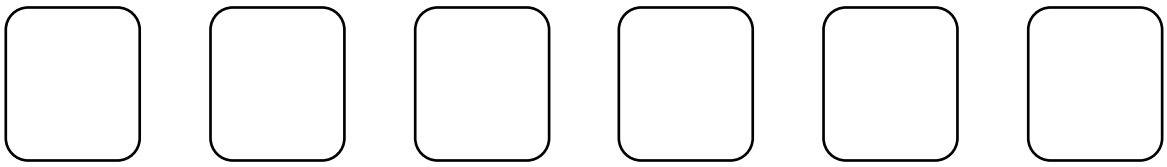
(c) A different factor of 24 is shown on each of 8 cards.



2 cards are chosen at random **without** replacement.

Find the probability that the **positive difference** between the two numbers on the chosen cards is
(i) 4;
(ii) an odd number;
(iii) a one-digit number.

(d) A different factor of 18 is shown on each of 6 cards.



In a game, a player chooses two of the above cards at random, **without** replacement.
The score for the game is the **largest** of the two chosen numbers.

(i) Draw a sample space diagram to show all the possible outcomes.
(ii) Find the probability that the score is 18.
(iii) A player wins the game if the score is 18.
If 120 people play the game once each, how many of them would you expect to win?
(iv) It costs 20p to play the game once.
The prize for winning the game is 50p.
If 120 people play the game once each, how much profit would you expect the game to make?

Evaluation

Key Words	Further Questions	What went well?	To reach my target grade I will…
			Grade ❑ Target ❑

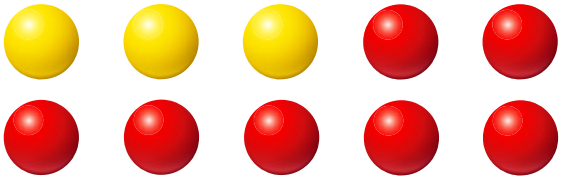
Tree Diagrams

Tree diagrams are used to show **combinations of two or more events**.
Each branch is labelled on the right with the **result** and in the middle with the **probability**.

It is possible to use tree diagrams to show independent events (intermediate tier) or dependent events (higher tier).

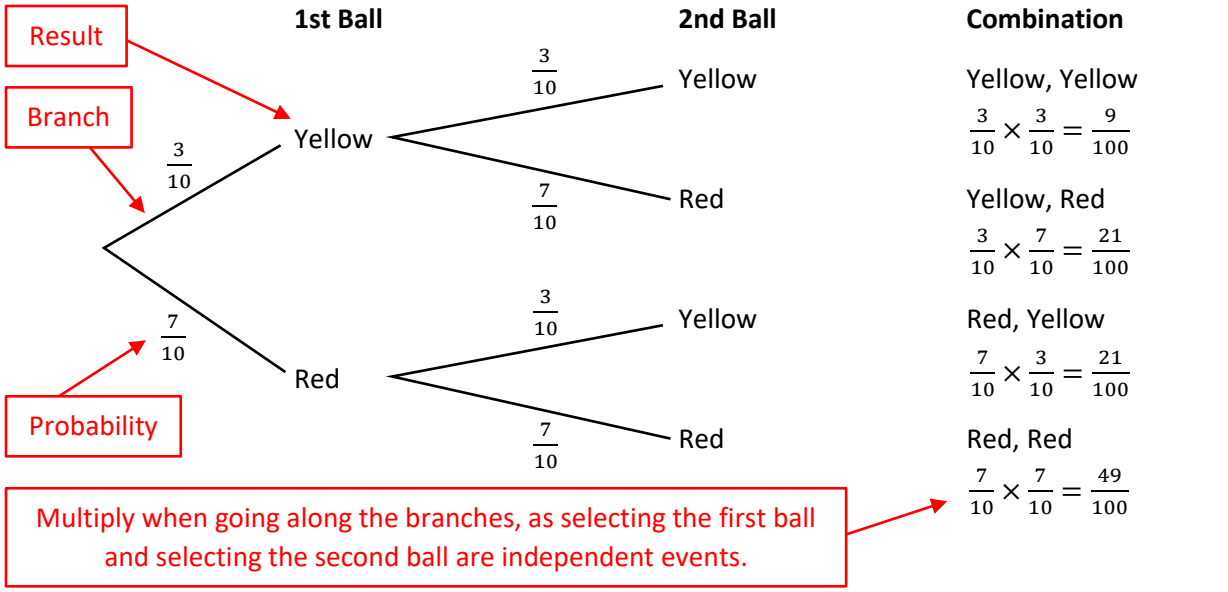
Example

A bag contains ten balls that are indentical apart from their colour. Three of the balls are yellow and the rest red. A ball is selected at random from the bag, its colour noted and returned to the bag. A second ball is chosen and its colour is also noted.



Use a tree diagram to calculate the probability that
(a) the two selected balls are yellow;
(b) the two selected balls have different colours;
(c) the two selected balls are not both yellow.

Answer: Step 1: Draw a tree diagram to illustrate the situation.



Step 2: Consider which paths in the tree diagram need to be considered in order to answer the questions.

(a) The first path in the tree diagram (yellow, yellow) shows the situation in which both selected balls are yellow. The probability for this is $\frac{9}{100}$.

(b) The paths that show balls of different colours are the second path (yellow, red) or the third path (red, yellow). Because these paths are mutually exclusive, it is possible to add the probabilities to obtain the final answer: $\frac{21}{100} + \frac{21}{100} = \frac{42}{100}$. Note that it would be possible to simplify this fraction to obtain $\frac{21}{50}$ but, unless the question notes otherwise, fractions do not have to be simplified in questions on probability.

(c) There are two ways of answering this question:
(i) Consider all paths in the tree diagram that do **not** give two yellow balls: $\frac{21}{100} + \frac{21}{100} + \frac{49}{100} = \frac{91}{100}$;
(ii) Consider the probability of selecting two yellow balls ($\frac{9}{100}$, the answer to part (a)) and subtract from 1:
$1 - \frac{9}{100} = \frac{91}{100}$.

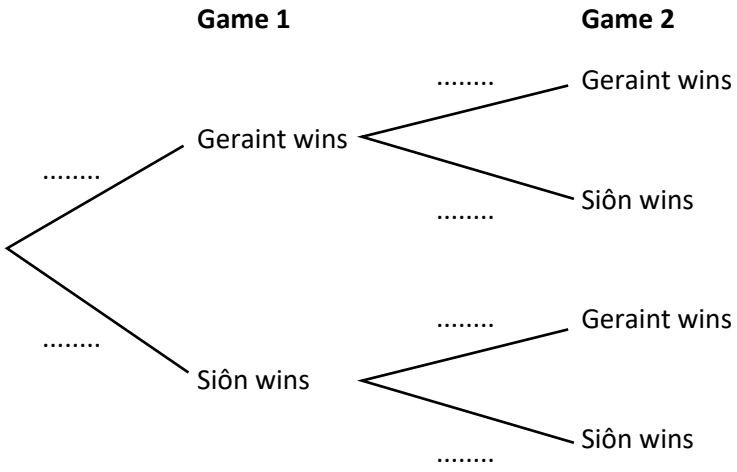
Exercise 13

Applying

I

Whenever Geraint and Siôn play a game of 'FIFA' on a games console, the probability that Gerains wins is 0.4.

(a) Complete the following tree diagram to show the probabilities of what can happen when Geraint and Siôn play two games of 'FIFA'.



(b) Calculate the probability that Siôn wins both games.

(c) Calculate the probability that Siôn wins exactly one of the games.

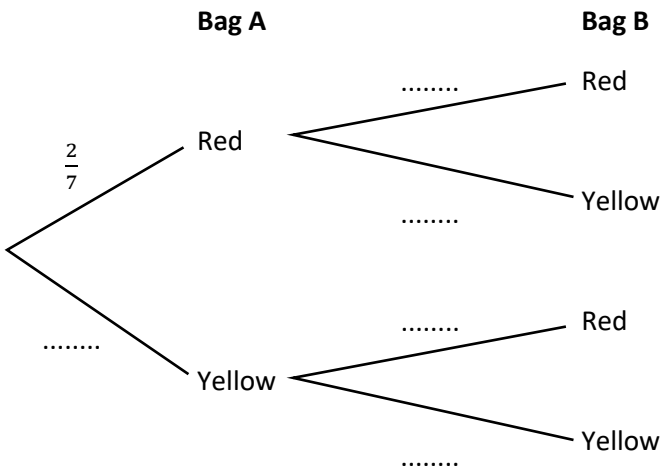
(d) Calculate the probability that Siôn wins neither of the games.

Exercise 14

I

There are two bags in a game, and both bags contain coloured balls.
Bag A contains 2 red balls and 5 yellow balls.
Bag B contains 3 red balls and 2 yellow balls.
A player randomly chooses one ball from each bag.

(a) Complete the following tree diagram.



(b) Find the probability of choosing two red balls.

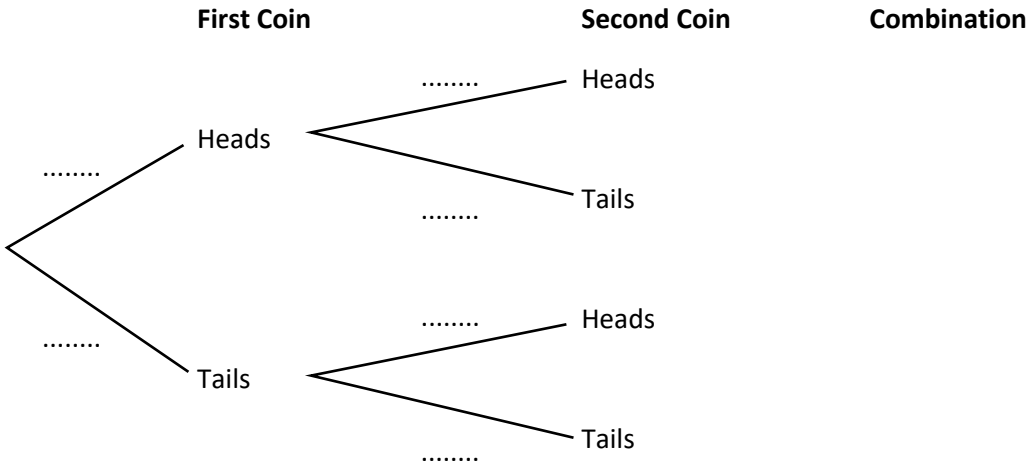
(c) Find the probability of choosing a ball of each colour.

(d) Find the probability of not choosing two red balls.

Exercise 15

Two biased coins are thrown. The probability of obtaining heads with the first coin is 70%.
The probability of obtaining heads with the second coin is 60%.

(a) Complete the following tree diagram.



(b) Calculate the probability that one coin shows 'Heads' and the other coin shows 'Tails'.

Example

The probability that Elin posts a picture on *Instagram* over the weekend is 0.4
The probability that Elin goes shopping over the weekend is independent of her posting a picture on Instagram over the weekend. The probability of Elin posting a picture on Instagram over the weekend, and going shopping over the weekend, is 0.12.

(a) Complete the following tree diagram.
(b) Find the probability that Elin posts a picture on Instagram over the weekend but does not go shopping over the weekend.

Answer: (a) The red text is the text that has been added.
Note in this example that the probability of one of the **combinations** has been given, so we must perform a **division** sum in order to calculate the probability of shopping over the weekend.

(b) 0.4 × 0.7 = 0.28 (the second path).

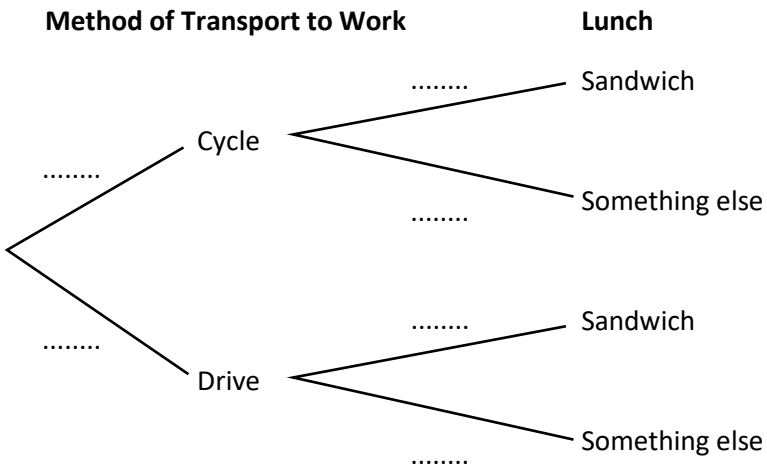
Exercise 16

I

Megan lives 5 miles from her work, and either cycles or drives to work.
The probability that she cycles to work is 0.3.
The probability that she cycles to work and has a sandwich for lunch is 0.24.
In Megan's case, her method of transport to work is independent of what she has for lunch.

(a) Find the probability that Megan has a sandwich for lunch.

(b) Complete the following tree diagram.



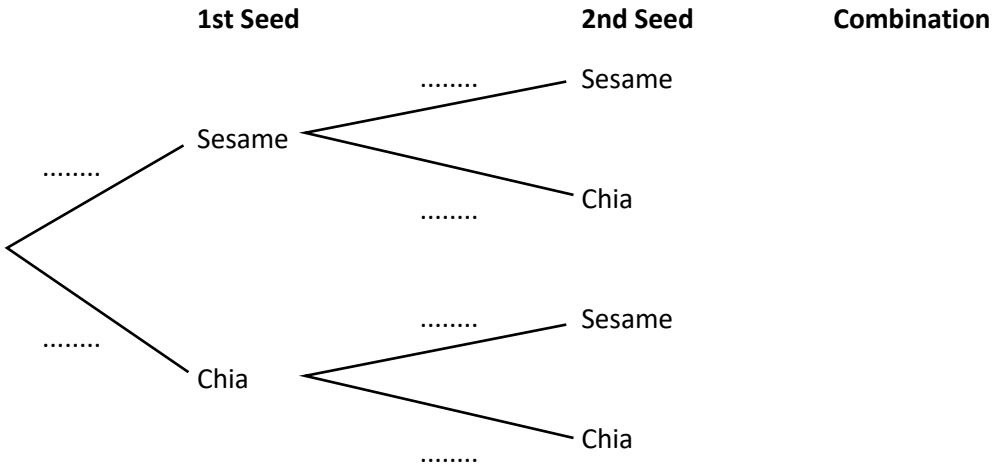
(c) Find the probability that Megan drives to work and eats something apart from a sandwich for lunch.

Exercise 17

I

A bag contains a large number of two types of seed, namely sesame and chia.
Two seeds are randomly chosen from the bag.
The probability that two sesame seeds are chosen is 0.49.

(a) Complete the following tree diagram.



(b) Calculate the probability that one seed of each type is selected.

(c) Calculate the probability that two chia seeds are chosen.

(d) Why are the words "large number" needed at the start of the question?

(e) If three seeds were chosen from the bag, what would be the probability of choosing 3 sesame seeds?

Dependent Events

Higher Tier

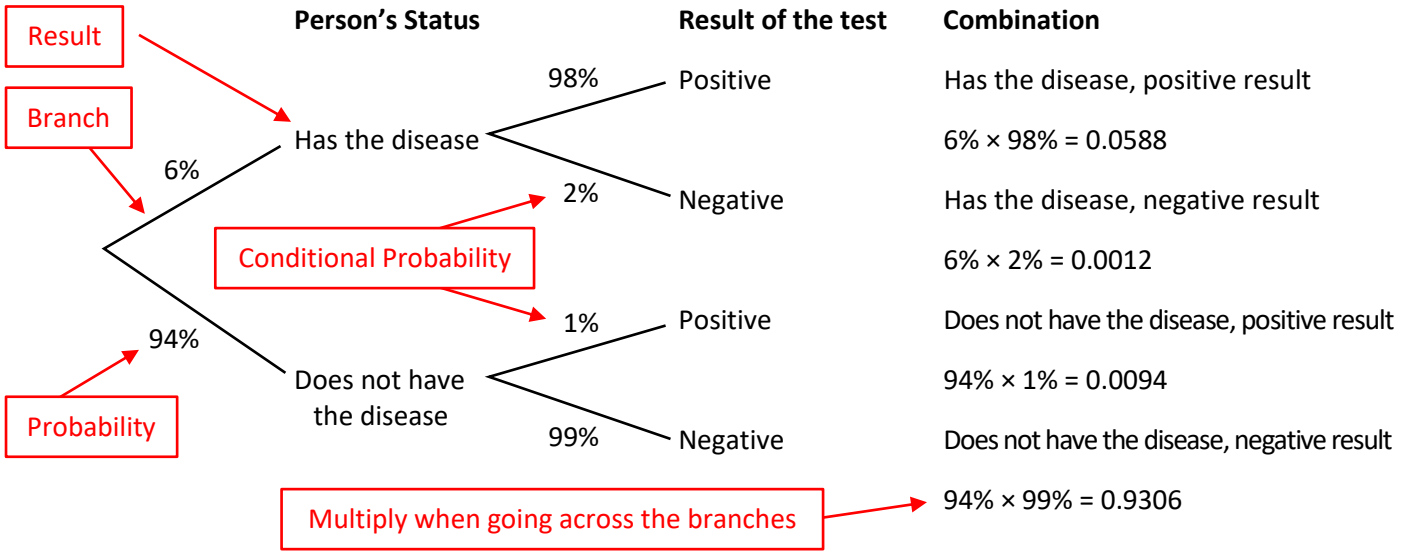
Example

A hospital tests patients for a specific disease. If the person has the disease, the test returns a "positive" result. If the person does not have the disease, the test returns a "negative" result. The test isn't perfect however:

- 98% of patients with the disease receive a "positive" result;
- 1% of patients without the disease receive a "positive" result;
- 6% of the population have the disease under consideration.

Use a tree diagram to calculate the probabilty
(a) that a randomly chosen person receives a "positive" result in the test;
(b) that the incorrect result is given to a person taking the test;
(c) that the correct result is given to a person taking the test.

Answer: Step 1: Draw a tree diagram to illustrate the situation.



Step 2: Consider which paths in the tree diagram need to be considered in order to answer the questions.

(a) Two paths give a positive result: either "has the disease, positive" (probability 0.0588) or "does not have the disease, positive" (probability 0.0094). We add these two probabilities to obtain the answer (they are mutually exclusive events): 0.0588 + 0.0094 = 0.0682.

(b) Two paths give an incorrect result: either "has the disease, negative" (probability 0.0012) or "does not have the disease, positive" (probability 0.0094). We add these two probabilities to obtain the answer (they are mutually exclusive events): 0.0012 + 0.0094 = 0.0106.

(c) There are two ways of answering this question:
(i) Consider all the paths in the tree diagram that give a correct result ("has the disease, positive" or "does not have the disease, negative"): 0.0588 + 0.9306 = 0.9894;
(ii) Consider the probability of obtaining an incorrect result (0.0106, the answer to part (b)) and subtract from 1: 1 – 0.0106 = 0.9894.

Challenge!

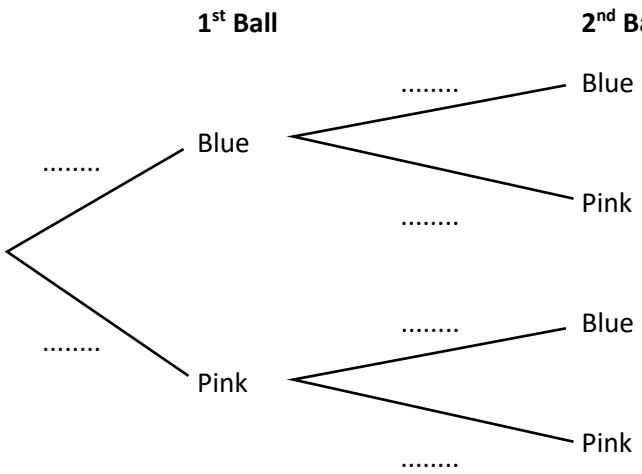
Use the internet to investigate the meaning of the terms *Type I Error* and *Type II Error*.
What is the connection between these statistical terms and the above example?

Exercise 18

H

A bag contains 4 blue balls and 6 pink balls.
Billy randomly chooses two balls from the bag **without replacement**.

(a) Complete the following tree diagram.



(b) Find the probability of choosing two pink balls.

(c) Find the probability of choosing one ball of each colour.

Exercise 19

H

A bag contains 7 yellow beads, 3 white beads and one black bead.
Two beads are chosen randomly from the bag **without replacement**.

(a) Calculate the probability that both beads are yellow.

(b) Calculate the probability that at least one white bead is chosen.

Exercise 20

H

A box contains 3 banana yogurts, 4 blueberry yogurts and 5 cherry yogurts.
Three yogurts are randomly chosen from the box **without replacement**.
Calculate the probability that at least one of the chosen yogurts is a cherry yogurt.

Evaluation

Key Words	Further Questions	What went well?	To reach my target grade I will…
			Grade ❑ Target ❑